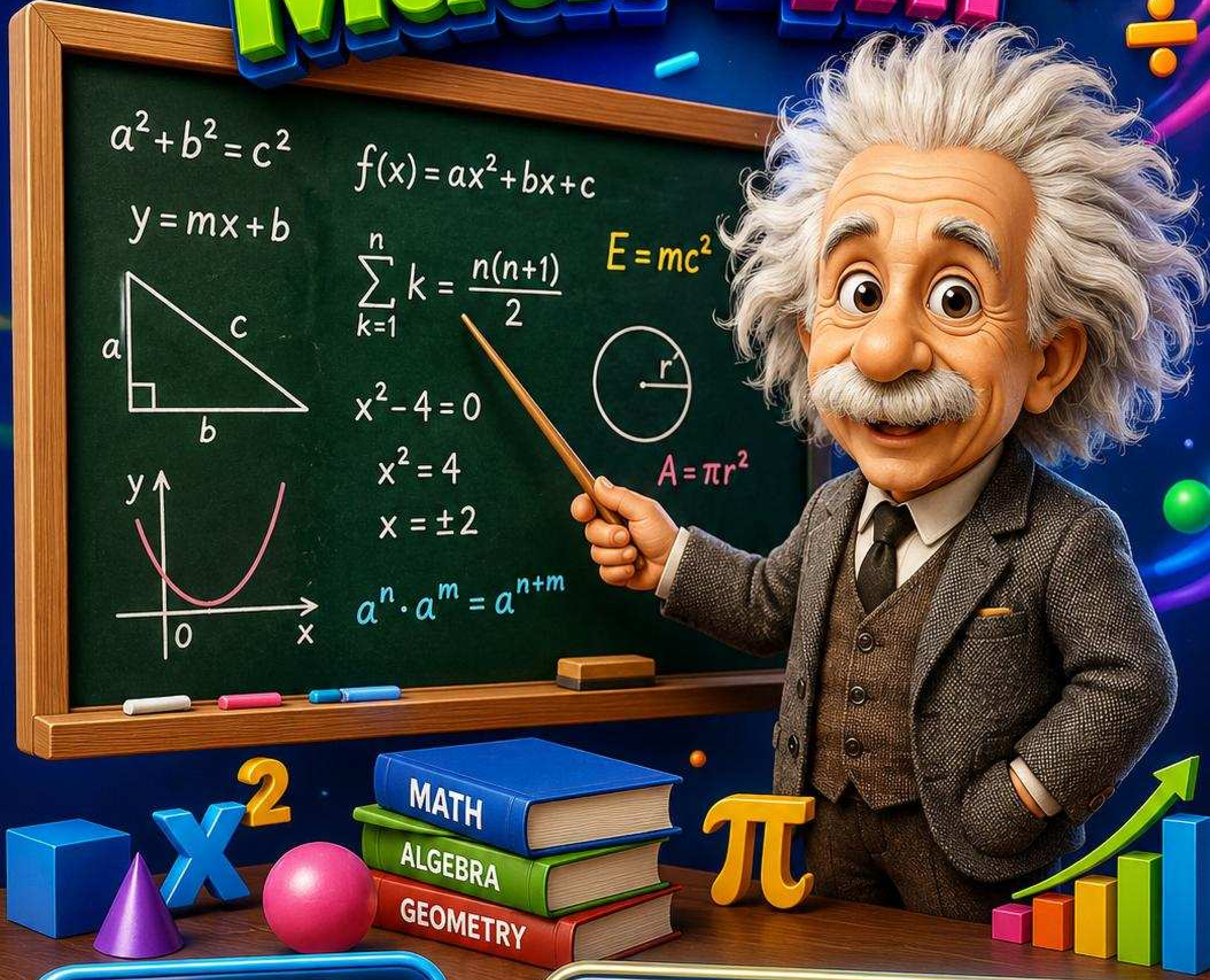
# Mathematics

# Math-4m




**Prep 3**

**First Term**

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Prepared By

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# Power of Power

$(a^m)^n$
a to power m
all to power n

## Let's Learn

**By the end of this lesson, you will be able to:**

- Understand the meaning of power of power.
- Use the rule $(a^m)^n = a^{m\times n}$.
- Simplify expressions using power of power.
- Solve problems using power of power in different forms.
- Apply the rule in real life situations.

## In Real Life

When something is repeated many times, we use powers. Sometimes, a power itself is raised to another power. This is called "**Power of Power**".

## The Basic Law

For any non-zero number $a$, and any integers $m$, n:

$$(a^m)^n = a^{m\times n}$$

**Where:**

- $a$ → the base (real number, and $a \neq 0$)
- $m$ → the power of the inner power
- $n$ → the power of the outer power
- $a^m$ → the inner power
- $(a^m)^n$ → the power of the power
- $a^{m\times n}$ → the simplified result

**Example of symbols**

$(5^3)^4$

- $n$ (outer power) → 4
- $m$ (inner power) → 3
- $a$ (base) → 5

## Explanation

When we have a power inside another power, we multiply the two powers and keep the same base.

**In other words:**

$a$ is multiplied by itself $m$ times, then the result is multiplied by itself $n$ times.
This is the same as multiplying $a$ by itself $(m \times n)$ times.

$(a^m)^n \longrightarrow a^{m\times n}$

$m \times n$
Multiply the exponents

## Examples

**Example 1**

Simplify $(2^3)^4$

**Solution:**

$(2^3)^4 = 2^{3\times 4}$
$= 2^{12}$
$= 4096$

**Example 2**

Simplify $(3^2)^5$

**Solution:**

$(3^2)^5 = 3^{2\times 5}$
$= 3^{10}$
$= 59049$

## Watch Out!

**Common Mistakes**

- Adding the powers instead of multiplying.
- Changing the base.
- Forgetting the parentheses.

Always keep the base the same and multiply the exponents.


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## More Examples

Let's see how the rule works with more examples.

**1** Simplify $(2^3)^2$

**Solution:**

$(2^3)^2 = 2^{3\times 2}$
$= 2^6$
$= 64$

**2** Simplify $(5^2)^3$

**Solution:**

$(5^2)^2 = 5^{2\times 3}$
$= 5^6$
$= 15625$

**3** Simplify $(10^4)^2$

**Solution:**

$(10^4)^2 = 10^{4\times 2}$
$= 10^8$
$= 100000000$

**4** Simplify $(7^3)^2$

**Solution:**

$(7^3)^2 = 7^{3\times 2}$
$= 7^6$
$= 117649$

**5** Simplify $(3^2)^4$

**Solution:**

$(3^2)^4 = 3^{2\times 4}$
$= 3^8$
$= 6561$

**6** Simplify $(a^5)^3$

**Solution:**

$(a^5)^3 = a^{5\times 3}$
$= a^{15}$

**7** Simplify $(x^4)^3$

**Solution:**

$(x^4)^3 = x^{4\times 3}$
$= x^{12}$

**8** Simplify $(y^3)^5$

**Solution:**

$(y^3)^5 = y^{3\times 5}$
$= y^{15}$

### Key Note

When you have a power of a power:

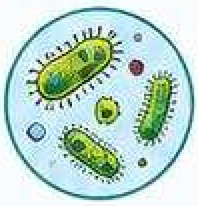
$$(a^m)^n$$
$$\downarrow$$
$$a^{m\times n}$$

**Multiply the exponents and keep the same base.**

## Real Life Application

Powers of power are used in many real situations.

**1** A bacteria multiplies by 10 every hour. If it starts with 10 bacteria, how many will there be after 3 hours?



**Solution:**

Number after 1 hour $= 10^1$

Number after 2 hours $= (10^1)^2 = 10^{1\times 2} = 10^2 = 100$

Number after 3 hours $= (10^1)^3 = 10^{1\times 3} = 10^3 = 1000$

**Answer:** There will be 1000 bacteria.

**2** A computer file increases in size by a factor of 2 every day. If it is 2 MB today, what will be its size after 4 days?

**Solution:**

Size after 1 day $= 2^1$

Size after 4 days $= (2^1)^4 = 2^{1\times 4} = 2^4 = 16$

**Answer:** The size after 4 days will be 16 MB.

## You Try

Apply the rule and simplify the following.

1. $(2^2)^3$ ______________
2. $(4^3)^2$ ______________
3. $(5^4)^2$ ______________
4. $(6^2)^4$ ______________
5. $(3^5)^3$ ______________
6. $(a^2)^5$ ______________
7. $(m^3)^2$ ______________
8. $(x^5)^2$ ______________
9. $(y^4)^3$ ______________
10. $(z^2)^6$ ______________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## More Examples

Let's solve more examples using the rule of power of power.

**1** Simplify $(2^4)^3$

Solution:
$(2^4)^3 = 2^{4\times3}$
$= 2^{12}$
$= 4096$

**2** Simplify $(3^3)^4$

Solution:
$(3^3)^4 = 3^{3\times4}$
$= 3^{12}$
$= 531441$

**3** Simplify $(5^2)^5$

Solution:
$(5^2)^5 = 5^{2\times5}$
$= 5^{10}$
$= 9765625$

**4** Simplify $(7^2)^3$

Solution:
$(7^2)^3 = 7^{2\times3}$
$= 7^6$
$= 117649$

**5** Simplify $(10^3)^2$

Solution:
$(10^3)^2 = 10^{3\times2}$
$= 10^6$
$= 1000000$

**6** Simplify $(a^5)^2$

Solution:
$(a^5)^2 = a^{5\times2}$
$= a^{10}$

**7** Simplify $(x^2)^4$

Solution:
$(x^2)^4 = x^{2\times4}$
$= x^8$

**8** Simplify $(y^3)^3$

Solution:
$(y^3)^3 = y^{3\times3}$
$= y^9$

**9** Simplify $(m^4)^3$

Solution:
$(m^4)^3 = m^{4\times3}$
$= m^{12}$

**10** Simplify $(z^6)^2$

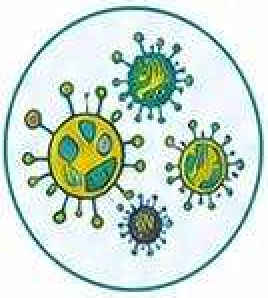
Solution:
$(z^6)^2 = z^{6\times2}$
$= z^{12}$

## Key Ideas

- When a power is raised to another power, multiply the exponents.
- Keep the base the same.
- Use this rule in expressions with numbers or letters.

## Real Life Application

**1** A virus multiplies by a factor of 10 every hour. If there are 10 viruses at the beginning, how many will there be after 4 hours?

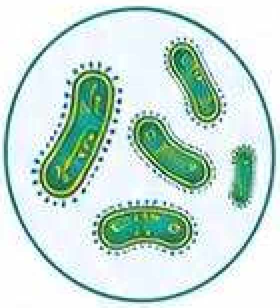


Solution:

After 1 hour $= 10^1 = 10$
After 2 hours $= 10^2 = 100$
After 3 hours $= 10^3 = 1000$
After 4 hours $= 10^4 = 10000$

Answer: There will be 10000 viruses.

**2** The population of bacteria doubles every day. If there are 500 bacteria today, how many will there be after 5 days?



Solution:

After 1 day $= 500 \times 2^1 = 1000$
After 2 days $= 500 \times 2^2 = 2000$
After 3 days $= 500 \times 2^3 = 4000$
After 4 days $= 500 \times 2^4 = 8000$
After 5 days $= 500 \times 2^5 = 16000$

Answer: There will be 16000 bacteria.

## You Try

Apply the rule and simplify the following.

1. $(2^3)^3$ ______
2. $(4^2)^3$ ______
3. $(5^3)^2$ ______
4. $(3^4)^2$ ______
5. $(6^2)^4$ ______
6. $(a^3)^4$ ______
7. $(x^4)^3$ ______
8. $(y^2)^5$ ______
9. $(m^3)^2$ ______
10. $(z^5)^2$ ______

**Think!**

If $(a^m)^n = a^{mn}$, $a = 2$ and $m = 3$, what is the value of $n$ if $2^{3n} = 2^{18}$ ?

______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## Examples (Advanced)

Let's practice more with different forms.

**1** Simplify $(2^3)^2 \times (2^4)^3$

**Solution:**

$= 2^{3\times2} \times 2^{4\times3}$

$= 2^6 \times 2^{12}$

$= 2^{6+12}$

$= 2^{18}$

$= 262144$

**2** Simplify $(3^2)^3 \div 3^4$

**Solution:**

$= 3^{2\times3} \div 3^4$

$= 3^6 \div 3^4$

$= 3^{6-4}$

$= 3^2$

$= 9$

**3** Simplify $(5^3)^2 \times 5^4$

**Solution:**

$= 5^{3\times2} \times 5^4$

$= 5^6 \times 5^4$

$= 5^{6+4}$

$= 5^{10}$

$= 9765625$

**4** Simplify $(7^2)^3 \div (7^3)^2$

**Solution:**

$= 7^{2\times3} \div 7^{3\times2}$

$= 7^6 \div 7^6$

$= 7^{6-6}$

$= 7^0$

$= 1$

**5** Simplify $\dfrac{(2^4)^3 \times (2^2)^2}{2^5}$

**Solution:**

$= \dfrac{2^{4\times3} \times 2^{2\times2}}{2^5}$

$= \dfrac{2^{12} \times 2^4}{2^5}$

$= \dfrac{2^{16}}{2^5} = 2^{16-5}$

$= 2^{11} = 2048$

**6** Simplify $\dfrac{(3^2)^4}{3^5}$

**Solution:**

$= \dfrac{3^{2\times4}}{3^5}$

$= \dfrac{3^8}{3^5} = 3^{8-5}$

$= 3^3 = 27$

**7** Simplify $\dfrac{(x^3)^2 \times x^4}{x^7}$

**Solution:**

$= \dfrac{x^{3\times2} \times x^4}{x^7}$

$= \dfrac{x^6 \times x^4}{x^7}$

$= \dfrac{x^{10}}{x^7} = x^{10-7}$

$= x^3$

**8** Simplify $\left(\dfrac{y^2}{y^5}\right)^3$

**Solution:**

$= (y^{2-5})^3$

$= (y^{-3})^3$

$= y^{-9} = \dfrac{1}{y^9}$

### Key Note

Use the laws of exponents carefully:

- Multiply powers → add exponents.
- Divide powers → subtract exponents.
- Power of a power → multiply exponents.
- The base always stays the same.

## Real Life Application

**1** A lake is covered with 2 water lilies. Each day, the number of lilies becomes 3 times the previous day. How many lilies will cover the lake after 6 days?

**Solution:**

Day 1 $= 2 = 2^1$

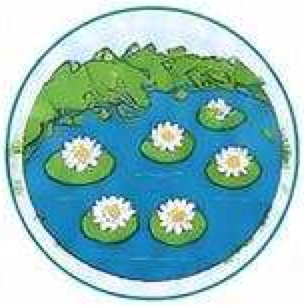
Day 2 $= 2 \times 3 = 2 \times 3^1 = 2 \times 3^1$

Day 3 $= 2 \times 3 \times 3 = 2 \times 3^2$

Day 4 $= 2 \times 3^3$

Day 5 $= 2 \times 3^4$

Day 6 $= 2 \times 3^5 = 2 \times 243 = 486$



**Answer:** After 6 days, 486 lilies will cover the lake.

**2** A small investment of 100 dollars earns compound interest at a rate of 5% per year, compounded annually. Find the value of the investment after 4 years.

**Solution:**

Year 1 $= 100(1 + 0.05)^1 = 100(1.05)^1$

Year 2 $= 100(1.05)^2$

Year 3 $= 100(1.05)^3$

Year 4 $= 100(1.05)^4 = 100(1.21550625)$

$= 121.550625$

**Answer:** The value after 4 years ≈ 121.55 dollars.

## Practice Time

Simplify the following.

**1** $(2^3)^2 \times (2^2)^3$ ______

**2** $(3^2)^3 \div 3^4$ ______

**3** $(5^2)^3 \times 5^4$ ______

**4** $(7^3)^2 \div (7^2)^3$ ______

**5** $(10^2)^3 \times (10)^4$ ______

**6** $\dfrac{(2^5)^2}{2^6}$ ______

**7** $\dfrac{(3^3)^2 \times 3^4}{3^5}$ ______

**8** $\dfrac{(x^2)^3 \times x^5}{x^4}$ ______

**9** $\left(\dfrac{a^3}{a^8}\right)^2$ ______

**10** $\left(\dfrac{m^2}{m^5}\right)^3$ ______

### Think!

If $(a^m)^n = a^{mn}$, and $2^3 = 8$, what is the value of $(2^3)^4 \div 2^2$ ?

______



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## Examples (Mixed Forms)

Let's solve mixed examples that combine more than one operation.

**1** Simplify $(2^3)^2 \times 2^4$

**Solution:**

$$(2^3)^2 \times 2^4 = 2^{3\times2} \times 2^4 = 2^6 \times 2^4 = 2^{6+4} = 2^{10} = 1024$$

**2** Simplify $(3^2)^3 \div 3^4$

**Solution:**

$$(3^2)^3 \div 3^4 = 3^{2\times3} \div 3^4 = 3^6 \div 3^4 = 3^{6-4} = 3^2 = 9$$

**3** Simplify $5^2 \times (5^3)^2$

**Solution:**

$$5^2 \times (5^3)^2 = 5^2 \times 5^{3\times2} = 5^2 \times 5^6 = 5^{2+6} = 5^8 = 390625$$

**4** Simplify $7^5 \div (7^2)^3$

**Solution:**

$$7^5 \div (7^2)^3 = 7^5 \div 7^{2\times3} = 7^5 \div 7^6 = 7^{5-6} = 7^{-1} = \frac{1}{7}$$

**5** Simplify $\frac{(2^4)^2 \times 2^3}{2^5}$

**Solution:**

$$\frac{(2^4)^2 \times 2^3}{2^5} = \frac{2^{4\times2} \times 2^3}{2^5} = \frac{2^8 \times 2^3}{2^5} = \frac{2^{8+3}}{2^5} = 2^{11-5} = 2^6 = 64$$

**6** Simplify $\frac{(3^2)^3 \div 3^4}{3^2}$

**Solution:**

$$\frac{(3^2)^3 \div 3^4}{3^2} = \frac{3^{2\times3} \div 3^4}{3^2} = \frac{3^6 \div 3^4}{3^2} = \frac{3^{6-4}}{3^2} = \frac{3^2}{3^2} = 1$$

**7** Simplify $(x^2)^3 \times x^4$

**Solution:**

$$(x^2)^3 \times x^4 = x^{2\times3} \times x^4 = x^6 \times x^4 = x^{6+4} = x^{10}$$

**8** Simplify $\frac{y^7}{(y^2)^2}$

**Solution:**

$$\frac{y^7}{(y^2)^2} = \frac{y^7}{y^{2\times2}} = \frac{y^7}{y^4} = y^{7-4} = y^3$$

### Key Ideas

- Solve inside parentheses first.
- Apply the power of a power rule.
- Multiply → add exponents.
- Divide → subtract exponents.
- Keep the base the same.

## Real Life Application

**1** A printer makes 2 copies in the first minute. Each minute, the number of copies is tripled. How many copies will be made in 6 minutes?

**Solution:**

Minute 1 $= 2 = 2^1$

Minute 2 $= 2 \times 3 = 2 \times 3^1 = 2^1 \times 3^1$

Minute 3 $= 2 \times 3^2 = 2^1 \times 3^2$

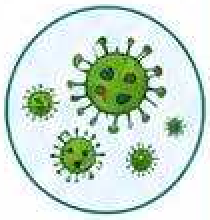
Minute 4 $= 2 \times 3^3 = 2^1 \times 3^3$

Minute 5 $= 2 \times 3^4 = 2^1 \times 3^4$

Minute 6 $= 2 \times 3^5 = 2 \times 243 = 486$

**Answer:** 486 copies will be made in 6 minutes.

**2** A computer virus increases by a factor of $2^3$ every hour. If it starts with 8 viruses, how many viruses will there be after 5 hours?



**Solution:**

After 1 hour $= 8 \times (2^3)^1 = 8 \times 2^3 = 8 \times 8 = 64$

After 2 hours $= 8 \times (2^3)^2 = 8 \times 2^6 = 8 \times 64 = 512$

After 3 hours $= 8 \times (2^3)^3 = 8 \times 2^9 = 8 \times 512 = 4096$

After 4 hours $= 8 \times (2^3)^4 = 8 \times 2^{12} = 8 \times 4096 = 32768$

After 5 hours $= 8 \times (2^3)^5 = 8 \times 2^{15} = 8 \times 32768 = 262144$

**Answer:** There will be 262144 viruses after 5 hours.

## Practice Time

Simplify the following.

1. $(2^2)^3 \times 2^2$ ______
2. $(3^3)^2 \div 3^4$ ______
3. $5^3 \times (5^2)^2$ ______
4. $7^6 \div (7^3)^2$ ______
5. $\frac{(2^3)^2 \times 2^4}{2^5}$ ______
6. $\frac{(3^2)^3 \div 3^2}{3^3}$ ______
7. $(x^2)^4 \times x^3$ ______
8. $\frac{y^9}{(y^3)^2}$ ______
9. $(a^3)^2 \div a$ ______
10. $\frac{(z^2)^5}{z^6}$ ______

### Challenge!

If $(a^m)^n = a^{mn}$ and $(b^2)^3 \times (b^3)^2 \div b^5 = b^k$, find the value of $k$.

______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## 1 Evaluate the following:

**Choose the correct answer from the given choices :**

① What is the value of $a^7$ if $a = 2$ ?

(a) $a^3$ (b) $a^{11}$ (c) $a^2$ (d) $a^{47}$

② What is the value of $(3^6)^0$ ?

(a) 24 (b) 1 (c) $3^6$ (d) 0

③ What is the value of $\left(\frac{x^3}{x^2}\right)^2$ for $x \neq 0$ ?

(a) $x^{31}$ (b) $x^{-2}$ (c) $x$ (d) $\frac{1}{x}$

④ What is the value of $a^8b^3$ if $a^2b = -2$ ?

(a) $-a^8b^2$ (b) $a^8b^2$ (c) $-a^5b^3$ (d) $a^8b^2$

⑤ What is the value of $x^2 \times (x^5)^2$ ?

(a) $x^{20}$ (b) $x^{14}$ (c) $x^{12}$ (d) $x^9$

⑥ What is the value of $2^4$ ?

(a) 15 (b) 27 (c) 32 (d) 125

⑦ If $2^4 = 5^x$, what is the value of $x$ ?

(a) 15 (b) 27 (c) 32 (d) 125

⑧ What is the value of $\frac{9a^4}{4b^2}$ if $\left(\frac{3a^2}{2b}\right)^2$ ?

(a) $\frac{3a^4}{2b^3}$ (b) $\frac{3a^4}{2b^2}$ (c) $\frac{9a^4}{4b^3}$ (d) $\frac{9a^4}{4b^3}$

**Ali's Rule**

$(a^2)^3 \times a^q = a^6 \times a^q = a^{10}$

**Khaled's Rule**

$(a^2)^3 \times a^q = a^6 \times a^q = a^{q+6} = a^{10}$

**Discover the Mistake**

Ali and Khaled solved $(a^2)^3 \times a^4$ as shown above.

Who is correct? Explain.

______________________

## 2 Apply the Concepts

**A Find each of the following in its simplest form:**

1. $(\sqrt{5})^3 \times (-\sqrt{5})^2$
2. $(a^2)^2 \times (a^3)^3$
3. $(a^4)^3 \times a^7$
4. $((-2)^3)^2 \times (-\sqrt{2})^{-4}$
5. $\frac{(a^4)^3}{(a^2)^5}$
6. $\frac{(a^4)^3}{(a^2)^5}$
8. $\frac{((-\sqrt{7})^2)^3 \times ((\sqrt{7})^3)^{-1}}{((\sqrt{7})^2)^{-2}}$

**B Express each of the following in its simplest form:**

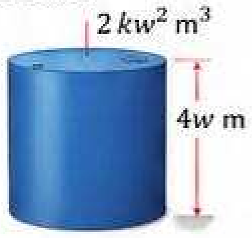
1. $(2r^2s^{-1})^{-2}$
2. $(-2m^5n^4)^3$
3. $(4a^4b^3c)^2$
4. $\frac{(c^3)^2 \times (-3c^7)^2}{(c^5)^2}$
5. $(-3x^2y^3z)^2 \times (-x^2y^3)^2$
6. $\frac{5qp^{-2}}{(q^{-1}p)^3}$
7. $\frac{(2a^5)^3 \times (3a^3)^2}{(6a^7)^2 \times (a^{-1})^3}$
8. $\frac{(16x^2y^{-2})^{-0}}{(4x^0y^{-6})^{-2}}$
9. $\frac{5^0c^2d^3f^{-3}}{2c^4d^{-4}f^3}$

**Real Life Skill**

1. A square garden has an area of $x^2$ m². Find the length of one side of the garden.

Answer : ________

2. A cylindrical water tank has a volume of $2kw^2$ m³ and a height of $4w$ m. Find the area of its base.




Answer : ________

## 3 More Practice

**A Evaluate, then find the value of $x$:**

1. $2^x = 5$

$x =$ ________

2. $3^4 = 81 = 9^x$

$x =$ ________

3. $5^x = 32$

$x =$ ________

4. $27^{x-1} = 9$

$x =$ ________

**B Prove that:**

1. $\frac{4^x \times 2^{x-4}}{16^{x-2}} = 1$
2. $\frac{(4)^x \times (49)^{x-1}}{(98)^x} = 1$
3. $\frac{27^{x-1} \times 8^x}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \frac{1}{27}$
4. $9^x + (27)^x$ when $3^x = 2$

$=$ ________

## 4 Extension Questions

1. If $4^x = 10^2$, find the value of $5^x$.

Answer : ________

2. If $a = 5^{30}$, $b = 3^{45}$, $c = 2^{60}$ arrange $a$, $b$, $c$ in descending order.

Answer : ________

3. If $a^m = a^{mn}$ and $(b^2)^3 \times (b^3)^2 \div b^5 = b^k$, find $m$ and $k$.

m = ______ , k = ______

4. If $16^x \div 2^{2x+1} = 8$, find $x$.

Answer : ________

5. If $25^x = 125$ and $x = 3^y$, find $y$.

Answer : ________



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## 1 Evaluate the following:

**A** *Choose the correct answer from the given choices :*

① What is the value of $(2^3)^4$ ?
(a) $2^7$ (b) $2^{12}$ (c) $2^{14}$ (d) $2^{24}$

② What is the value of $(3^2)^3 \div 3^4$ ?
(a) $3^2$ (b) $3^3$ (c) $3^4$ (d) $3^6$

③ If $(5^x)^2 = 5^8$, what is the value of $x$ ?
(a) 2 (b) 4 (c) 6 (d) 8

④ What is the value of $a^5 \times (a^3)^2 \div a^4$ ?
(a) $a^3$ (b) $a^5$ (c) $a^7$ (d) $a^9$

⑤ What is the value of $(10^2 \div 10^8)^3$ ?
(a) $10^{-18}$ (b) $10^{-6}$ (c) $10^6$ (d) $10^{18}$

⑥ What is the value of $(x^2y^3)^3 \div (xy)^2$ ?
(a) $x^4y^7$ (b) $x^4y^5$ (c) $x^5y^7$ (d) $x^5y^9$

⑦ If $2^x = 64$, what is the value of $2^{x-2}$ ?
(a) 8 (b) 16 (c) 32 (d) 64

⑧ What is the value of $\frac{(2a^2)^3 \times (3a)^2}{6a^4}$ ?
(a) $6a^2$ (b) $9a^2$ (c) $12a^2$ (d) $18a^2$

**Power of a Power Rule**
If $a$ and $m$, $n$ are integers,
$(a^m)^n = a^{m \times n}$

**Power of Product Rule**
If $a$, $b$ and $n$ are integers,
$(ab)^n = a^n b^n$

**Think Smart!**
Break the expression into smaller parts and apply the rules step by step.

## 2 Apply the Concepts

**A** *Find each of the following in its simplest form:*

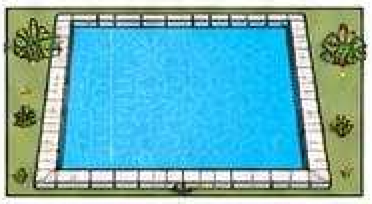
1. $(-3)^4 \times (-3)^2$ ______
2. $(2^2)^5$ ______
3. $(a^2)^3 \times a^7$ ______
4. $(x^3)^2 \div x^4$ ______
4. $(5^2 \times 5^3)^2$ ______
5. $(m^4 \div m^2)^3$ ______
7. $(3a^2b^3)^2$ ______
8. $(x^2y)^3 \times xy^2$ ______

**B** *Express each of the following in its simplest form:*

1. $(2^3 \times 4^2)^2$ ______
2. $(5a^2b)^3$ ______
3. $(3x^2y^4 \div xy^2)^2$ ______
4. $(6m^2n)^2 \div (2mn)^3$ ______
5. $(7p^5)^0$ ______
6. $(a^2b^5)^4 \div (ab^2)^3$ ______
7. $(12x^4y^3 \div 3x^2y)^2$ ______
8. $(-2a^3)^2 \times (-3a^2)^3$ ______

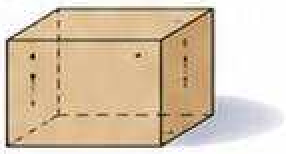
### Real Life Skill

1. A square swimming pool has an area of 25 $m^2$. What is the length of one side of the pool?



Answer : ______

2. A cube-shaped box has a volume of 64 $cm^3$. What is the length of one edge of the box?



Answer : ______

## 3 More Practice

**A** *Find the value of x:*

1. $2^{x+1} = 32$
$x =$ ______
2. $3^{2x} = 81$
$x =$ ______
3. $(5^x)^2 = 125$
$x =$ ______
4. $4^x \div 4^3 = 16$
$x =$ ______
5. $(2^x)^3 = 64$
$x =$ ______
6. $27^{x-1} = 9$
$x =$ ______

**B** *Simplify the following:*

1. $(2^2 \times 3^3)^2$ ______
2. $(a^3b^2)^2 \times ab^4$ ______
6. $(x^2y^3)^2 \div (xy)^3$ ______
4. $(8a^2b^3 \div 2ab)^2$ ______
5. $(3^2 \times 5^2 \times 7^2)^0$ ______
6. $(p^4q \div p^2q^5)^3$ ______

## 4 Extension Questions

1. If $(2^x)^3 = 128$, find the value of $x$.
Answer : ______

2. If $a = 2^5$, $b = 3^4$ and $c = 5^2$, arrange $a$, $b$, $c$ in descending order.
Answer : ______

3. If $(a^m)^n = a^{mn}$ and $(a^2b)^3 = a^6b^3$, find the value of $m$ and $n$.
m = ______ , n = ______

4. If $(3^2 \times 5)^x = 15^2$, find the value of $x$.
Answer : ______

5. If $2^x = 5$ and $3^y = 2$, find the value of $2^{x+y}$.
Answer : ______



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## 1 Evaluate the following:

**A** *Choose the correct answer from the given choices :*

① What is the value of $(2^3)^4$ ?

(a) $2^7$ (b) $2^{12}$ (c) $2^{14}$ (d) $2^{24}$

② What is the value of $5^2 \times 5^3 \times 5$ ?

(a) $5^2$ (b) $5^3$ (c) $5^5$ (d) $5^6$

③ What is the value of $(-3)^4 \times (-3)^2$ ?

(a) $-3^2$ (b) $3^6$ (c) $-3^6$ (d) $3^8$

④ If $9^x = 27$, what is the value of $x$ ?

(a) 2 (b) 3 (c) 4 (d) 6

⑤ What is the value of $(10^3 \div 10)^2$ ?

(a) $10^2$ (b) $10^4$ (c) $10^6$ (d) $10^8$

⑥ What is the value of $(x^2y^3)^2 \times (xy)^3$ ?

(a) $x^4y^9$ (b) $x^4y^6$ (c) $x^5y^9$ (d) $x^7y^9$

⑦ If $2^x = 128$, what is the value of $2^{x-3}$ ?

(a) 8 (b) 16 (c) 32 (d) 64

⑧ What is the value of $\dfrac{(3a^2b)^3 \times (2ab^2)^2}{(6a^2b^3)^2}$ ?

(a) $3a^2b^2$ (b) $3ab^2$ (c) $3a^2b^3$ (d) $3ab$

**Quotient of Powers Rule**

If $a \neq 0$ and $m$, $n$ are integers,

$$a^m \div a^n = a^{m-n}$$

**Power of a Power Rule**

If $a$ and $m$, $n$ are integers,

$$(a^m)^n = a^{m \times n}$$

**Remember!**

When multiplying or dividing powers with the same base, keep the base and add or subtract the exponents.

## 2 Apply the Concepts

**A** *Find each of the following in its simplest form:*

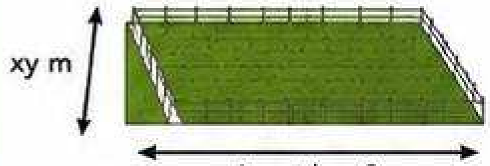
1. $(-2)^5 \times (-2)^3$ ______
2. $(3^2)^4$ ______
3. $(a^3)^2 \times a^5$ ______
4. $(x^4)^3 \div x^5$ ______
5. $(4^2 \times 4^3)^2$ ______
6. $(m^3 \div m)^2$ ______
7. $(2a^2b^3)^3$ ______
8. $(x^3y^2)^2 \times xy^4$ ______

**B** *Express each of the following in its simplest form:*

1. $(2^4 \div 2^2)^3$ ______
2. $(5a^2)^2 \times (2a^3)^3$ ______
3. $(3x^2y^3 \div xy)^2$ ______
4. $(6m^2n)^3 \div (3mn^2)^2$ ______
5. $(7p^4q)^0$ ______
6. $(a^2b^3)^2 \div (ab)^4$ ______
7. $(8x^4y^2 \div 4x^2y)^2$ ______
8. $(-3a^2)^2 \times (-2a^3)^2$ ______

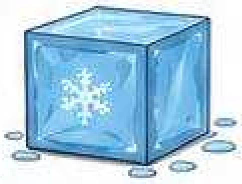
### Real Life Skill

1. A rectangular field has an area of $x^2y^3m^2$. If its width is $xy$ m, what is its length?




Answer : ______

2. A cube-shaped ice box has a volume of $125a^3b^6$ cm³. What is the length of one edge?



Answer : ______

## 3 More Practice

**A** *Find the value of x:*

1. $2^x = 64$
   $x =$ ______
2. $3^x = 243$
   $x =$ ______
3. $(5^x)^2 = 625$
   $x =$ ______
4. $4^x \div 4^2 = 16$
   $x =$ ______
5. $9^x \times 9 = 729$
   $x =$ ______
6. $2^{x-1} = 32$
   $x =$ ______

**B** *Simplify the following:*

1. $(2^3 \times 2^{-1})^2$ ______
2. $(a^2b^3)^3 \times (ab^2)^2$ ______
6. $(3x^2y)^2 \div (xy^2)^3$ ______
4. $(4a^2b)^2 \div 2ab$ ______
5. $(5^2 \times 5^{-3})^0$ ______
6. $(p^3q^2)^2 \times p^2q^3$ ______

## 4 Extension Questions

1. If $4^x = 256$, find the value of $4^{x-2}$.
   Answer : ______
2. If $a = 2^4$, $b = 3^2$ and $c = 2^3 \times 3$, arrange $a$, $b$, $c$ in ascending order.
   Answer : ______
3. If $(a^m)^n = a^{mn}$ and $(a^3b^2)^2 = a^6b^4$, find the value of $m$ and $n$.
   $m =$ ______, $n =$ ______
4. If $(x^2y^3)^2 \times (xy)^3 = x^py^q$, find the values of $p$ and $q$.
   $p =$ ______, $q =$ ______
5. If $2^x = 8$ and $5^y = 25$, find the value of $2^{x-1} \times 5^{y+1}$.
   Answer : ______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## 1 Concept Check

**A Fill in the blanks:**

1. $3^5 \times 3^2 = 3$ ______
2. $a^7 \div a^3 =$ ______
3. $(x^4)^3 =$ ______
4. $2^5 \times 4^5 =$ ______
5. $\left(\frac{6^2}{6^5}\right) = 6$ ______
6. $27^x = 3^9 \Rightarrow x =$ ______

**B Write each of the following in a single power of the given base:**

1. $5^6 \times 5^3 \times 5^2$ (base 5) = ______
2. $2^8 \div 2^3$ (base 2) = ______
3. $7^4 \times 7^{-2} \times 7^3$ (base 7) = ______
4. $a^9 \div a^4 \times a^2$ (base $a$) = ______

**C Compare using >, < or = :**

1. $2^5$ ◯ $2^3$
2. $3^2 \times 3^4$ ◯ $3^7$
3. $5^6$ ◯ $25^3$
4. $8^2$ ◯ $4^3$

**D Match the expressions in column (A) with their equivalent in column (B):**

| Column (A) | Column (B) |
|---|---|
| 1. $(2^3)^4$ | a. $4^2$ |
| 2. $3^5 \div 3^{-2}$ | b. $2^{12}$ |
| 3. $(5^{-2})^3$ | c. $5^{-6}$ |
| 4. $4^3 \times 4^{-1}$ | d. $3^7$ |

**Think!**

If $x^m = 2^5$ and $x^n = 2^3$, find $x^{m-n}$.

Explain your answer.

______

______

## 2 Problem Solving

1. A bacteria cell divides into 8 cells every hour. If there is 1 cell at the beginning, how many cells will there be after 6 hours?

Answer : ______

2. A tiny rocket travels $3^4$ km in the first second. If it travels the same distance tripled each second, how far does it travel in the $3^{rd}$ second?

Answer : ______

3. A factory produces $2^6$ items on Monday. If the production is doubled each day, how many items will be produced on Thursday?

Answer : ______

4. The side of a square is $3^5$ cm. Find the area of the square.

Answer : ______

**Real Life Connection**

A microscope can magnify an object by a factor of $10^3$ each time you increase the power by one level.

If the magnification at level 1 is $10^3$ times, what is the magnification at level 4?

Answer : ______

## 3 Challenge Yourself

**Choose the correct answer:**

1. Which of the following is equal to $\left(\frac{2^4 \times 3^2}{2^2}\right)^2$ ?

a) $2^4 \times 3^4$  b) $2^8 \times 3^2$  c) $2^4 \times 3^2$  d) $2^6 \times 3^4$

2. Which expression is equal to $5^3 \div 5^{-4} \times 5^2$ ?

a) $5^{-1}$  b) $5^1$  c) $5^9$  d) $5^5$

3. If $9^x = 27^2$, then $x$ equals:

a) 2  b) 3  c) 4  d) 6

4. Which of the following is **NOT** equal to $(2a^3b^2)^2$ ?

a) $4a^6b^4$  b) $2^2a^6b^4$  c) $2a^6b^4$

5. Which is equal to $\frac{(x^3y^{-2})^4}{x^5y^3}$ ?

a) $x^7y^{-5}$  b) $x^7y^{-11}$  c) $x^7y^{-8}$  d) $x^{-7}y^{-11}$

6. If $4^m = 2^{15}$, then the value of $m$ is:

a) 5  b) 10  c) 15  d) 30

## 4 Mixed Practice

**A Simplify:**

1. $(3^2)^3 \times 3^{-4}$
2. $6^4 \div 6^2 \times 6^{-1}$
3. $(2^3 \times 5^2)^2$
4. $\frac{(7^3)^2 \times 7^{-4}}{7^3}$
5. $(x^2y^{-1})^3 \times (xy^2)^2$
6. $\frac{(a^4b^3)^2 \div (a^2b^{-1})^3}{(a^3b)^2}$

**B Find the value:**

1. $2^x = 64$
2. $3^{x-1} = 27$
3. $5^x \times 5^2 = 5^7$
4. $\frac{7^x}{7^3} = 49$

**C Arrange in ascending order:**

1. $2^5$ , $4^2$ , 8 , $2^4$

______

2. $3^3$ , 9 , 27 , $3^4$

______

**D Write as a single power:**

1. $2^3 \times 2^5 \times 2 \div 2^2 =$ ______
2. $a^4 \times a^{-2} \div a^3 =$ ______



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## Homework ........ Keep Practicing!

### 1 Evaluate each of the following.

1. $2^4 =$ ______
2. $3^3 =$ ______
3. $5^2 =$ ______
4. $7^1 =$ ______
5. $4^3 =$ ______
6. $2^6 =$ ______
7. $3^0 =$ ______
8. $10^2 =$ ______
9. $(2^2)^3 =$ ______
10. $(3^3)^2 =$ ______
11. $(5^2)^4 =$ ______
12. $(7^3)^1 =$ ______
13. $2^3 \times 2^4 =$ ______
14. $3^5 \div 3^2 =$ ______
15. $5^2 \times 5^3 =$ ______
16. $7^6 \div 7^2 =$ ______
17. $8^2 \times 2^3 =$ ______
18. $9^3 \div 3^2 =$ ______
19. $11^2 \times 11^3 =$ ______
20. $12^5 \div 12^2 =$ ______

### 2 Simplify each expression.

1. $(a^2)^3 =$ ______
2. $(x^3)^2 =$ ______
3. $(b^4)^2 =$ ______
4. $(y^5)^1 =$ ______
5. $a^3 \times a^4 =$ ______
6. $x^5 \div x^2 =$ ______
7. $b^6 \times b^3 =$ ______
8. $y^9 \div y^4 =$ ______
9. $2^3 \times 2^2 =$ ______
10. $3^7 \div 3^3 =$ ______
11. $5^4 \times 5 =$ ______
12. $7^8 \div 7^2 =$ ______
13. $(a^2)^3 \times a =$ ______
14. $(x^3)^2 \div x^4 =$ ______
15. $(m^2)^4 \times m^3 =$ ______
16. $(n^5)^2 \div n^3 =$ ______

### 3 Write each of the following as a single power.

1. $2^3 \times 2^5 =$ ______
2. $3^6 \div 3^2 =$ ______
3. $5^4 \times 5^3 =$ ______
4. $7^9 \div 7^4 =$ ______
5. $a^2 \times a^5 =$ ______
6. $b^7 \div b^3 =$ ______
7. $x^4 \times x^4 =$ ______
8. $y^8 \div y^2 =$ ______
9. $2^3 \times 2^2 \times 2^4 =$ ______
10. $3^6 \div 3^2 \times 3^3 =$ ______
11. $a^2 \times a^{-1} =$ ______
12. $b^7 \div b^4 \times b^2 =$ ______

### 4 Solve the following problems.

1. A newspaper doubles the number of copies printed each day. If $2^5$ copies are printed on the first day, how many copies will be printed on the sixth day?
   Answer: ______
2. A computer virus increases by a factor of 3 every hour. If it starts with 2 viruses, how many viruses will there be after 5 hours?
   Answer: ______
3. A certain plant grows in height by a factor of 2 each week. If its height was $2^2$ cm in the first week, what will be its height in the 5ᵗʰ week?
   Answer: ______
4. A printer makes $2^4$ pages in the first minute. Each minute, the number of pages it makes is multiplied by 2. How many pages will it make in 4 minutes?
   Answer: ______

### 5 Choose the correct answer.

1. $(2^3)^2$ is equal to:
   a) $2^5$ b) $2^6$ c) $2^8$ d) $2^{12}$
2. $3^4 \times 3^{-2}$ is equal to:
   a) $3^2$ b) $3^6$ c) $3^{-2}$ d) $3^8$
3. If $5^x = 125$, then $x$ equals:
   a) 2 b) 3 c) 4 d) 5
4. $(a^2b^3)^2$ is equal to:
   a) $a^4b^6$ b) $a^2b^6$ c) $a^4b^5$ d) $a^3b^6$
5. $2^m \times 2^n$ is equal to:
   a) $2^{m+n}$ b) $2^{m-n}$ c) $2^{m\times n}$ d) $2^{m/n}$
6. $(x^3)^4 \div x^5$ is equal to:
   a) $x^7$ b) $x^8$ c) $x^{12}$ d) $x^9$

### 6 Think and Answer

1. If $2^a = 32$ and $2^b = 8$, find the value of $a - b$.
   ______
2. Find the smallest natural number $n$ such that $3^n > 100$.
   ______
3. If $4^x = 64$ and $8^y = 64$, compare $x$ and $y$.
   ______


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# Homework

Keep Practicing!

## 1 Evaluate each of the following.

1. $a^2$ when $a = 3$
   $a^2 =$ ______
2. $b^3$ when $b = -2$
   $b^3 =$ ______
3. $(2x)^2$ when $x = 5$
   $(2x)^2 =$ ______
4. $y^2$ when $y = -4$
   $y^2 =$ ______
5. $(3m)^3$ when $m = 2$
   $(3m)^3 =$ ______
6. $(k^2)^3$ when $k = -3$
   $(k^2)^3 =$ ______
7. $a^3 - b^2$ when $a = 2$ , $b = 5$
   $a^3 - b^2 =$ ______
8. $2x^2 + y^3$ when $x = 3$ , $y = -1$
   $2x^2 + y^3 =$ ______
9. $3p^2q - q^3$ when $p = 2$ , $q = -2$
   $3p^2q - q^3 =$ ______
10. $a^2b$ when $a = -2$ , $b = 3$
    $a^2b =$ ______
11. $2m^2n^2$ when $m = -1$ , $n = 4$
    $2m^2n^2 =$ ______
12. $x^3y^2$ when $x = 2$ , $y = -3$
    $x^3y^2 =$ ______

## 2 Simplify each of the following expressions.

1. $a^2 \times a^5 =$ ______
2. $b^7 \div b^3 =$ ______
3. $x^4 \times x^2 \times x =$ ______
4. $y^9 \div y^4 \times y^2 =$ ______
5. $(m^3)^2 =$ ______
6. $(a^2b)^3 =$ ______
7. $(2x^2y)^2 =$ ______
8. $(-3a^3b^2)^2 =$ ______
9. $p^6 \div p^2 \times p^3 =$ ______
10. $c^5 \times c^3 \div c^4 =$ ______
11. $(d^2)^3 \times d^4 =$ ______
12. $(e^3f)^2 \div (ef^2) =$ ______
13. $(xy^2)^3 =$ ______
14. $(x^2y^3) \div (xy)^2 =$ ______
15. $(2a^2)^3 \times (a^4)^2 =$ ______
16. $(-x^3y)^2 \times (x^2y^2) =$ ______

## 3 Write each of the following as a single power.

1. $a^3 \times a^4 =$ ______
2. $b^8 \div b^5 =$ ______
3. $x^6 \times x \times x^3 =$ ______
4. $y^7 \div y \times y^2 =$ ______
5. $(a^2)^4 =$ ______
6. $(b^3)^2 =$ ______
7. $(x^2y^3)^2 =$ ______
8. $(ab^2)^3 =$ ______
9. $p^4 \times p^3 \div p^2 =$ ______
10. $q^9 \div q^3 \times q =$ ______
11. $(m^2n)^3 \div (mn) =$ ______
12. $(2x^2y)^3 \div (2xy)^2 =$ ______

## 4 Express each of the following in its simplest form.

1. $(2a^4 \times 3a^2) \div (6a^3) =$ ______
2. $(9x^3 \times 2x) \div (3x^2) =$ ______
3. $(4b^5 \times 5b^2) \div (2b^3) =$ ______
4. $(6m^4n^2 \times 3mn) \div (9m^2n) =$ ______
5. $(8x^2y^3) \div (2xy) =$ ______
6. $(15a^3b^2) \div (3ab^2) =$ ______
7. $(12p^4q \times 5pq^2) \div (10p^2q) =$ ______
8. $(14m^2n^3) \div (7mn) =$ ______
9. $(4x^2y)^2 \div (2xy) =$ ______
10. $(3a^3b)^2 \times (2ab^2) =$ ______
11. $(2p^2q^3)^3 \div (4pq) =$ ______
12. $(-3x^2y)^2 \times (-2xy) =$ ______

## 5 Solve the following problems.

1. A rectangle has a length of $3x^2$ m and a width of $2x$ m. Find the area of the rectangle in terms of $x$.
   Answer : ______
2. The volume of a cube is $8a^3$ $cm^3$. Find the length of one edge of the cube in terms of $a$.
   Answer : ______
3. The number of bacteria in a dish is $n$. If the number triples every hour, write an expression for the number of bacteria after 4 hours.
   Answer : ______
4. A machine produces $p^2$ items in one minute. If the number of items doubles every minute, write an expression for the number of items produced in 5 minutes.
   Answer : ______

## 6 Challenge Yourself

1. Which of the following is equal to $(a^2b^3)^2 \div (ab)$ ?
   a) $ab^5$ b) $a^3b^5$ c) $a^3b^6$ d) $a^5b^5$
2. If $x^m \times x^3 = x^7$, what is the value of $m$ ?
   a) 1 b) 3 c) 4 d) 7
3. Which expression is NOT equal to $x^6y^3$ ?
   a) $(x^2)^3\, y^3$ b) $x^3 \times x^2y^3$
   c) $(xy)^3\, x^3$ d) $x^4y^3 \times x^2$
4. If $2^a = 32$ and $2^b = 8$, what is the value of $a - b$ ?
   a) 1 b) 2 c) 3 d) 4
5. Which of the following is equal to $\frac{(x^3y^2)^2 \times (xy)^3}{x^4y^3}$ ?
   a) $x^2y^2$ b) $x^3y^2$
   c) $x^2y^3$ d) $x^3y^3$
6. If $(-2m^2n)^3 = -8m^6n^3$, what is the value of $m^6n^3$ ?
   a) 1 b) 2 c) $-1$ d) $-2$


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# Exponential Equations

Bacteria under a microscope

## Get Inspired!

In nature, bacteria multiply rapidly. Some types can double every hour under ideal conditions using binary fission.

This rapid growth can be modeled using exponential equations.

If a single bacterium grows in this way, how many bacteria will be present after several hours?

It may reach 1024 bacteria!

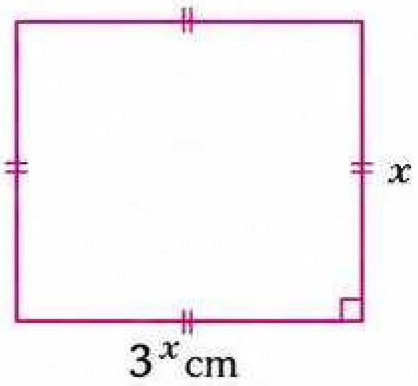
### Learning Outcomes

- Solve exponential equations.
- Use the laws of exponents.
- Solve real-life problems using exponential equations.

In this lesson, we will learn what exponential equations are and how to solve them using the laws of exponents.

### Think!

If the area of the square opposite equals $63$ cm$^2$, what is the value of $x$?




### Key Vocabulary

- Exponential equation
- Base
- Exponent
- Equality
- Solution of an equation

## Learn

### Basic Rule

An exponential equation is an equation that contains a variable in the exponent.

| Examples of exponential equations: | Examples of non-exponential equations: |
|---|---|
| • $x^2 = 9$ | • $2^x = 8$ |
| • $x - 7 = 3^2$ | • $3^{x-4} = \frac{1}{9}$ |

Solving an exponential equation means finding the value(s) of the variable that make the two sides of the equation equal.

### Remember

You can use repeated multiplication to write numbers in exponential form.

| | |
|---|---|
| 125 | 5 |
| 25 | 5 |
| 5 | 5 |
| 1 | |

$125 = 5^3$

### Solving exponential equations

Solve the following exponential equation using the equality property.

If $a = b$, then:

1) If $a^m = a^n$, and $a > 0$, $a \neq 1$, then $m = n$

2) If $a^m = b^m$, and $a > 0$, $a \neq 1$, then $a = b$

Examples:

- If $3^x = 3^4$, and $3 > 0$, $3 \neq 1$, then $x = 4$
- If $5^x = 5^7$, and $5 > 0$, $5 \neq 1$, then $x = 7$



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## Examples

Solve each of the following exponential equations using the equality property.

**1** $2^x = 16$

① $2^x = 16$
② $2^x = 2^4$ $(16 = 2^4)$
③ $x = 4$
$\therefore$ $x = 4$

**2** $3^x = 81$

① $3^x = 81$
② $3^x = 3^4$ $(81 = 3^4)$
③ $x = 4$
$\therefore$ $x = 4$

**3** $5^x = 125$

① $5^x = 125$
② $5^x = 5^3$ $(125 = 5^3)$
③ $x = 3$
$\therefore$ $x = 3$

**4** $7^x = 49$

① $7^x = 49$
② $7^x = 7^2$ $(49 = 7^2)$
③ $x = 2$
$\therefore$ $x = 2$

**5** $10^x = 1000$

① $10^x = 1000$
② $10^x = 10^3$ $(1000 = 10^3)$
③ $x = 3$
$\therefore$ $x = 3$

**6** $2^{x-1} = 8$

① $2^{x-1} = 8$
② $2^{x-1} = 2^3$ $(8 = 2^3)$
③ $x - 1 = 3$
④ $x = 4$
$\therefore$ $x = 4$

**7** $3^{x+2} = 27$

① $3^{x+2} = 27$
② $3^{x+2} = 3^3$ $(27 = 3^3)$
③ $x + 2 = 3$
④ $x = 1$
$\therefore$ $x = 1$

**8** $5^{2x} = 125$

① $5^{2x} = 125$
② $5^{2x} = 5^3$ $(125 = 5^3)$
③ $2x = 3$
④ $x = \frac{3}{2}$
$\therefore$ $x = \frac{3}{2}$

**9** $4^{x-2} = 16$

① $4^{x-2} = 16$
② $4^{x-2} = 4^2$ $(16 = 4^2)$
③ $x - 2 = 2$
④ $x = 4$
$\therefore$ $x = 4$

### Key Idea

If the bases of the two sides are the same, we can equate the exponents.

$$a^m = a^n \quad , \; a > 0, \; a \neq 1$$
$$\Rightarrow \quad m = n$$

### Common Mistakes

✗ Adding the exponents instead of equating them.

Example:

$3^x = 3^2$

**Wrong:** $x = 3 + 2 = 5$

**Correct:** $x = 2$

✗ Ignoring the base of each side.

Example:

$2^x = 3^2$

**Wrong:** $x = 2$

**Correct:** Not possible because the bases are different.

✗ Forgetting the order of operations with exponents containing expressions.

Example:

$2^{x+1} = 8$

**Wrong:** $x + 1 = 8$

**Correct:** $2^{x+1} = 2^3 \Rightarrow x + 1 = 3$

## Practice Yourself

Solve each equation using the equality property.

1. $2^x = 64$ ________
2. $3^x = 243$ ________
3. $5^x = 1$ ________
4. $7^x = 1$ ________
5. $10^x = 10{,}000$ ________
6. $2^{x-1} = 4$ ________
7. $3^{x+1} = 81$ ________
8. $5^{2x} = 25$ ________
9. $4^{x-1} = 2$ ________
10. $9^{x+1} = 27$ ________
11. $2^{x+3} = 16$ ________
12. $5^{x-2} = 25$ ________
13. $3^{2x} = 27$ ________
14. $7^{x-1} = 7^2$ ________
15. $10^{x+2} = 100$ ________

### Remember

✓ The bases must be the same.
✓ The bases must be positive.
✓ The bases must not equal 1.
✓ Then, equate the exponents.

### Quick Check

Fill in the blank:

1) If $2^x = 32$, then $x =$ ______.
2) If $5^x = 625$, then $x =$ ______.
3) If $10^{x-1} = 100$, then $x =$ ______.



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Solve each of the following exponential equations.

## More Examples

**1** $2^{x+3} = 16$

① $2^{x+3} = 16$

② $2^{x+3} = 2^4$ $(16 = 2^4)$

③ $x + 3 = 4$

④ $x = 4 - 3$

$\therefore$ $x = 1$

**2** $3^{x-1} = 81$

① $3^{x-1} = 81$

② $3^{x-1} = 3^4$ $(81 = 3^4)$

③ $x - 1 = 4$

④ $x = 4 + 1$

$\therefore$ $x = 5$

**3** $5^{2x} = 625$

① $5^{2x} = 625$

② $5^{2x} = 5^4$ $(625 = 5^4)$

③ $2x = 4$

④ $x = \frac{4}{2}$

$\therefore$ $x = 2$

**4** $7^{x-2} = 49$

① $7^{x-2} = 49$

② $7^{x-2} = 7^2$ $(49 = 7^2)$

③ $x - 2 = 2$

④ $x = 2 + 2$

$\therefore$ $x = 4$

**5** $4^{x+1} = 64$

① $4^{x+1} = 64$

② $4^{x+1} = 4^3$ $(64 = 4^3)$

③ $x + 1 = 3$

④ $x = 3 - 1$

$\therefore$ $x = 2$

**6** $8^{x-1} = \frac{1}{8}$

① $8^{x-1} = \frac{1}{8}$

② $8^{x-1} = 8^{-1}$ $\left(\frac{1}{8} = 8^{-1}\right)$

③ $x - 1 = -1$

④ $x = -1 + 1$

$\therefore$ $x = 0$

**7** $9^{x+2} = 729$

① $9^{x+2} = 729$

② $9^{x+2} = 9^3$ $(729 = 9^3)$

③ $x + 2 = 3$

④ $x = 3 - 2$

$\therefore$ $x = 1$

**8** $2^{2x-1} = 32$

① $2^{2x-1} = 32$

② $2^{2x-1} = 2^5$ $(32 = 2^5)$

③ $2x - 1 = 5$

④ $2x = 6$

⑤ $x = 3$

$\therefore$ $x = 3$

## Special Cases

1) $a^0 = 1$ (where $a \neq 0$)

- Examples:

$3^x = 1 \Rightarrow 3^x = 3^0 \Rightarrow x = 0$

$7^{x+2} = 1 \Rightarrow 7^{x+2} = 7^0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

2) $a^1 = a$

- Examples:

$5^x = 5 \Rightarrow 5^x = 5^1 \Rightarrow x = 1$

$2^{x-3} = 2 \Rightarrow 2^{x-3} = 2^1 \Rightarrow x - 3 = 1 \Rightarrow x = 4$

## Practice Yourself

Solve each equation using the equality property.

**1** $2^{x+2} = 64$ ________

**2** $3^{x-2} = 27$ ________

**3** $5^{x+1} = 125$ ________

**4** $7^{x+1} = 343$ ________

**5** $4^{2x} = 256$ ________

**6** $9^{x-1} = 81$ ________

**7** $2^{3x} = 64$ ________

**8** $5^{x-2} = 5$ ________

**9** $3^x = 1$ ________

**10** $7^{x+3} = 1$ ________

**11** $2^{x-3} = 8$ ________

**12** $9^{x+2} = 81$ ________

**13** $4^{x-1} = 16$ ________

**14** $10^{x-1} = 100$ ________

**15** $8^{x+1} = 2$ ________

### Key Idea

Make the bases the same using the laws of exponents, then equate the exponents.

**Laws of Exponents (Useful)**

1) $a^m \cdot a^n = a^{m+n}$

2) $\frac{a^m}{a^n} = a^{m-n}, \ a \neq 0$

3) $(a^m)^n = a^{mn}$

4) $a^0 = 1, \ a \neq 0$

5) $a^{-m} = \frac{1}{a^m}, \ a \neq 0$

### Common Mistakes

1) Forgetting to write both sides with the same base.

Example:

$4^{x+1} = 16$

Wrong: $4^{x+1} = 16 \Rightarrow x + 1 = 16$

Correct: $4^{x+1} = 2^4 = 4^2 \Rightarrow x + 1 = 2$

2) Ignoring negative exponents.

Example:

$2^x = \frac{1}{8}$

Wrong: $2^x = 8^{-1} = 8 \Rightarrow x = 3$

Correct: $2^x = 2^{-3} \Rightarrow x = -3$

3) Mistake in solving the linear equation after equating exponents.

Example:

$3^{x-2} = 27$

Wrong: $x - 2 = 3 \Rightarrow x = 1$

Correct: $x - 2 = 3 \Rightarrow x = 5$

### Think!

If $2^{x+1} = 32$, what is the value of $x$? ________

### Quick Check

Fill in the blank:

1) If $3^x = 27$, then $x =$ ______.

2) If $5^{x+1} = 125$, then $x =$ ______.

3) If $2^{x-1} = 1$, then $x =$ ______.

4) If $4^{2x} = 16$, then $x =$ ______.

5) If $7^{x+2} = 49$, then $x =$ ______.



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**2** $2^x = 16^{3-x}$

$\therefore\ 2^x = (2^4)^{3-x}$

$\therefore\ 2^x = 2^{4(3-x)}$

$\therefore\ 2^x = 2^{12-4x}$

$\therefore\ 2^x = 12 - 4x$

$\therefore\ x^2 + 4x - 12 = 0$

$\therefore\ (x+6)(x-2) = 0$

$\therefore\ x + 6 = 0$ or $x - 2 = 0$

$x = -6$ | $x = 2$

**4** $: \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{4}{9}\right)^{2x-7}$

$\therefore\ \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{2^2}{3^2}\right)^{2x-7}$

$\therefore\ \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{2}{3}\right)^{2(2x-7)}$

$\therefore\ \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{2}{3}\right)^{4x-14}$

$\therefore\ 6 - 3x = 4x - 14$

$\therefore\ 5 - 3x = -4x + 14$

$\therefore\ 4x - 3x = 14 - 5$

$\therefore\ x = 9$

**3** $\left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{4}\right)^{2x-3}$

$\therefore\ \left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{2^2}\right)^{2x-3}$

$\therefore\ \left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{2}\right)^{2(2x-3)}$

$\therefore\ \left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{2}\right)^{4x-6}$

$\therefore\ 5 - x = 4x - 6$

$\therefore\ 5 + 6 = 4x + x$

$\therefore\ 11 = 5x$

$\therefore\ x = \frac{11}{5}$

**Note :**

- $x = 0$ : if $a \neq 1,\ a > 0$ then $a^x = 1$ • For example : $5^0 = 1$
- $x = 0$ : if $a \neq b,\ b > 0,\ a > 0$ then $a^x = b^x$ • For example : $3^0 = 7^0$

### Example 3
Solve both equations with unequal bases in $\mathbb{R}$.

**1** $2^{x+1} = 3^{x+1}$

$\therefore\ 2^{x+1} = 3^{x+1}$

$\therefore\ x + 1 = 0$

$x = -1$

**2** $4^{x^2-9} = 1$

$\therefore\ x^2 - 9 = 0$

$\therefore\ x^2 = 9$

$\therefore\ x = \pm\sqrt{9}$

$x = \pm 3$

**3** $4^{x^2-9} = 1$

$\therefore\ x^2 - 9 = 0$

$\therefore\ x^2 = 9$

$\therefore\ x = \pm\sqrt{9}$

$x = \pm 3$

### Example 4
Find $n$ in each of the following :

**1** $\frac{12^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ \frac{12^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ \frac{(3 \times 2^2)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ \frac{3^{n-1} \times 2^{2(n-1)}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ 2^{2n-2-n+1} = 1$

$\therefore\ 2^{n-1} = 1$

$\therefore\ n - 1 = 0$

$\therefore\ n = 1$

**2** $\frac{2^n \times 9^{n+1}}{18^n} = 3^n$

$\therefore\ \frac{2^n \times 9^{n+1}}{18^n} = 3^n$

$\therefore\ \frac{2^n \times (3^2)^{n+1}}{(2 \times 3^2)^n} = 3^n$

$\therefore\ \frac{2^n \times 3^{2(n+1)}}{2^n \times 3^{2n}} = 3^n$

$\therefore\ 3^{2n+2-2n} = 3^n$

$\therefore\ 3^2 = 3^n$

$\therefore\ n = 2$

### Note 3

Solve both exponential equations in $\mathbb{R}$.

$$x^2 \times 2^x = 10$$

$$\left(\frac{1}{2}\right)^{2x-1} = 5^{1-2x}$$

### Note 4

Find $n$ in each of the following :

$$\frac{x^n \times 9^{1-x}}{6^{2x}} = 1$$

$$\frac{2^x \times 3^x}{12^x} = \frac{1}{2}$$

### Key Point

- When the bases are the same, equate the exponents.
- When the bases are different, try to simplify and rewrite using the same base.
- **Remember :** $a^0 = 1$ (where $a \neq 0$).
- $a^n = b^n$ with $a, b > 0$ and $a \neq b$ implies $n = 0$.

### Math Tip

Always simplify expressions before equating the exponents. Factorization and laws of exponents make the work easier.



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## Practice Yourself

Solve the following equations.

1. $2^{x+1} = 3^{x+1}$ ______
2. $4^{x-2} = 1$ ______
3. $5^{3x} = 125$ ______
4. $3^{x+2} = 27$ ______
5. $6^{2x-1} = 36$ ______
6. $9^{x-1} = 1$ ______
7. $2^x + 2^x = 12$ ______
8. $3^{2x-1} = 9$ ______
9. $5^{x-1} = \frac{1}{25}$ ______
10. $7^2 = 7^{x+3}$ ______
11. $4^{x-1} = 2$ ______
12. $8^{x+1} = 16$ ______
13. $3^{x-2} = \frac{1}{9}$ ______
14. $11^x = 11^3$ ______
15. $2^{3x-2} = 32$ ______

### B Find $n$ in each of the following.

1. $\frac{2^n \times 3^{n-1}}{6^n} = 1$ ______
2. $\frac{5^{n+1}}{5^{n-2}} = 125$ ______
3. $\frac{3^n \times 9^{n-1}}{27^n} = 1$ ______
4. $\frac{2^{2n-1}}{2^{n+2}} = 32$ ______
5. $\frac{7^{n+1}}{7^{n-1}} = 49$ ______
6. $\frac{4^n \times 2^{n-1}}{8^n} = 1$ ______
7. $\frac{9^{n+2}}{9^n} = 81$ ______
8. $\frac{5^{2n} \times 25^{n-1}}{125^n} = 1$ ______
9. $\frac{2^{n+3}}{2^{n-2}} = 32$ ______
10. $\frac{3^{2n+1}}{9^n} = 27$ ______

### C Word Problems (Real Life).

1. A bacteria cell divides into two cells every hour. If there is one cell at the beginning, how many cells will there be after 6 hours?
Answer: ______

2. The amount of money in a savings account doubles every year. If you have EGP 2500 now, how much will you have after 5 years?
Answer: ______

3. A certain drug loses 20% of its amount every hour. If the initial amount is 100 mg, find the amount remaining after 3 hours.
Answer: ______

4. The side of a square is doubled. By what factor does the area of the square increase?
Answer: ______

5. The population of a town increases by 10% every year. If the population is 20,000 now, what will it be after 3 years?
Answer: ______

### Remember

- ✓ When the bases are the same, equate the exponents.
- ✓ When the bases are different, try to write them with the same base using the laws of exponents.
- ✓ $a^0 = 1$, where $a \neq 0$.
- ✓ $a^n = b^n$ and $a, b > 0$, $a \neq b$ implies $n = 0$.
- ✓ $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$.

### Common Mistakes

**✗ 1.** Not making the bases the same.

Example:

$2^x = 3^x$

**Wrong:** $x = x$

**Correct:** Since $2 \neq 3$, then $x = 0$

**✗ 2.** Adding exponents.

Example:

$2^{3x} = 2^{x+2}$

**Wrong:** $3x = x + 2 \Rightarrow 2x = 2$

**Correct:** $3x = x + 2 \Rightarrow 2x = 2$

$\Rightarrow x = 1$

**✗ 3.** Forgetting negative exponent.

Example:

$2^{-x} = 4$

**Wrong:** $-x = 4 \Rightarrow x = -4$

**Correct:** $2^{-x} = 2^2$

$\Rightarrow -x = 2 \Rightarrow x = -2$

### Think!

If $3^{2x} = 27$, what is the value of $2x$ ?

______

## Challenge Yourself

1. Solve for $x$ and $y$:

$$\begin{cases} 2^x \times 3^y = 72 \\ 2^{x-y} = 4 \end{cases}$$

Answer: $x =$ ______, $y =$ ______

2. Find the value of $n$ such that

$$\frac{4^{n-1} \times 16^{n+2}}{2^{3n-1}} = 8^{2n+1}$$

Answer: $n =$ ______

3. If $a^3 = 2$ and $b^2 = 3$ find the value of:

$$(a^2 b)^6$$

Answer: ______

### Quick Check

Fill in the blank:

1) $2^x = 16 \Rightarrow x =$ ______
2) $3^x = 81 \Rightarrow x =$ ______
3) $5^x = 1 \Rightarrow x =$ ______



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## Practice on the Lesson

**A** Solve for $x$ in each of the following equations.

1. $2^x = 16$ ________
2. $3^x = 81$ ________
3. $5^x = 125$ ________
4. $7^x = 49$ ________
5. $2^{x+1} = 8$ ________
6. $3^{x-1} = 9$ ________
7. $5^{x+2} = 125$ ________
8. $2^{x-2} = 32$ ________
9. $4^x = 64$ ________
10. $10^x = 1000$ ________
11. $9^{x+1} = 81$ ________
12. $2^{x-1} = 8$ ________
13. $3^{x+2} = 27$ ________
14. $5^{2x} = 125$ ________
15. $4^{x-2} = 16$ ________
16. $7^{x-1} = 7^2$ ________

**B** Find $n$ in each of the following.

1. $\frac{12^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$ ________
2. $\frac{2^n \times 9^{n+1}}{18^n} = 3^n$ ________
3. $4^{1-2n} = 2$ ________
4. $(3)^{2x+1} = \frac{125}{27}$ ________
5. $3^{2x-3} = \frac{1}{27}$ ________
6. $(2)^{2x+1} = \frac{125}{32}$ ________
7. $5^{2x-1} = 125$ ________
8. $\left(\frac{1}{4}\right)^{x-2} = 128$ ________
9. $9 \times 5^{x-1} = 25 \times 3^{x-1}$ ________
10. $7^{x+2} = 2^x \times 2^{-x}$ ________
11. $32^{x-3} = 8^{x+1}$ ________
12. $\frac{8^x \times 9^4}{18^x} = 64$ ________
13. $8^{x-1} \times 4^{2x+1} = 64$ ________
14. $\frac{36^{x+1}}{6^{x-1}} = \frac{1}{216}$ ________
15. $\frac{3^{2x+1}}{3^{x-1}} = 27$ ________
16. $\frac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^n} = 343$ ________

**C** Solve the following equations. (Different types)

1. $2^x + 2^{x+1} = 24$ ________
2. $3^{x+1} - 3^x = 54$ ________
3. $5^x + 5^{x-1} = 30$ ________
4. $2^{x+2} - 2^x = 28$ ________
5. $4^x + 3 \times 4^x = 64$ ________
6. $7^{x+1} + 7^x = 64$ ________
7. $6^x - 6^{x-1} = 30$ ________
8. $9^x + 2 \times 9^x = 243$ ________
9. $2^x \times 3^x = 216$ ________
10. $\frac{5^x}{5^{x-1}} = 125$ ________
11. $\frac{3^{2x}}{3^x} = 27$ ________
12. $(2^x)^3 = 512$ ________
13. $2^{x+1} + 2^{x-1} = 40$
14. $5^{x+2} - 125 = 0$
15. $\frac{8^x}{4^x} = 16$
16. $\left(\frac{1}{2}\right)^{2x-3} = 32$

**D** Word Problems (Real Life)

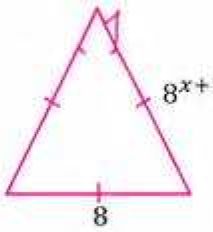
1. A bacteria cell divides into two cells every hour. If there is one cell at the beginning, how many cells will there be after 6 hours?
Answer : ________
2. The amount of money in a savings account doubles every year. If you have EGP 2500 now, how much will you have after 5 years?
Answer : ________
3. A certain drug loses 20% its amount every hour. If the initial amount is 100 mg, find the amount remaining after 3 hours.
Answer : ________
4. The side of a square is doubled. By what factor does the area of the square increase?
Answer : ________
5. The population of a town increases by 10% every year. If the population is 20,000 now, what will it be after 3 years?
Answer : ________

**E** Solve for $x$ in each of the following equations.

1. $2^{x^2} = 16$ ________
2. $3^{x^2} = 81$ ________
3. $5^{x^2} = 625$ ________
4. $7^{x^2} = 49$ ________
5. $2^{(x-1)^2} = 32$ ________
6. $3^{(x+1)^2} = 27$ ________
7. $4^{(x+2)^2} = 64$ ________
8. $10^{x^2-1} = 1000$ ________

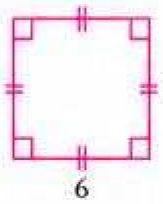
**F** Solve both equations in $\mathbb{R}$.

1. $2^x \times 2^x = 10$ ________
2. $\left(\frac{1}{2}\right)^{2x-1} = 5^{1-2x}$ ________
3. $x^2 \times 2^x = 1$ ________
4. $\left(\frac{1}{4}\right)^{2x-1} = 5^{1-2x}$ ________
5.




Find the value of $x$.

6.




Find the value of $x$.

**G** Find the value of $x$ in each of the following.

1. $2^{x+1} + 2^{x+1} + 2^{x+1} = 112$ ________
2. $2 \times 3^{x+1} - 3^x = 5^{x+1} - 2 \times 5^x$ ________
3. $4^x - 5 \times 2^x + 4 = 0$ ________
4. $9^x - 4 \times 3^x + 3 = 0$ ________
5. $2^{2x} - 5 \times 2^x + 6 = 0$ ________
6. $5^{x+1} - 26 \times 5^x + 25 = 0$ ________



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## More Practice on Exponential Equations

**A** Solve for $x$ in each of the following equations.

1. $3^{x+2} = 27$
2. $2^{x-1} = 16$
3. $4^{x+1} = 64$
4. $5^{x-2} = 25$
5. $7^{x+1} = 343$
6. $3^{2x} = 81$
7. $5^{2x} = 125$
8. $2^{3x} = 64$
9. $7^{2x-1} = 49$
10. $4^{2x+2} = 256$
11. $6^{2x-1} = 36$
12. $9^{x+1} = 243$
13. $8^{x-2} = 32$
14. $3^{x+3} = 243$
15. $2^{4x} = 256$
16. $5^{3x} = 125$
17. $2^{x+3} = 64$
18. $10^{x-1} = 1000$
19. $7^{x-1} = 7^2$
20. $3^{x+2} = 1$

**B** Find $n$ in each of the following.

1. $\frac{2^{n+1}}{2^{n-1}} = 4$
2. $\frac{5^{n+2}}{5^n} = 25$
3. $\frac{3^{2n}}{3^{n-1}} = 27$
4. $\frac{7^{n-1}}{7^{n+2}} = \frac{1}{343}$
5. $\frac{(3^2)^{n+1}}{3^{2n-1}} = 9$
6. $\frac{2^{3n+2}}{2^n} = 32$
7. $\frac{5^{2n+1}}{5^{n-1}} = 125$
8. $\frac{2^n \times 3^{n+1}}{6^{n-1}} = 6$
9. $\frac{4^{n+2}}{2^{n-1}} = 64$
10. $\frac{9^n}{3^{n+2}} = \frac{1}{27}$
11. $\frac{7^{n+1} \times 7^{n-2}}{7^{2n}} = 7$
12. $\frac{2^{2n} \times 8^n}{4^{n-1}} = 64$
13. $\frac{5^{n+2} \times 5^{n-1}}{5^{2n}} = 125$
14. $\frac{(2^3)^n}{2^{2n-1}} = 8$
15. $\frac{3^{3n+1}}{9^{n-1}} = 81$

**C** Solve the following equations.

1. $2^{x+2} \times 2^{x-1} = 64$
2. $3^{x+1} \times 3^{x-2} = 81$
3. $5^{x+2} \times 5^x = 625$
4. $7^{x-1} \times 7^{x+2} = 343$
5. $2^{x+3} \div 2^{x-1} = 16$
6. $3^{2x} \div 3^{x-2} = 27$
7. $5^{x+1} \div 5^{x-2} = 125$
8. $7^{2x+1} \div 7^{x-1} = 49$
9. $(2^x)^3 = 8$
10. $(3^x)^2 = 27$
11. $(5^{x-1})^2 = 625$
12. $(7^{x+1})^2 = 2401$
13. $2^x + 2^x + 2^x = 56$
14. $3^x + 3^x = 18$
15. $5^x - 5^x = 0$
16. $7^x - 2 \times 7^x + 7^x = 28$

**D** Word Problems (Real Life)

1. A computer virus doubles every hour. If there were 500 viruses at the beginning, how many viruses will there be after 6 hours?
Answer : ________
2. The amount of money in a bank account triples every year. If the account has EGP 1500 now, how much will it have after 4 years?
Answer : ________
3. A drug loses 25% of its amount every day in the body. If the initial amount is 160 mg, find the amount remaining after 3 days.
Answer : ________
4. A square field has an area of 81 m². If the side of another square is twice the side, what is its area?
Answer : ________
5. The population of a village increases by 20% every year. If the current population is 12,000, what will it be after 3 years?
Answer : ________

**E** Solve for $x$ in each of the following equations.

1. $4^x + 4^x = 160$
2. $3^x + 3^x + 3^x = 81$
3. $2^x - 2^{x-1} = 8$
4. $5^x + 5^{x-1} = 30$
5. $2^x + 2^{x+1} + 2^{x+2} = 112$
6. $4^x - 4^x = 0$
7. $7^{x+1} - 7^x = 42$
8. $3^{x+2} - 3^{x+1} = 54$
9. $5^x + 2 \times 5^x = 90$
10. $2^{x+1} - 5 \times 2^x = -12$

**F** Solve for $x$ in each of the following equations.

1. $\left(\frac{1}{2}\right)^{x-2} = 16$
2. $\left(\frac{1}{3}\right)^{2x-1} = \frac{1}{27}$
3. $\left(\frac{2}{5}\right)^{x+1} = \frac{4}{25}$
4. $\left(\frac{4}{7}\right)^{2x} = \frac{16}{49}$
5. $\left(\frac{3}{2}\right)^{x-1} = \frac{9}{4}$
6. $\left(\frac{5}{2}\right)^{2x+1} = \frac{125}{8}$

**G** Challenge Yourself

1. If $2^x + 2^{x+1} + 2^{x+2} + 2^{x+3} = 240$, find $x$.
2. If $3^x - 3^{x-1} = 54$, find $x$.
3. If $5^{x+2} + 5^{x+1} = 186$, find $x$.
4. If $(2^x)^2 + (2^x)^3 = 96$, find $x$.
5. If $7^x + 7^{x+1} - 7^{x-1} = 294$, find $x$.



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## More Practice on Exponential Equations

### A Choose the correct answer :

1. If $2^{x+1} = 16$, then $x =$ ........
   a) 2 b) 3 c) 4 d) 5
2. If $5^{2x-1} = 125$, then $x =$ .........
   a) 1 b) 2 c) 3 d) 4
3. If $7^{x} = 49$, then $7^{x+2} =$ .........
   a) 49 b) 98 c) 343 d) 2401
4. If $3^{a} \times 3^{2} = 3^{7}$, then $a =$ .........
   a) 4 b) 5 c) 6 d) 7
5. If $2^{x-1} \times 2^{x+2} = 2^{8}$, then $x =$ ........
   a) 2 b) 3 c) 4 d) 5

### B Fill in the blank :

1. $4^{x} = 64 \Rightarrow x =$ ________
2. $9^{2x} = 81 \Rightarrow x =$ ________
3. $2^{x+3} = 32 \Rightarrow x =$ ________
4. $5^{x-2} = \frac{1}{25} \Rightarrow x =$ ________
5. $3^{2x+1} = 243 \Rightarrow x =$ ________
6. $7^{x} = \frac{1}{49} \Rightarrow x =$ ________
7. $6^{x+2} = 216 \Rightarrow x =$ ________
8. $8^{x-3} = \frac{1}{8} \Rightarrow x =$ ________

### C Match the following :

| | |
|---|---|
| 1. $2^{x+2} = 32$ | a. $x = -1$ |
| 2. $3^{2x} = 81$ | b. $x = 4$ |
| 3. $5^{x-1} = \frac{1}{25}$ | c. $x = 3$ |
| 4. $7^{x+1} = 49$ | d. $x = 2$ |
| 5. $2^{2x} = 256$ | e. $x = 1$ |
| 6. $4^{x-1} = 2$ | f. $x = -2$ |

### D Write as a single power, then solve for $x$ :

1. $2^{x+1} \times 2^{3x-1} = 2^{10}$ $x =$ ________
2. $3^{2x} \times 3^{x-2} = 3^{11}$ $x =$ ________
3. $5^{x+2} \div 5^{2x-1} = 5^{3}$ $x =$ ________
4. $7^{x+3} \times 7^{x-1} \div 7^{2x} = 49$ $x =$ ________

### E Solve for $x$ :

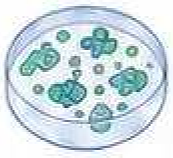
1. $\left(\frac{1}{3}\right)^{2x-1} = 27$ $x =$ ________
2. $\left(\frac{2}{5}\right)^{x+2} = \frac{4}{125}$ $x =$ ________
3. $\left(\frac{3}{4}\right)^{x-1} = \frac{16}{9}$ $x =$ ________
4. $\left(\frac{5}{2}\right)^{2x} = \frac{125}{8}$ $x =$ ________

### F Solve for $x$ in each of the following :

1. $9^{x-1} + 9^{x+1} = 90$ $x =$ ________
2. $3^{x+2} - 3^{x} = 72$ $x =$ ________
3. $2^{2x} + 2^{2x+1} = 48$ $x =$ ________
4. $5^{x+1} + 4 \cdot 5^{x} = 150$ $x =$ ________

### G Word Problems (Real Life)

1. The number of bacteria in a culture is $2^{n}$ at the beginning. After 3 hours, it becomes $32 \times$ the initial number. Find the value of $n$.

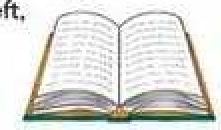


**Answer :** ________________

2. A certain amount of money is invested at a bank with annual interest compounded yearly. If the amount triples every year, how many years will it take for the investment to become 243 times the initial amount?

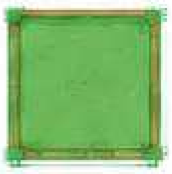
**Answer :** ________________

3. A book contains $2^{x}$ pages. Ali read $\frac{1}{4}$ of the book on the first day and $\frac{1}{2}$ of the remaining pages on the second day. If he still has 32 pages left, find $x$.



**Answer :** ________________

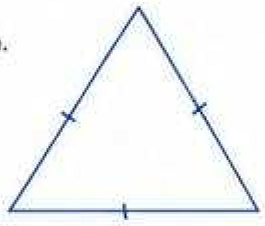
4. A square field has an area of $4^{n}$ $m^{2}$. If the length of its side is increased by 2 meters, the new area becomes $4^{n+2}$ $m^{2}$. Find the value of $n$.



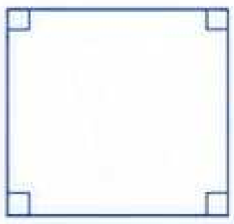
**Answer :** ________________

### H Geometry and Exponential Equations

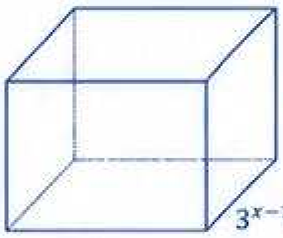
1. In the triangle, all sides have the same length.
   Perimeter $= 3 \times 7^{x-1}$ cm
   If the perimeter $= 147$ cm,
   find the value of $x$.
   $x =$ ________




2. In the square, the area $= 2^{2x+4}$ $cm^{2}$
   If the length of one side $= 2^{x+2}$ cm,
   find the value of $x$.
   $x =$ ________




3. The volume of the rectangular box is $2^{2x} \times 3^{x-1}$ $cm^{3}$.
   If its length $= 2^{x}$ cm and width $= 3^{x-1}$ cm,
   find the height.
   Height = ________ cm




### I True or False :

1. If $a^{x} = a^{y}$ and $a > 0$, $a \neq 1$, then $x = y$. ( )
2. $10^{x} \div 10^{y} = 10^{x-y}$ ( )
3. If $2^{x} = 2^{4}$, then $x = 2$. ( )
4. If $3^{x} = 27$, then $x = 3$. ( )
5. If $a^{x} = 1$ and $a \neq 1$, then $x = 0$. ( )
6. If $a^{x} = 1$ and $a \neq 1$, then $x = 1$. ( )



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# Unit 1 Assessment

## A Choose the correct answer:

1. What is the value of $a^2 \times a^5$ ?
   a) $a^{10}$ b) $a^7$ c) $a^3$ d) $a^2$

2. If $\frac{3^x \times 27^x}{81^x} = 2^{2x}$, then $x =$ ?
   a) 0 b) 1 c) 2 d) $-1$

3. If $9^{x+1} = 3^{2x-1}$, then $x =$ ?
   a) -2 b) -1 c) 1 d) 2

4. If $4^x = 8$, then $x =$ ?
   a) $\frac{1}{2}$ b) 1 c) 2 d) $\frac{3}{2}$

5. The value of $(-2a^2)^3$ is:
   a) $-6a^6$ b) $-8a^6$ c) $-8a^5$ d) $8a^6$

6. If $x^3 \times x^2 \times x^5 = x^k$, then $k =$ ?
   a) 5 b) 8 c) 10 d) 15

7. If $2^{-x+3} = 16$, then $x =$ ?
   a) -1 b) 1 c) 2 d) 3

8. If $3^{2x-1} = \frac{1}{27}$, then $x =$ ?
   a) -2 b) $-1$ c) 1 d) 2

9. If $5^x = 125$, then $x =$ ?
   a) 2 b) 3 c) 4 d) 5

10. The value of $\left(\frac{a^3}{a}\right)^2$ is:
    a) $a$ b) $a^2$ c) $a^3$ d) $a^6$

## B Complete the following:

1. $a^3 \times a^4 =$ ..........
2. $\frac{2^6}{2^3} =$ ..........
3. $(x^2)^5 =$ ..........
4. $9^x = 81 \Rightarrow x =$ ..........
5. $4^{-2} =$ ..........
6. $27^{\frac{2}{3}} =$ ..........
7. $a^{-3} = \frac{1}{\text{..........}}$
8. $3^x \times 3^2 = 3^{\text{.....}}$
9. $\frac{5^4}{5^{\text{.....}}} = 5^2$
10. $x^7 \div x^3 = x^{\text{.....}}$

## C Answer the following:

1. Calculate the value of $2^5 \times 2^{-3}$.
   ..........
2. Simplify: $3a^2b \times 2ab^3$.
   ..........
3. If $4^x = 64$, find the value of $2^x$.
   ..........
4. Write the number 0.001 as a power of 10.
   ..........
5. If $x^2 \times x^m = x^9$, find $m$.
   ..........

**Assessment Tips**
- Read each question carefully.
- Use the laws of exponents.
- Work step by step.
- Check your answers.



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# Unit 1 Assessment

## D Solve the following:

1. $2^x = 32$ ..........
2. $3^{x+1} = 27$ ..........
3. $5^{x-2} = 25$ ..........
4. $4^{2x} = 64$ ..........
5. $7^x = \frac{1}{49}$ ..........
6. $9^x = 81$ ..........
7. $2^x = \frac{1}{8}$ ..........
8. $5^{2x} = 125$ ..........
9. $3^{x-1} = \frac{1}{9}$ ..........
10. $8^x = 2^9$ ..........

## E True or False:

1. $a^m \times a^n = a^{m+n}$ *True / False*
2. $\frac{a^m}{a^n} = a^{m-n}$ *True / False*
3. $(a^m)^n = a^{m+n}$ *True / False*
4. $a^0 = 1$ where $a \neq 0$ *True / False*
5. $a^{-n} = -a^n$ *True / False*
6. $1^a = a$ *True / False*
7. If $2^x = 16$, then $x = 4$ *True / False*
8. $3^{-2} = \frac{1}{9}$ *True / False*
9. If $5^x = 125$, then $x = 3$ *True / False*
10. $(-2)^2 = -4$ *True / False*

## F Match the two columns:

| Column (A) | Column (B) |
|---|---|
| 1. $2^3 \times 2^4$ • | • a. $\frac{1}{9}$ |
| 2. $\frac{5^6}{5^2}$ • | • b. 1 |
| 3. $(3^2)^3$ • | • c. $2^7$ |
| 4. $2^{-3}$ • | • d. $5^4$ |
| 5. $7^0$ • | • e. $3^6$ |

**Remember**

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$ $(a \neq 0)$
- $a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$

## G Solve the word problems:

1. A bacteria doubles every hour. If we start with 1 bacteria, how many bacteria will we have after 6 hours?

..........

2. A car travels $3^4$ km on Monday and $3^5$ km on Tuesday. How many kilometers did it travel in total?

..........

3. A company's profit was $2^6$ dollars in January. In February, it was $2^2$ times the profit of January. What was the profit in February?

..........

4. The length of a rectangle is $5^3$ cm and its width is $5^2$ cm. Find the area of the rectangle.

..........



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# Unit 1 Assessment

## H Complete the following:

1. $7^x = 343 \Rightarrow x =$ ..........
2. $5^{2x} = 25 \Rightarrow x =$ ..........
3. $2^{-x} = \frac{1}{8} \Rightarrow x =$ ..........
4. $8^x \times 8^2 = 8^5 \Rightarrow x =$ ..........
5. $(3^2)^x = 3^6 \Rightarrow x =$ ..........
6. $9^x \div 9^{-1} = 9^3 \Rightarrow x =$ ..........
7. $\left(\frac{2^3}{2^2}\right)^x = 2^4 \Rightarrow x =$ ..........
8. $a^4 \times a^x = a^9 \Rightarrow x =$ ..........
9. $\frac{5^x}{5^2} = 5^3 \Rightarrow x =$ ..........
10. $(2a^2b)^3 = 2^x a^6 b^3 \Rightarrow x =$ ..........

## I Choose the correct answer:

1. The value of $16^2$ is:
   a) 4 b) 16 c) 64 d) 256
2. If $3^4 = 81$, then the value of $3^3$ is:
   a) 27 b) 81 c) 9 d) 243
3. $4^3$ equals:
   a) 7 b) 12 c) 16 d) 64
4. If $2^3 = 8$ and $2^x = 32$, then $x$ equals:
   a) 4 b) 3 c) 2 d) 5
5. The value of $(2^4)^2$ is:
   a) $2^6$ b) $2^7$ c) $2^8$ d) $2^9$
6. The value of $(-3)^3 \times (-3)^2$ is:
   a) $-3^5$ b) $3^5$ c) $-3^6$ d) $3^6$
7. If $5^x = 125$, then $x + 1$ equals:
   a) 2 b) 3 c) 4 d) 5
8. The value of $a^0 + a^2$ where $a \neq 0$ is:
   a) 0 b) 1 c) $a^2$ d) $1 + a^2$
9. $(2a)^3$ is equal to:
   a) $2a^3$ b) $6a^3$ c) $8a^3$ d) $2^3a^3$
10. The value of $\frac{1}{3^{-2}}$ equals:
    a) $\frac{1}{9}$ b) 9 c) 3 d) $\frac{1}{3}$

## J Answer the following:

1. Find the value of $4^{-2} + 2^3$.
   ..........
2. Simplify: $\frac{2^5 \times 2^{-3}}{2^2}$.
   ..........
3. If $10^3 = 1000$, find the value of $x$: $10^x = 100$.
   ..........
4. Write 10000 as a power of 10.
   ..........
5. If $3^x = 27$ and $3^y = 81$, find $3^{x-y}$.
   ..........

**Study Tip**
Keep the rules of exponents in mind.
Practice regularly to become faster and more confident.

### Test Yourself ???

**1 Multiple Choice**
Which of the following equals $2^3 \times 2^{-2}$?
a) $2^1$ b) $2^5$ c) $2^3$ d) $2^{-1}$

**2 Complete**
$7^x = 343 \Rightarrow x =$ ______

**3 True or False**
$2^4 = (2^2)^2$ ( ) True ( ) False

**4 Find the Value**
Find the value of $\frac{(3^2)^3}{3^4}$.
..........

### Challenge

If $a^2 = 9$ and $a^3 = 27$, find $a^5$.
..........



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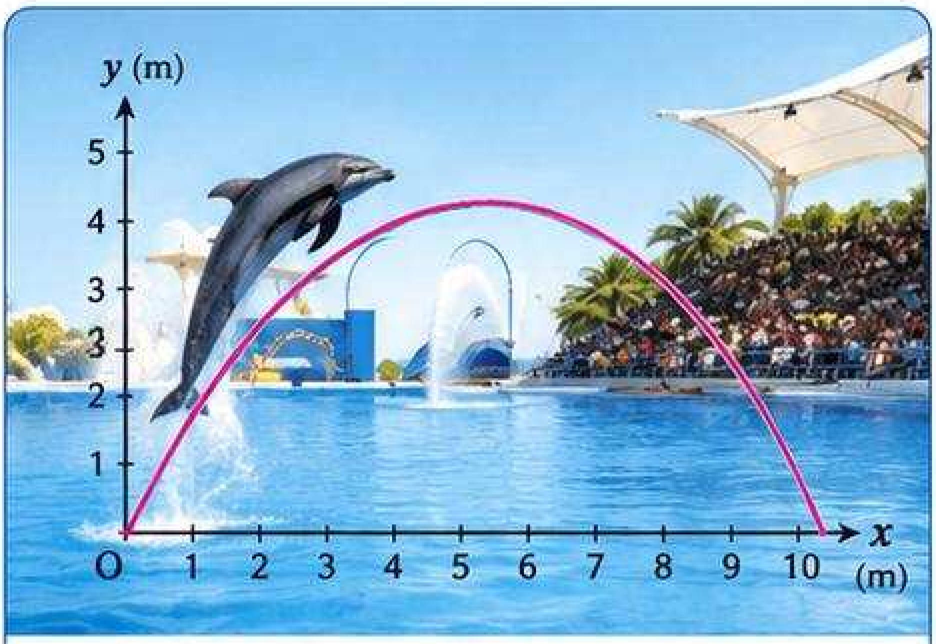
# Quadratic Function




One of the dolphins' trajectories

## Motivation!

Dolphins are among the smartest marine animals. They jump in shows and pass through hoops at high speed and jump above the water surface.

The path followed by a dolphin in the air is in the shape of a parabola.

This type of path is modeled using a quadratic function which describes the relationship between the height of the dolphin and the horizontal distance it travels.

If a dolphin moves according to the function: $f(x) = -0.2x^2 + 2x$

What is the maximum height the dolphin can reach?

## Learning Outcomes

**By the end of this lesson you will be able to:**

- Define the quadratic function.
- Identify the general form of a quadratic function.
- Identify the leading coefficient and the $y$–intercept.
- Find the domain and range of a quadratic function.
- Graph a quadratic function.
- Find the maximum or minimum value of a quadratic function.
- Use the quadratic function to solve real–life problems.
- Handle some simple problems in daily life.

## Let's Learn!

In this lesson, we will learn the concept of quadratic function and its properties, how to represent it graphically and how to solve real–life problems.

## Key Terms

- Quadratic function
- Parabola
- Maximum value
- Minimum value
- Y–intercept
- Vertex form

## Main Rule

**The general form of a quadratic function is:**

$$f(x) = ax^2 + bx + c$$

**where: $a \neq 0$**

- $a$ : the coefficient of $x^2$ (leading coefficient).
- $b$ : the coefficient of $x$.
- $c$ : the constant term.

**Explanation of symbols**

$x$ : the independent variable.

$f(x)$ or $y$ : the dependent variable.

$a$ : determines the opening direction of the parabola.

✓ If $a > 0$ → opens upward

✓ If $a < 0$ → opens downward

## Prior Knowledge

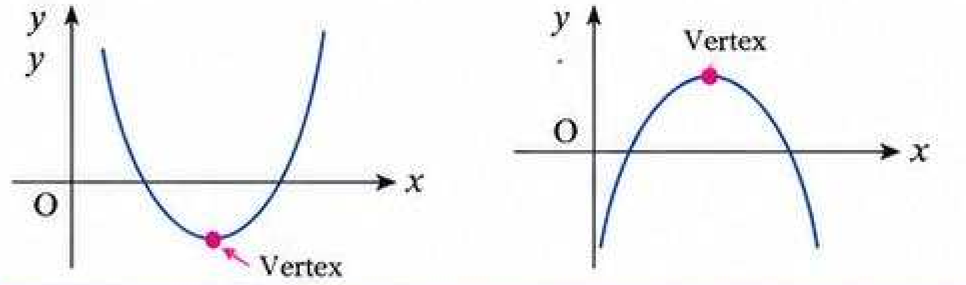
**You should know:**

- Real numbers $\mathbf{R}$.
- Plotting points on coordinate plane.
- Values of $x$ and $y$.
- Linear function of the form $f(x) = ax + b + c$
- Finding $f(0)$ when $x = 0$.

## Explanation

The graph of a quadratic function is called a **parabola**.

It has a special point called the **vertex**. The parabola may open upward (u–shape) or downward (n–shape).




## Common Mistakes

- ✗ Forgetting that $a \neq 0$.
- ✗ Confusing between the signs of $a$ and the opening direction.
- ✗ Thinking the vertex is always at the origin (it is not).
- ✗ Forgetting that the domain of any quadratic function is all real numbers.

## Examples

**1** $f(x) = 2x^2 + 3x - 1$

Here: $a = 2$ , $b = 3$ , $c = -1$

Since $a > 0$

→ opens upward.

**2** $f(x) = -x^2 + 4x + 5$

Here: $a = -1$ , $b = 4$ , $c = 5$

Since $a < 0$

→ opens downward.



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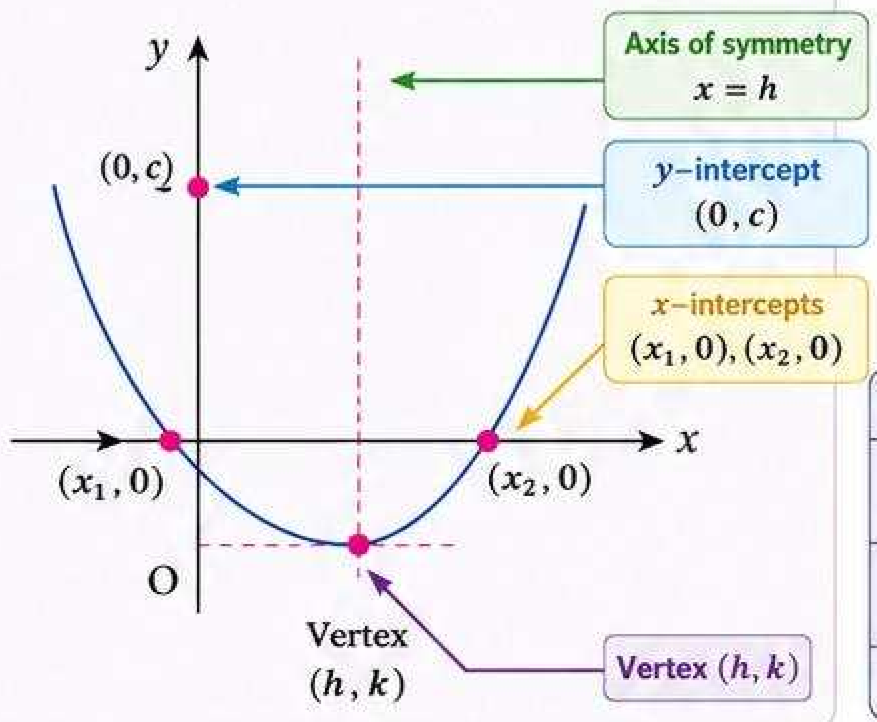
# Quadratic Function

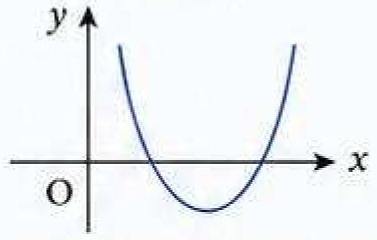
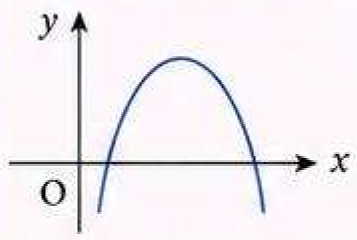
## Shape and Important Elements

The graph of a quadratic function is a parabola.

It has several important elements:




**Parabola can open in two directions**

| | If $a > 0$ | If $a < 0$ |
|---|---|---|
| | Parabola opens upward (U-shape)  | Parabola opens downward (n-shape)  |
| **Vertex** | Minimum point | Maximum point |
| **The smallest ($y$−value)** | At the vertex | Not defined |
| **The largest ($y$−value)** | Not defined | At the vertex |
| **Range** | $y \geq k$ | $y \leq k$ |

## Vertex Form

Every quadratic function can be written in vertex form:

$$f(x) = a(x - h)^2 + k$$

where $(h, k)$ is the vertex of the parabola.

The axis of symmetry is: $x = h$

**Note!**

- ✓ If $k > 0$, the vertex is above the $x$−axis.
- ✓ If $k < 0$, the vertex is below the $x$−axis.
- ✓ If $k = 0$, the vertex lies on the $x$−axis.

## Examples

**1** Find the vertex and axis of symmetry of:

$$f(x) = (x - 2)^2 + 3$$

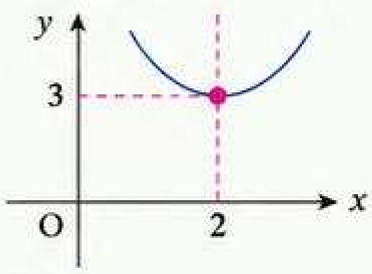
**Solution:**

Compare with $a(x - h)^2 + k$

Here: $h = 2$, $k = 3$

Vertex: $(2, 3)$

Axis of symmetry: $x = 2$




**2** Find the vertex and axis of symmetry of:

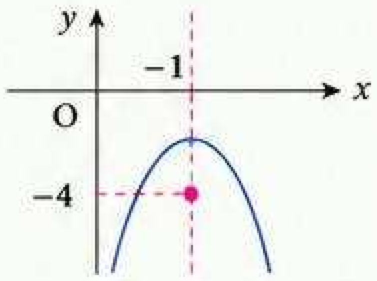
$$f(x) = -2(x + 1)^2 - 4$$

**Solution:**

Here: $h = -1$, $k = -4$

Vertex: $(-1, -4)$

Axis of symmetry: $x = -1$




**3** Find the vertex and axis of symmetry of:

$$f(x) = 3x^2$$

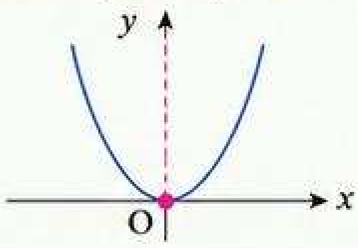
**Solution:**

$f(x) = 3(x - 0)^2 + 0$

Here: $h = 0$, $k = 0$

Vertex: $(0, 0)$

Axis of symmetry: $x = 0$




**4** Find the vertex and axis of symmetry of:

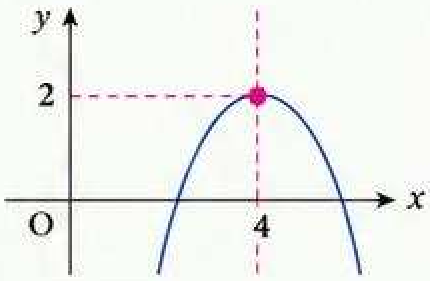
$$f(x) = -(x - 4)^2 + 2$$

**Solution:**

Here: $h = 4$, $k = 2$

Vertex: $(4, 2)$

Axis of symmetry: $x = 4$




## How to Find the Vertex from General Form

For $f(x) = ax^2 + bx + c$ $(a \neq 0)$:

1. Find the $x$−coordinate of the vertex:

$$h = -\frac{b}{2a}$$

2. Substitute $h$ in $f(x)$ to find $k = f(h)$.
3. The vertex is $(h, k)$.

**Example:**

Find the vertex of $f(x) = 2x^2 - 8x + 1$.

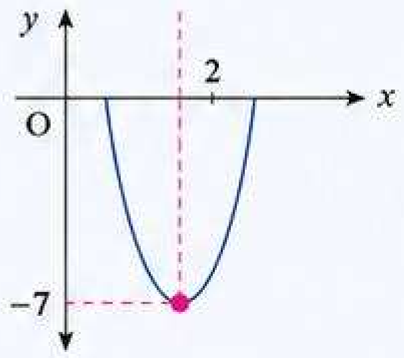
**Solution:**

$a = 2$, $b = -8$, $c = 1$

$h = -\frac{b}{2a} = -\frac{-8}{2 \times 2} = \frac{8}{4} = 2$

$k = f(2) = 2(2)^2 - 8(2) + 1 = 8 - 16 + 1 = -7$

Vertex $= (2, -7)$



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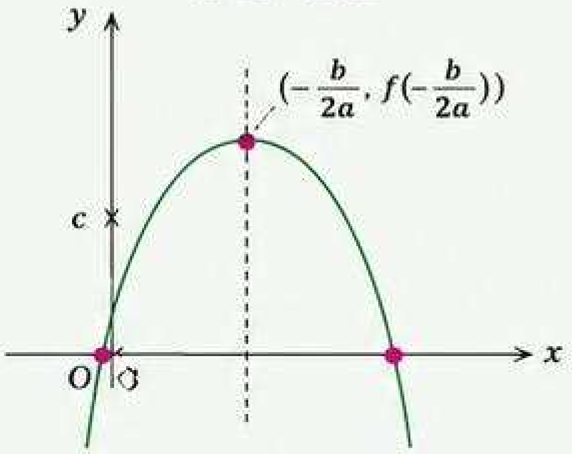
# Representing Quadratic Function in the Coordinate Plane

**If $a < 0$**

The curve opens downward, and the maximum value occurs at the vertex.

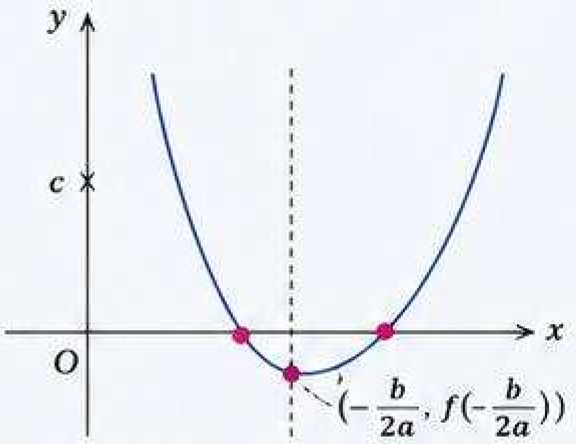



$x$-intercepts: $(x_1, 0)$ , $(x_2, 0)$

The maximum value is $f\left(-\frac{b}{2a}\right)$

**If $a > 0$**

The curve opens upward, and the minimum value occurs at the vertex.




$x$-intercepts: $(x_1, 0)$ , $(x_2, 0)$

The minimum value is $f\left(-\frac{b}{2a}\right)$

**Remember!**

The quadratic curve is symmetric about the axis of symmetry $x = -\frac{b}{2a}$, and the vertex is either the highest point (if $a < 0$) or the lowest point (if $a > 0$).

**Evaluation of Self**

**Example 1**

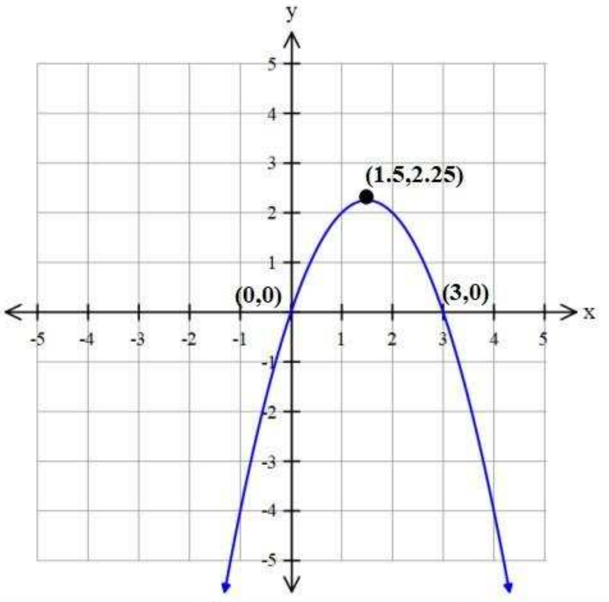
Represent the function
$f(x) = 3x - x^2$
on the coordinate plane.

## Example 1 $f(x) = 3x - x^2$

1. Find $a$, $b$, $c$ : $f(x) = -x^2 + 3x + 0$
   $a = -1$, $b = 3$, $c = 0$
2. Axis of symmetry: $x = -\frac{b}{2a} = -\frac{3}{2(-1)} = \frac{3}{2} = 1.5$
3. Vertex:
   $y = f(1.5) = -(1.5)^2 + 3(1.5) = -2.25 + 4.5 = 2.25$
   Vertex $(1.5, 2.25)$
4. $y$-intercept: $f(0) = 0 \Rightarrow (0, 0)$
5. $x$-intercepts: $-x^2 + 3x = 0 \Rightarrow x(x-3) = 0$
   $\Rightarrow x = 0$ or $x = 3$
   $x$-intercepts: $(0, 0)$ , $(3, 0)$
6. Value table:

| $x$ | $-1$ | $0$ | $1$ | $1.5$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-4$ | $0$ | $2$ | $2.25$ | $2$ | $0$ | $-4$ |

7. Plot the points and draw the curve.



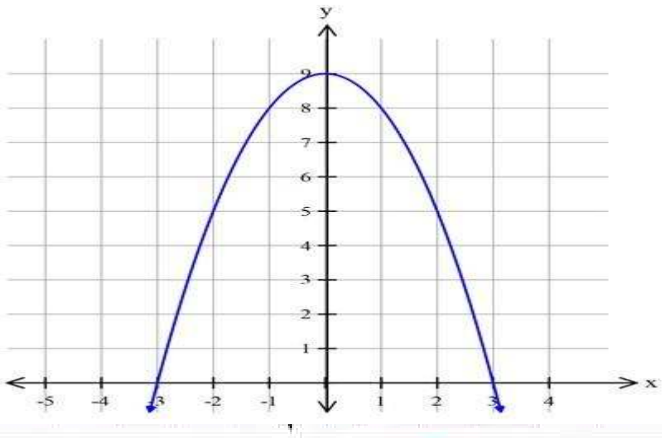

**Technological Employment**

- You can use many programs to represent quadratic functions such as GeoGebra.
- Try drawing the curve of Example 1 using GeoGebra.

## Example 2 $g(x) = -x^2 + 9$

1. Find $a$, $b$, $c$ : $g(x) = -x^2 + 0x + 9$
   $a = -1$, $b = 0$, $c = 9$
2. Axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(-1)} = 0$
3. Vertex: $y = g(0) = -(0)^2 + 9 = 9$
   Vertex $(0, 9)$
4. $y$-intercept: $g(0) = 9 \Rightarrow (0, 9)$
5. $x$-intercepts: $-x^2 + 9 = 0 \Rightarrow x^2 = 9$
   $\Rightarrow x = 3$ or $x = -3$
   $x$-intercepts: $(-3, 0)$ , $(3, 0)$
6. Value table:

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $g(x)$ | $0$ | $5$ | $8$ | $9$ | $8$ | $5$ | $0$ |




**Think & Discuss**

1. Is the point $(6, 21)$ on the curve of $f(x) = -x^2 + 6x - 5$ ?
2. Is the point $(-1, -10)$ on the curve of $g(x) = -x^2 + 9$ ?

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**Summary**

- The quadratic function $f(x) = ax^2 + bx + c$ has a parabolic graph.
- If $a > 0$, the curve opens upward $\rightarrow$ minimum at the vertex.
- If $a < 0$, the curve opens downward $\rightarrow$ maximum at the vertex.
- Axis of symmetry: $x = -\frac{b}{2a}$ , Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

**Common Mistakes**

- ✖ Confusing the signs of $a$ and the opening direction.
- ✖ Forgetting to calculate the axis of symmetry.
- ✖ Thinking the vertex is always a maximum point.
- ✖ Forgetting to test the function at several points before drawing the curve.



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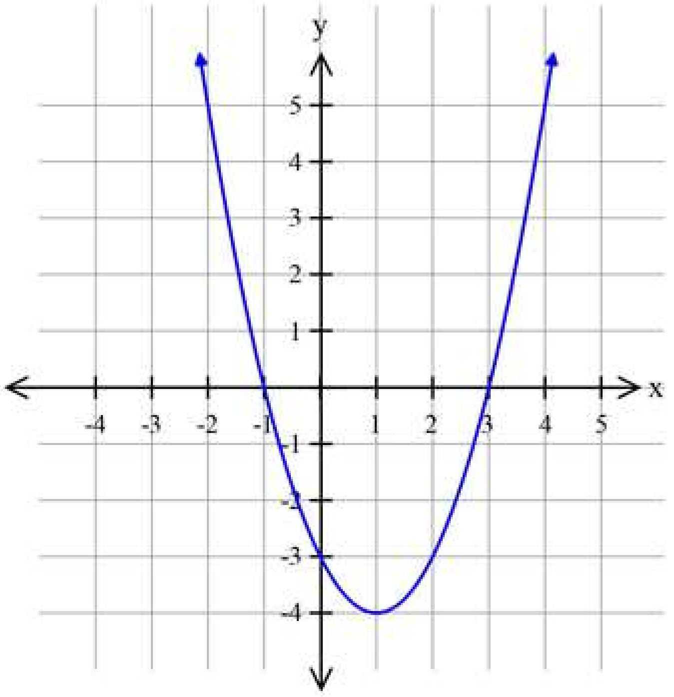
## Representing Quadratic Function in the Coordinate Plane (Examples)

### Example 3 $f(x) = x^2 - 2x - 3$

1. Find $a, b, c$: $f(x) = x^2 - 2x - 3$
   $a = 1, \quad b = -2, \quad c = -3$
2. Axis of symmetry: $x = -\dfrac{b}{2a} = -\dfrac{-2}{2(1)} = 1$
3. Vertex: $y = f(1) = 1^2 - 2(1) - 3 = 1 - 2 - 3 = -4$
   Vertex $(1, -4)$
4. $y$-intercept: $f(0) = -3 \Rightarrow (0, -3)$
5. $x$-intercepts: $x^2 - 2x - 3 = 0$
   $(x-3)(x+1) = 0$
   $x = 3$ or $x = -1$
   $x$-intercepts: $(3, 0), \ (-1, 0)$
6. Value table:

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $5$ | $0$ | $-3$ | $-4$ | $-3$ | $0$ | $5$ |

7. Plot the points and draw the curve.




**Note!**

To sketch the graph of $y = ax^2 + bx + c$:

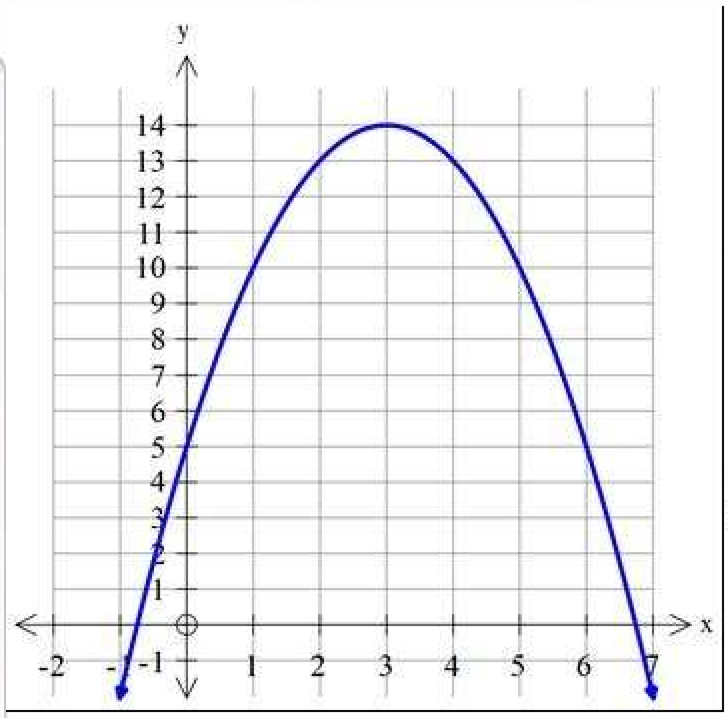
- ✓ Find the axis of symmetry.
- ✓ Find the vertex.
- ✓ Find the $y$-intercept.
- ✓ Find the $x$-intercepts.
- ✓ Make a value table.
- ✓ Plot the points and draw the curve.

### Example 4 $g(x) = -x^2 + 6x + 5$

1. Find $a, b, c$: $g(x) = -x^2 + 6x + 5$
   $a = -1, \quad b = 6, \quad c = 5$
2. Axis of symmetry: $x = -\dfrac{b}{2a} = -\dfrac{6}{2(-1)} = 3$
3. Vertex: $y = g(3) = -(3)^2 + 6(3) + 5 = -9 + 18 + 5 = 14$
   Vertex $(3, 14)$
4. $y$-intercept: $g(0) = 5 \Rightarrow (0, 5)$
5. $x$-intercepts: $-x^2 + 6x + 5 = 0$
   $\Rightarrow x^2 - 6x - 5 = 0$
   $(x-1)(x-5) = 0$
   $x = 1$ or $x = 5$
   $x$-intercepts: $(1, 0), \ (5, 0)$
6. Value table:

| $x$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ |
|---|---|---|---|---|---|---|---|
| $g(x)$ | $5$ | $0$ | $9$ | $14$ | $9$ | $0$ | $-7$ |

7. Plot the points and draw the curve.




**Key Points**

From the graph we can determine:

1. The axis of symmetry.
2. The vertex.
3. The maximum or minimum value.
4. The $x$-intercepts.
5. The $y$-intercept.

## Practice Yourself

For each of the following functions, complete the table and draw the graph:

**1** $f(x) = x^2 + 4x - 5$

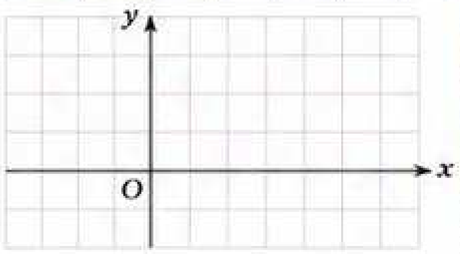
Axis of symmetry: ..................

Vertex: ..................

$y$-intercept: ..................

$x$-intercepts: .................. , ..................

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |




**2** $g(x) = x^2 - 4x$

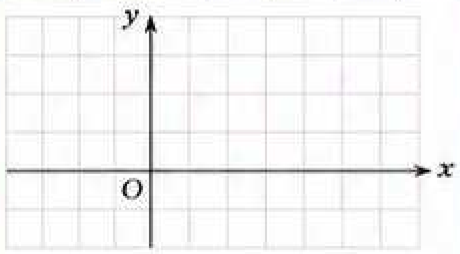
Axis of symmetry: ..................

Vertex: ..................

$y$-intercept: ..................

$x$-intercepts: .................. , ..................

| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|
| $g(x)$ | | | | | | |




**3** $h(x) = -x^2 + 2x + 3$

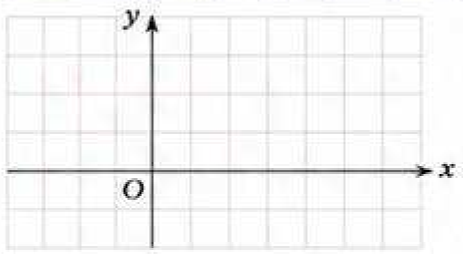
Axis of symmetry: ..................

Vertex: ..................

$y$-intercept: ..................

$x$-intercepts: .................. , ..................

| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|
| $h(x)$ | | | | | | |




**4** $p(x) = -x^2 - 2x + 8$

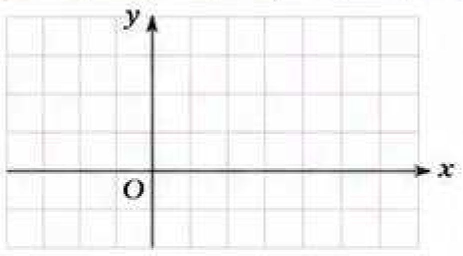
Axis of symmetry: ..................

Vertex: ..................

$y$-intercept: ..................

$x$-intercepts: .................. , ..................

| $x$ | $-4$ | $-2$ | $0$ | $2$ | $4$ | $6$ |
|---|---|---|---|---|---|---|
| $p(x)$ | | | | | | |




**Common Mistakes**

- ✗ Forgetting the sign of $a$ (openness of the parabola).
- ✗ Confusing the axis of symmetry with the $y$-axis.
- ✗ Not calculating the vertex correctly.
- ✗ Plotting too few points to get the right shape.

**Summary**

- The graph of a quadratic function is a parabola.
- Axis of symmetry: $x = -\dfrac{b}{2a}$
- Vertex: $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$
- If $a > 0$ opens upward (minimum). If $a < 0$ opens downward (maximum).



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## Quadratic Function in Vertex Form

The vertex form of a quadratic function is:

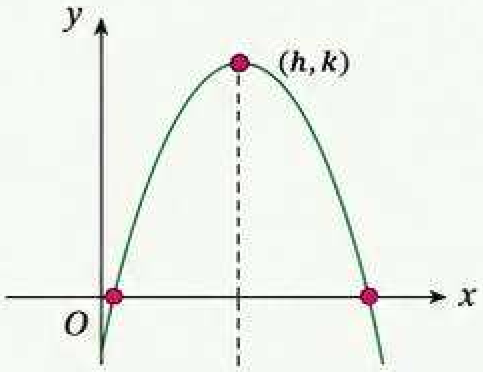
$$f(x) = a(x-h)^2 + k \quad (a \neq 0)$$

where $(h, k)$ is the vertex and $a, b, c$ are real numbers.

### Representing Quadratic Function in Vertex Form

**If $a < 0$**

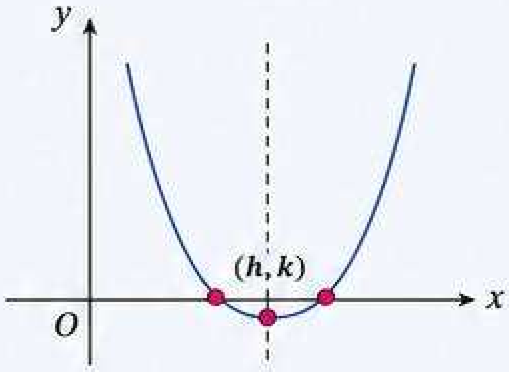
The curve opens downward and the maximum value occurs at the vertex.




The maximum value is $k = f(h)$

**If $a > 0$**

The curve opens upward and the minimum value occurs at the vertex.




The minimum value is $k = f(h)$

**Remember!**

For $f(x) = a(x-h)^2 + k$ :

- ✔ Vertex $= (h, k)$
- ✔ Axis of symmetry: $x = h$
- ✔ If $a > 0$ → opens upward (minimum at the vertex).
- ✔ If $a < 0$ → opens downward (maximum at the vertex).
- ✔ Maximum or minimum value $= k$.

**Think & Discuss**

1. If the vertex of a quadratic function is $(2, -3)$ and $a < 0$, is the maximum value $-3$ or the minimum value $-3$ ? Why?

.................................................

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2. Can the vertex of a quadratic function lie on the $x$-axis ? Give an example.

.................................................

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### Example 1 $f(x) = 5 - 2(x+1)^2$

Represent the function on the coordinate plane.

1. **Identify $a, h, k$.**

$f(x) = 5 - 2(x+1)^2 = -2(x+1)^2 + 5$

So: $a = -2, \quad h = -1, \quad k = 5$

2. **Find the vertex and axis of symmetry.**

Vertex $= (h, k) = (-1, 5)$

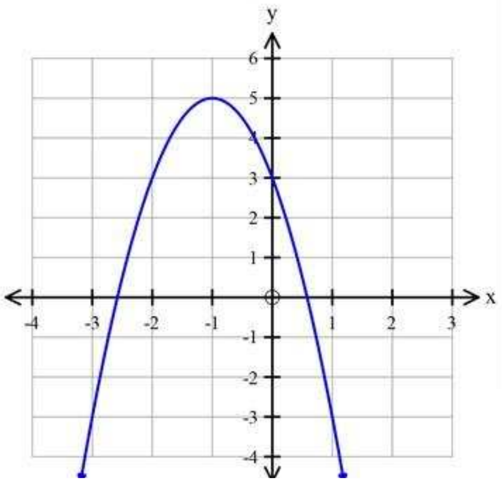
Axis of symmetry: $x = h = -1$

3. **Find some points.**

Let $u = x - h$, so $x + 1 = u$.

| $u$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ |
| $f(x) = -2u^2 + 5$ | $-3$ | $3$ | $5$ | $3$ | $-3$ |
| $(x, f(x))$ | $(-3, -3)$ | $(-2, 3)$ | $(-1, 5)$ | $(0, 3)$ | $(1, -3)$ |

4. **Plot the points and draw the curve.**




### Example 2 $g(x) = 2(x-3)^2 + 1$

1. **Identify $a, h, k$.**

$g(x) = 2(x-3)^2 + 1$

So: $a = 2, \quad h = 3, \quad k = 1$

2. **Vertex and axis of symmetry.**

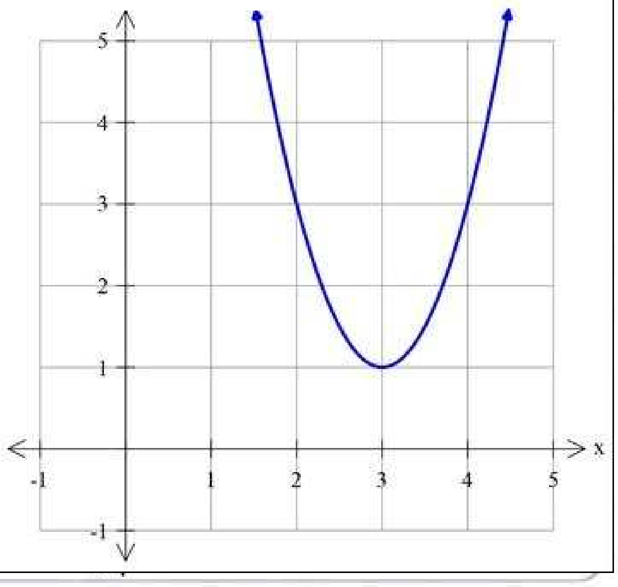
Vertex $= (3, 1)$

Axis of symmetry: $x = 3$

3. **Find some points.**

Let $u = x - 3$.

| $u$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $x$ | $1$ | $2$ | $3$ | $4$ | $5$ |
| $g(x) = 2u^2 + 1$ | $9$ | $3$ | $1$ | $3$ | $9$ |
| $(x, g(x))$ | $(1, 9)$ | $(2, 3)$ | $(3, 1)$ | $(4, 3)$ | $(5, 9)$ |




**Common Mistakes**

- ✖ Confusing $h$ with $k$ (vertex is $(h, k)$, not $(k, h)$).
- ✖ Forgetting that the axis of symmetry is $x = h$.
- ✖ Forgetting to substitute $x = h$ to find the maximum or minimum value.
- ✖ Plotting points that are not symmetric about $x = h$.

**Summary**

- Vertex form: $f(x) = a(x-h)^2 + k$.
- Vertex: $(h, k)$ , Axis of symmetry: $x = h$.
- If $a > 0$ opens upward → minimum value $= k$.
- If $a < 0$ opens downward → maximum value $= k$.
- To sketch the graph, find the vertex, axis of symmetry, some points, plot them and draw the curve.



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# Graphing and Analyzing Quadratic Function in Vertex Form

## Steps to Graph $f(x) = a(x-h)^2 + k$

1. Find the vertex $(h, k)$.
2. Draw the axis of symmetry $x = h$.
3. Determine the direction of the parabola:
   - ✔ If $a > 0$ → opens upward.
   - ✔ If $a < 0$ → opens downward.
4. Find some symmetrical points about $x = h$.
5. Plot the points and draw the curve.

## Key Ideas

1. The vertex gives the highest or lowest point.
2. The axis of symmetry passes through the vertex.
3. The further $|a|$ is from $0$, the narrower the parabola.
4. To find symmetric points, choose values for $u = x - h$ such as $-2, -1, 0, 1, 2$.

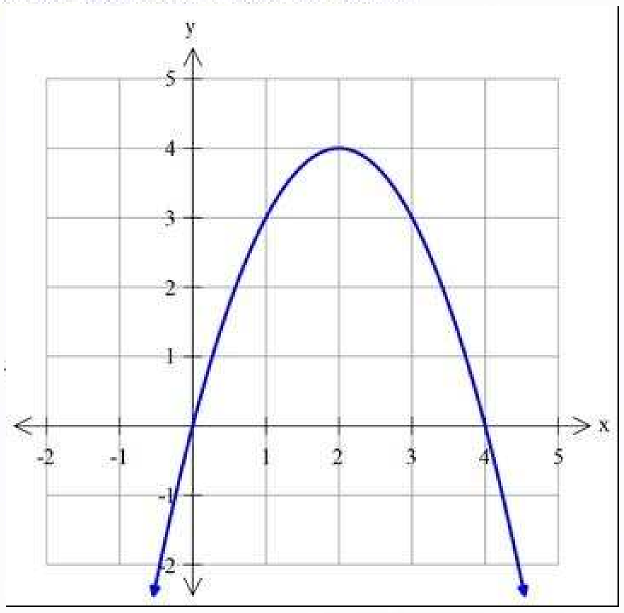
## Example 3 $f(x) = -(x-2)^2 + 4$

Graph the function and determine the maximum or minimum value.

1. Vertex: $(h, k) = (2, 4)$
2. Axis of symmetry: $x = 2$
3. Since $a = -1 < 0$ → opens downward
   The maximum value is $k = 4$ at $x = 2$.
4. Find some symmetric points.
   Let $u = x - 2$.

| $u$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $x = u + 2$ | $0$ | $1$ | $2$ | $3$ | $4$ |
| $f(x) = -(x-2)^2 + 4$ | $0$ | $3$ | $4$ | $3$ | $0$ |
| $(x, f(x))$ | $(0, 0)$ | $(1, 3)$ | $(2, 4)$ | $(3, 3)$ | $(4, 0)$ |

5. Plot the points and draw the curve.

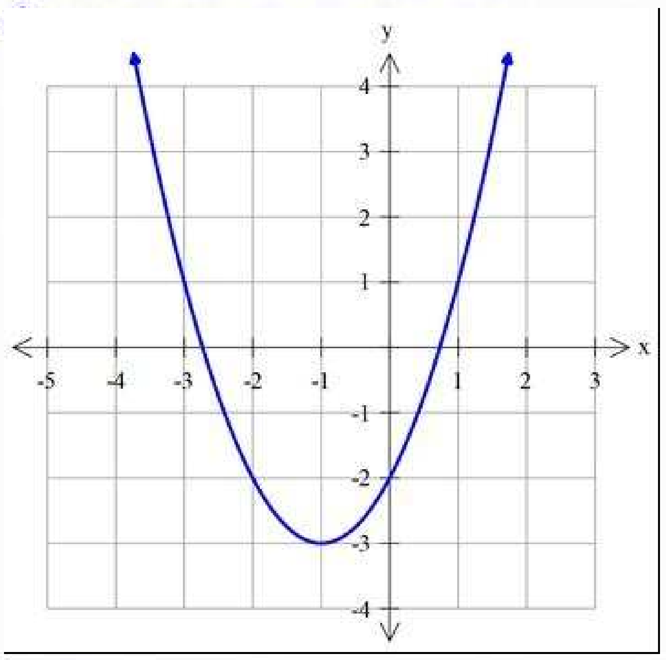



## Example 4 $g(x) = (x+1)^2 - 3$

Graph the function and determine the maximum or minimum value.

1. Vertex: $(h, k) = (-1, -3)$
2. Axis of symmetry: $x = -1$
3. Since $a = 1 > 0$ → opens upward
   The minimum value is $k = -3$ at $x = -1$.
4. Find some symmetric points.
   Let $u = x + 1$.

| $u$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $x = u - 1$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ |
| $g(x) = (x+1)^2 - 3$ | $1$ | $-2$ | $-3$ | $-2$ | $1$ |
| $(x, g(x))$ | $(-3, 1)$ | $(-2, -2)$ | $(-1, -3)$ | $(0, -2)$ | $(1, 1)$ |




## Note!

- ✔ The vertex is the turning point of the parabola.
- ✔ If the parabola opens upward, the vertex is the minimum point.
- ✔ If the parabola opens downward, the vertex is the maximum point.
- ✔ All points on the same side of the axis of symmetry have the same $y$-value.

## Think & Discuss

① For $f(x) = (x-5)^2 + 2$, what are the vertex and axis of symmetry?

..................................................

..................................................

..................................................

② For $g(x) = -(x+3)^2 + 1$, is the vertex a maximum or a minimum point? Why?

..................................................

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..................................................

## Practice Yourself

For each of the following functions:

1. Find the vertex and axis of symmetry.
2. Determine whether the parabola opens.
3. Flo furthee m'alnter the minıboin value.
4. Plot some points and draw the curve.

**1** $f(x) = (x-4)^2 + 1$

Vertex: ..................................
Axis of symmetry: ....................
Opens: ......................................
Max or Min value: ....................

**2** $g(x) = -(x+2)^2 + 5$

Vertex: ..................................
Axis of symmetry: ....................
Opens: ......................................
Max or Min value: ....................

**3** $h(x) = 2(x-1)^2 - 3$

Vertex: ..................................
Axis of symmetry: ....................
Opens: ......................................
Max or Min value: ....................

**4** $p(x) = -2(x+3)^2 - 1$

Vertex: ..................................
Axis of symmetry: ....................
Opens: ......................................
Max or Min value: ....................

## Common Mistakes

- ✖ Writing the vertex as $(a, b)$ instead of $(h, k)$.
- ✖ Forgetting that the axis of symmetry is $x = h$.
- ✖ Confusing maximum and minimum values.
- ✖ Not choosing symmetric points about the axis of symmetry.

## Summary

- Vertex form: $f(x) = a(x-h)^2 + k$.
- Vertex: $(h, k)$ , Axis of symmetry: $x = h$.
- If $a > 0$ opens upward → minimum value $= k$.
- If $a < 0$ opens downward → maximum value $= k$.
- Use symmetric points about $x = h$ to draw the parabola.



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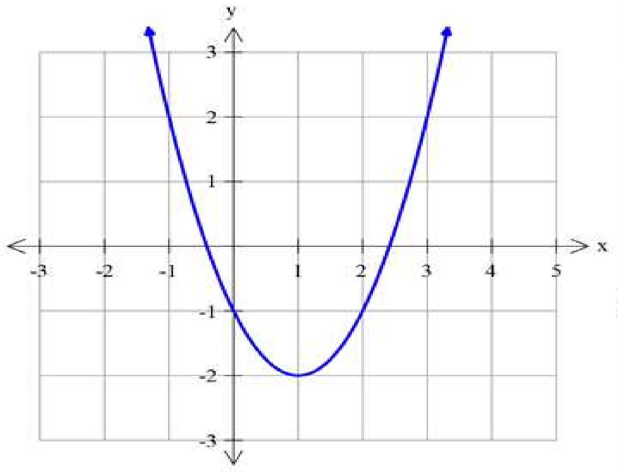
## Finding the Quadratic Function from its Graph

If the graph of a quadratic function in vertex form is given, we can write the function as: $f(x) = a(x-h)^2 + k$

where $(h, k)$ is the vertex. Use another point on the graph to find $a$.

### Example 5

Find the quadratic function represented by the graph.



**Solution:**

1. Vertex: $(h, k) = (1, -2)$

   So, $f(x) = a(x-1)^2 - 2$
2. Use the point $(3, 2)$ to find $a$.

   $2 = a(3-1)^2 - 2$

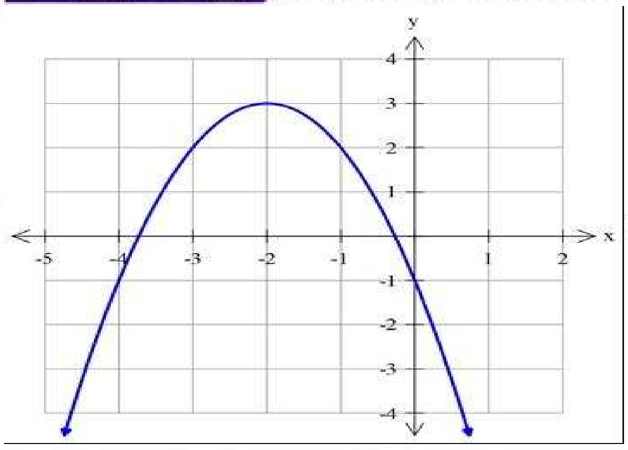
   $2 = a(2)^2 - 2$

   $2 = 4a - 2 \Rightarrow 4a = 4 \Rightarrow a = 1$
3. Therefore, the quadratic function is:

$$f(x) = (x-1)^2 - 2$$

### Example 6

Find the quadratic function represented by the graph.



**Solution:**

1. Vertex: $(h, k) = (-2, 3)$

   So, $f(x) = a(x+2)^2 + 3$
2. Use the point $(0, -1)$ to find $a$.

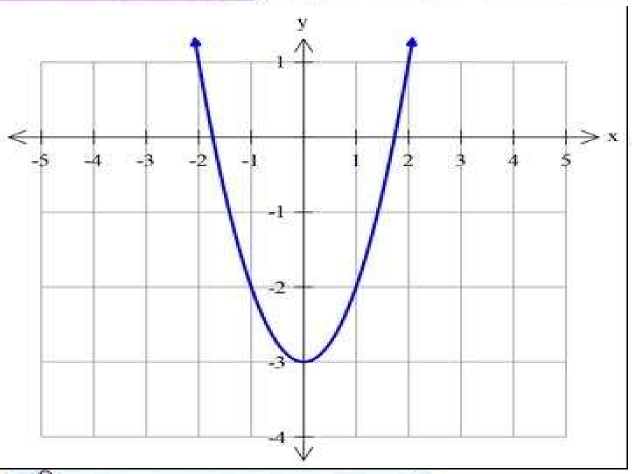
   $-1 = a(0+2)^2 + 3$

   $-1 = 4a + 3 \Rightarrow 4a = -4 \Rightarrow a = -1$
3. Therefore, the quadratic function is:

$$f(x) = -(x+2)^2 + 3$$

### Example 7

Find the quadratic function represented by the graph.



**Solution:**

1. Vertex: $(h, k) = (0, -3)$

   So, $f(x) = a(x-0)^2 - 3 = ax^2 - 3$
2. Use the point $(2, 1)$ to find $a$.

   $1 = a(2)^2 - 3$

   $1 = 4a - 3 \Rightarrow 4a = 4 \Rightarrow a = 1$
3. Therefore, the quadratic function is:

$$f(x) = x^2 - 3$$

### Remember!

- ✔ Vertex form: $f(x) = a(x-h)^2 + k$
- ✔ $(h, k)$ is the vertex.
- ✔ Substitute any other point $(x, y)$ from the graph to find $a$.
- ✔ Then write the quadratic function.

### Key Steps

1. Identify the vertex $(h, k)$ from the graph.
2. Choose another point $(x_1, y_1)$ on the graph.
3. Substitute in $y_1 = a(x_1 - h)^2 + k$ to find $a$.
4. Write the function $f(x) = a(x-h)^2 + k$.

### Think & Discuss

① If the graph has vertex $(2, -1)$ and passes through the point $(4, 3)$, find the quadratic function.

........................................

........................................

② If the graph has vertex $(-1, 4)$ and passes through the point $(0, 1)$, find the quadratic function.

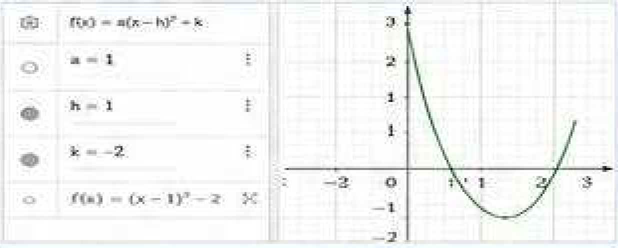
........................................

........................................

### Technology Corner

You can use GeoGebra to:

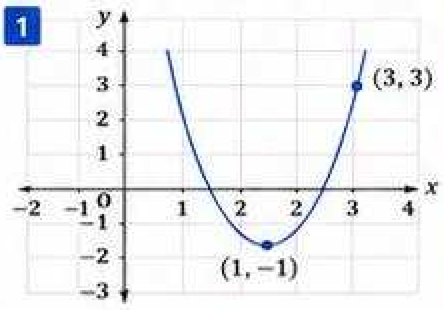
- Enter a function in vertex form.
- Change the values of $a$, $h$, $k$.
- Observe how the graph changes.
- Find the vertex and axis of symmetry.



### Practice Yourself

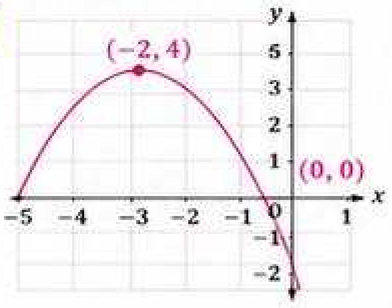
For each graph, find the quadratic function in vertex form.

**1**



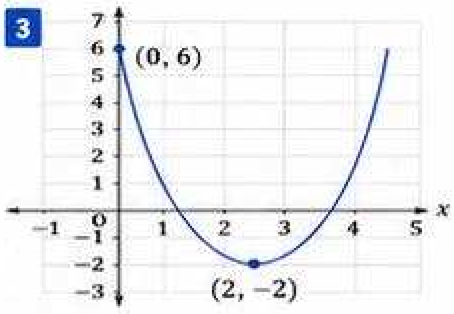

$f(x) =$ ........................

**2**



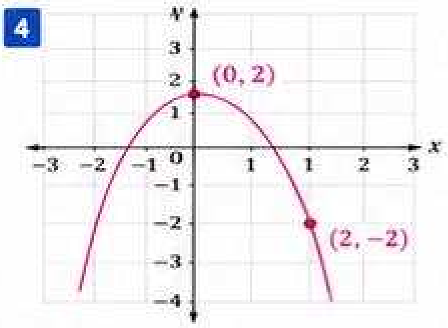

$f(x) =$ ........................

**3**




$f(x) =$ ........................

**4**




$f(x) =$ ........................

### Common Mistakes

- ✖ Using the wrong sign for $a$ (upward or downward opening).
- ✖ Forgetting to use the vertex $(h, k)$ in $a(x-h)^2 + k$.
- ✖ Substituting the vertex instead of another point to find $a$.
- ✖ Not simplifying calculations when solving for $a$.

### Summary

- Vertex form: $f(x) = a(x-h)^2 + k$.
- Vertex: $(h, k)$, Axis of symmetry: $x = h$
- If $a > 0$ opens upward (minimum value $= k$).
- If $a < 0$ opens downward (maximum value $= k$).
- Use another point on the graph to find $a$ and write the function.



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## Practice & Master

### 1 Choose the correct answer :

① The vertex of the function $f(x) = 2(x-3)^2 - 5$ is :

(a) $(3\ ,2)$ (b) $(3\ ,-5)$ (c) $(-3\ ,-5)$ (d) $(-3\ ,5)$

② The axis of symmetry of $f(x) = -x^2 + 6x - 4$ is :

(a) $x = 3$ (b) $x = -3$ (c) $x = 6$ (d) $x = -6$

③ If the parabola opens upward, then :

(a) $a < 0$ (b) $a > 0$ (c) $a = 0$ (d) $k < 0$

④ The minimum value of $f(x) = (x-2)^2 + 4$ equals :

(a) $-4$ (b) $2$ (c) $4$ (d) $6$

⑤ The maximum value of $f(x) = -(x+1)^2 + 7$ equals :

(a) $-7$ (b) $1$ (c) $7$ (d) $8$

⑥ The $y$-intercept of $f(x) = 2x^2 - 4x + 6$ is :

(a) $(0\ ,6)$ (b) $(0\ ,-6)$ (c) $(6\ ,0)$ (d) $(2\ ,0)$

⑦ If the vertex of a quadratic function is $(-2\ ,5)$, then its axis of symmetry is :

(a) $x = -2$ (b) $x = 2$ (c) $y = -2$ (d) $y = 2$

⑧ If $f(x) = (x-a)^2 + k$ has vertex $(1\ ,-3)$, then $a + k = ?$

(a) $2$ (b) $-2$ (c) $-4$ (d) $4$

### 2 Complete the following :

① The vertex of $f(x) = -(x+4)^2 + 9$ is (.............. , ..............).

② The axis of symmetry of $g(x) = 3(x-1)^2 - 2$ is .............. .

③ If $f(x) = x^2 - 2x + c$ has vertex $(1\ ,0)$, then $c =$ .............. .

④ The maximum value of $h(x) = -2(x-3)^2 + 4$ equals .............. .

⑤ If the vertex of $f(x) = a(x-h)^2 + k$ is $(2\ ,-5)$, then $h =$ .............. and $k =$ .............. .

⑥ The $y$-intercept of $f(x) = -x^2 + 8x$ is .............. .

⑦ The minimum value of $f(x) = (x+1)^2 - 7$ equals .............. .

⑧ If $f(x) = -(x-a)^2 + 5$ has maximum value 5, then $a =$ .............. .

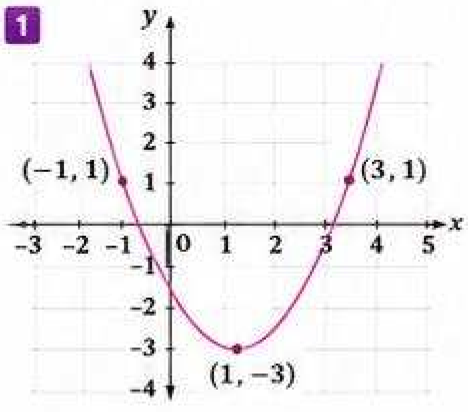
### 3 For each function, find the vertex $(h\ ,k)$ and the axis of symmetry :

1. $f(x) = x^2 - 4x + 1$
   Vertex $(h\ ,k) =$ ( ......... , ......... )
   Axis of symmetry: ..................
2. $g(x) = 2x^2 + 8x - 3$
   Vertex $(h\ ,k) =$ ( ......... , ......... )
   Axis of symmetry: ..................
3. $h(x) = -x^2 + 6x + 2$
   Vertex $(h\ ,k) =$ ( ......... , ......... )
   Axis of symmetry: ..................
4. $p(x) = 3x^2 - 12x + 7$
   Vertex $(h\ ,k) =$ ( ......... , ......... )
   Axis of symmetry: ..................

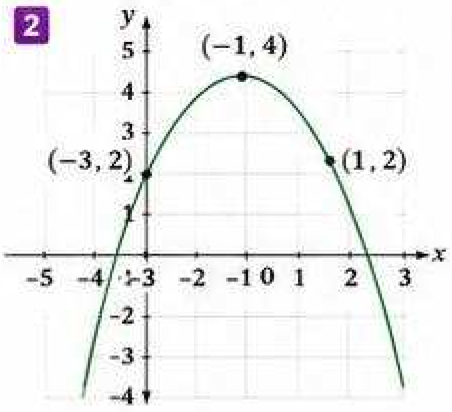
### 4 Determine whether the parabola opens upward or downward, then find the maximum or minimum value.

1. $f(x) = 2(x-3)^2 - 1$
   Opens: ..............................
   Axis or Min value: .................
2. $g(x) = -(x+2)^2 + 6$
   Opens: ..............................
   Max or Min value: .................
3. $h(x) = x^2 + 4x + 5$
   Opens: ..............................
   Max or Min value: .................
4. $p(x) = -2(x-1)^2 - 3$
   Opens: ..............................
   Max or Min value: .................

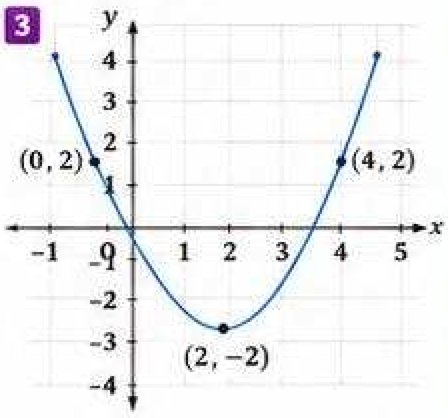
### 5 Find the quadratic function in vertex form that represents each graph :

1.


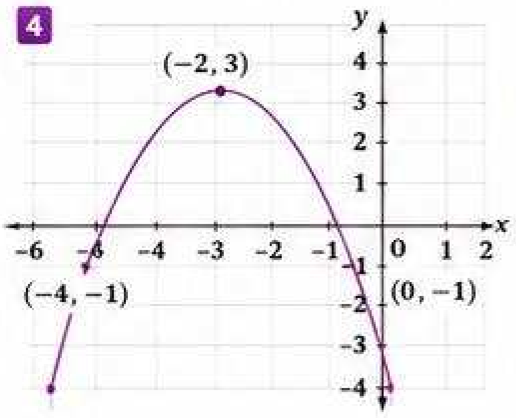

$f(x) =$ ......................................

2.



$g(x) =$ ......................................

3.



$h(x) =$ ......................................

4.



$p(x) =$ ......................................

### 6 Answer the following :

① The vertex of a quadratic function is $(3\ ,-2)$ and it passes through the point $(1\ ,6)$.
Find the function in vertex form.

..............................................................................

② The graph of $f(x) = ax^2 + bx + c$ has vertex $(-1\ ,\ 4)$ and passes through the point $(2\ ,-2)$.
Find the value of $a$, then write the function in vertex form.

..............................................................................

..............................................................................

③ A ball is thrown in the air from a height of 2 m above the ground with an initial velocity. Its height after $t$ seconds is given by: $h(t) = -4.9t^2 + 19.6t + 2$, where $h$ is the height in meters.

(1) Draw the graph of the function for $0 \le t \le 4$.

(2) Find the maximum height and the time at which it occurs.

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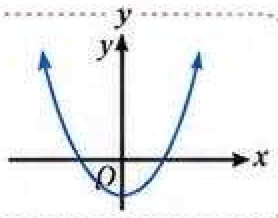
# Lesson 1 Quadratic Function



## Exercises

### A Choose the correct answer :

1. The vertex of the parabola $f(x) = 2(x-3)^2 + 1$ is :
   (a) $(3, 1)$ (b) $(2, 1)$ (c) $(2, 3)$ (d) $(-3, 1)$

2. If $(h, k)$ is the vertex of the parabola $f(x) = 2x^2 - 8x + 7$, then $h$ equals :
   (a) 2 (b) 4 (c) 6 (d) 8

3. The graph of $f(x) = x^2 - 3x - 5$ intersects the $x$-axis at :
   (a) $(0, -5)$ and $(0, 5)$ (b) $(0, -3)$ and $(0, 3)$
   (c) $(-5, 0)$ and $(5, 0)$ (d) $(-3, 0)$ and $(3, 0)$

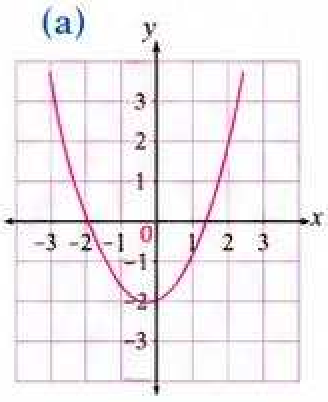
4. The minimum value of $f(x) = x^2 - 6x + 14$ is :
   (a) $-6$ (b) 5 (c) 9 (d) 14

5. If $f(x) = (a+4)x^2 + x^2 - 5x + 6$ is a quadratic function, then $a$ equals :
   (a) $-4$ (b) $-5$ (c) 5 (d) 6

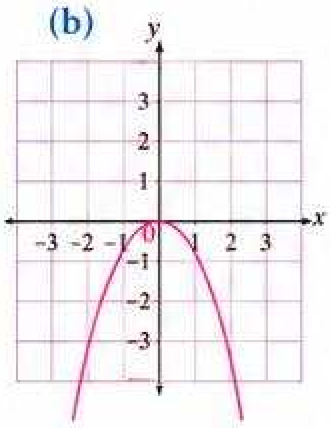
6. If $f(x) = ax^2 + 12x + 7$ has a maximum value, then $a$ equals :
   (a) 3 (b) 2 (c) $-2$ (d) $-3$

7. Which of the following represents the function $f(x) = x^2 - 2x - 3$ ?

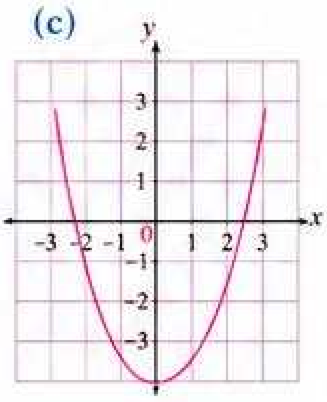
(a)



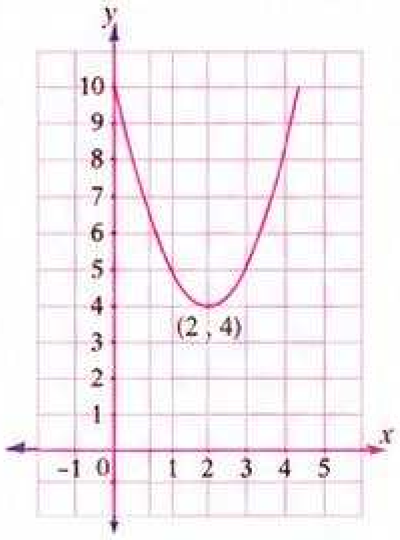
(b)



(c)



8. If the graph opposite is the graph of $f(x)$, which of the following is $f(x)$ ?
   (a) $f(x) = (x+2)^2 + 4$
   (b) $f(x) = (x+4)^2 + 2$
   (c) $f(x) = (x-2)^2 + 4$
   (d) $f(x) = (x-4)^2 + 2$




### B Complete the following :

1. The general form of a quadratic function is
   $f(x) =$ .............. + ........ $x$ + ..........., $a \neq 0$
2. The axis of symmetry of $f(x) = x^2 - 4x + 7$ is
   $x =$ ............
3. The vertex of $f(x) = 2x^2 + 8x + 1$ is ( ........ , ........ )
4. The minimum value of $f(x) = x^2 + 4x - 3$ equals ........
5. The graph of $f(x) = -(x-1)^2 + 4$ intersects the $y$-axis at the point ( ........ , ........ )
6. If the vertex of a parabola is $(3, -2)$, then the axis of symmetry is $x =$ ............

### C Write true (T) or false (F) :

1. The graph of a quadratic function is always a parabola. ( ........ )
2. The axis of symmetry of any parabola is horizontal. ( ........ )
3. If $a > 0$ in $f(x) = ax^2 + bx + c$, then the parabola opens upward. ( ........ )
4. The vertex is the highest point of the parabola when $a > 0$. ( ........ )
5. The graph of $f(x) = x^2 + 2x + 1$ touches the $x$-axis at one point. ( ........ )

### D Solve the following :

1. Find the vertex and the axis of symmetry of $f(x) = x^2 - 6x + 5$.
   ..............................................................................
2. Find the minimum value of $f(x) = x^2 + 4x - 1$.
   ..............................................................................
3. Find the value of $k$ if the vertex of $f(x) = x^2 + kx + 3$ is $(-2, -1)$.
   ..............................................................................
4. Find the $x$-intercepts of $f(x) = x^2 - 5x + 6$.
   ..............................................................................
5. Find the $y$-intercept of $f(x) = -2x^2 + 4x - 1$.
   ..............................................................................
6. Sketch the graph of $f(x) = x^2 - 2x - 3$.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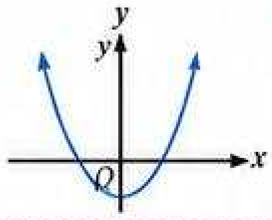
# Lesson 1 Quadratic Function




## Exercises

### A Choose the correct answer :

① The vertex of the parabola $f(x) = -2x^2 + 8x - 3$ is :

(a) $(2, 5)$ (b) $(2, 3)$ (c) $(-2, 5)$ (d) $(2, -3)$

② The axis of symmetry of $f(x) = 3x^2 - 6x + 2$ is :

(a) $x = -1$ (b) $x = 1$ (c) $x = -3$ (d) $x = 3$

③ The graph of $f(x) = x^2 + 2x - 3$ intersects the $x$-axis at :

(a) $(-1, 0)$ and $(3, 0)$ (b) $(1, 0)$ and $(-3, 0)$

(c) $(0, -3)$ and $(0, 3)$ (d) $(-1, 0)$ and $(-3, 0)$

④ If the vertex of a parabola is $(4, -1)$, then the minimum value of the function equals :

(a) $-1$ (b) $1$ (c) $4$ (d) $-4$

⑤ If $f(x) = ax^2 + 4x + c$ has a minimum value, then $a$ is :

(a) $> 0$ (b) $< 0$ (c) $= 0$ (d) any real number

⑥ The graph of $f(x) = -x^2 + 4$ intersects the $y$-axis at :

(a) $(0, 0)$ (b) $(0, 4)$ (c) $(0, -4)$ (d) $(4, 0)$

⑦ Which of the following functions has vertex $(1, 3)$ ?

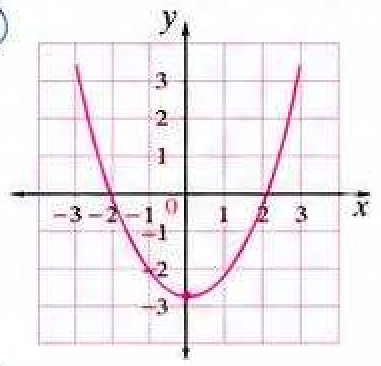
(a) $f(x) = (x-1)^2 + 3$ (b) $f(x) = (x+1)^2 + 3$

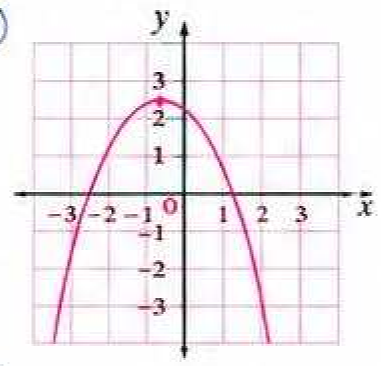
(c) $f(x) = (x-1)^2 - 3$ (d) $f(x) = (x+1)^2 - 3$

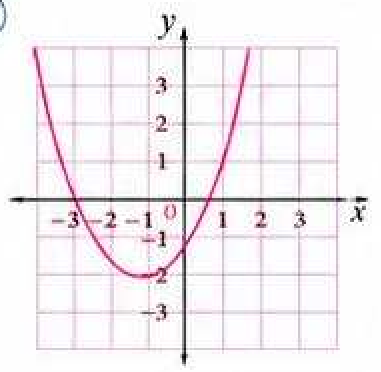
### D In each figure, state :

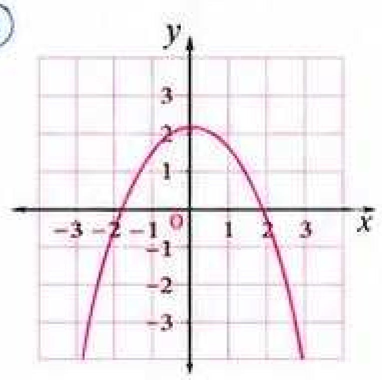
(1) The vertex (2) The axis of symmetry

(3) The $x$-intercepts (4) The $y$-intercept

①



②



③



④



### B Complete the following :

① The general form of a quadratic function is

$f(x) =$ .......... $x^2 +$ .......... $x +$ .......... , $a \neq 0$

② The axis of symmetry of $f(x) = 2x^2 + 10x + 7$ is $x =$ ........

③ The vertex of $f(x) = -x^2 + 6x - 2$ is ( ........ , ........ )

④ The maximum value of $f(x) = -2x^2 + 4x + 1$ equals ........

⑤ If $f(x) = x^2 - 4x + c$ touches the $x$-axis at one point, then $c =$ .........

⑥ The axis of symmetry of $f(x) = ax^2 + bx + c$ is $x =$ ...........

### C Match each function with its vertex :

| | |
|---|---|
| 1) $f(x) = x^2 - 4x + 1$ | (a) $(3, -2)$ |
| 2) $f(x) = -x^2 + 6x - 5$ | (b) $(-2, 3)$ |
| 3) $f(x) = (x+2)^2 + 3$ | (c) $(2, -4)$ |
| 4) $f(x) = (x-3)^2 - 2$ | (d) $(3, 4)$ |

### E Solve the following word problems :

① A ball is thrown upward from the top of a tower 16 m high with an initial velocity of 8 m/s. The height of the ball above the ground after $t$ seconds is given by $h(t) = -4t^2 + 8t + 16$.

(a) After how many seconds does the ball reach its maximum height ?

(b) What is the maximum height reached by the ball ?

(c) After how many seconds does the ball hit the ground ?

② A bridge has the shape of a parabola. The height of the bridge above the river is given by $h(x) = -x^2 + 6x$, where $x$ is the horizontal distance in meters from one bank.

(a) What is the maximum height of the bridge ?

(b) At what distance(s) from the first bank does the bridge touch the water surface ?

③ The profit (in hundred pounds) from selling $x$ items is given by $P(x) = -x^2 + 10x - 9$.

(a) How many items should be sold to get maximum profit ?

(b) What is the maximum profit ?

(c) For what values of $x$ is the profit zero ?



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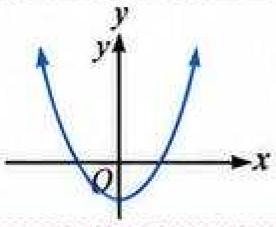
# Lesson 1 Quadratic Function




## Exercises

### A Fill in the blanks :

1. The general form of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq$ .........
2. The graph of $f(x) = 3x^2 - 2x + 1$ opens ........ because $a >$ .......
3. The axis of symmetry of $f(x) = x^2 + 6x - 5$ is $x =$ .........
4. The vertex of a downward parabola is the ........... point.
5. The maximum value of $f(x) = -x^2 + 2x + 4$ is .........
6. The graph of $f(x)$ does not intersect the $x$-axis if its discriminant $b^2 - 4ac$ is .........

### B Put (✓) next to each correct statement :

1. A quadratic function has exactly two $x$-intercepts. ☐
2. The axis of symmetry is a vertical line. ☐
3. If $a < 0$, the parabola opens downward. ☐
4. The vertex lies on the axis of symmetry. ☐
5. If the vertex is $(h, k)$, then the axis of symmetry is $y = h$. ☐
6. The graph of $f(x) = x^2 + 4x + 4$ touches the $x$-axis at one point. ☐

### C Match Column (A) with Column (B) :

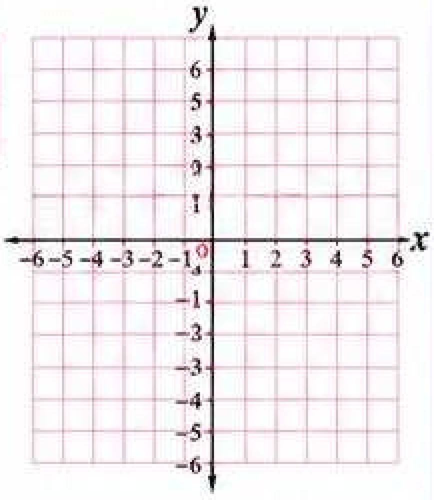
| Column (A) | Column (B) |
|---|---|
| 1. $f(x) = (x-2)^2 + 3$ | a. Vertex $= (2, 3)$ |
| 2. $f(x) = -(x+1)^2 + 4$ | b. Axis of symmetry: $x = -1$ |
| 3. $f(x) = x^2 - 4x$ | c. $x$-intercepts: $(0, 0)$ and $(4, 0)$ |
| 4. $f(x) = x^2 + 6x + 8$ | d. Maximum value $= 4$ |
| 5. $f(x) = -x^2 + 5x - 6$ | e. $x$-intercepts: $(2, 0)$ and $(3, 0)$ |

### D For the function $f(x) = x^2 - 4x + 3$:

1. Complete the table :

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

2. Use the table to :
   - (a) Determine the vertex.
   - (b) Find the axis of symmetry.
   - (c) Find the $x$-intercepts.
   - (d) Sketch the graph of the function.




### E State whether each function has:

(i) two real roots (ii) one real root (iii) no real roots

by using the discriminant $b^2 - 4ac$.

1. $f(x) = x^2 - 5x + 6$ ....................
2. $f(x) = 2x^2 + 4x + 5$ ....................
3. $f(x) = x^2 + 2x + 1$ ....................
4. $f(x) = x^2 - 6x + 9$ ....................
5. $f(x) = 3x^2 + x - 4$ ....................

### F Solve the following problems :

1. A rectangular garden has a length 4 m more than its width. If the area of the garden is 77 $m^2$, write a quadratic equation to find the width of the garden, then solve it. (**Hint**: Area = length × width)
2. A stone is dropped from the top of a cliff 20 m high. The height of the stone above the ground after $t$ seconds is given by $h(t) = -5t^2 + 20$.
   - (a) After how many seconds does the stone hit the ground ?
   - (b) What is the maximum height reached by the stone above the ground ?
3. The profit (in thousands of pounds) from selling $x$ items of a product is given by $P(x) = -x^2 + 12x - 20$.
   - (a) How many items should be sold to get maximum profit ?
   - (b) What is the maximum profit ?
   - (c) For what values of $x$ is the profit zero ?
4. A ball is thrown downward from the top of a building 80 m high with an initial speed of 10 m/s. The height of the ball above the ground after $t$ seconds is given by $h(t) = -5t^2 - 10t + 80$.
   - (a) After how many seconds does the ball reach the ground ?
   - (b) What is the height of the ball after 2 seconds ?
5. Find a quadratic function $f(x)$ that satisfies the following conditions:
   - The vertex is $(2, -3)$.
   - The graph passes through the point $(0, 5)$.



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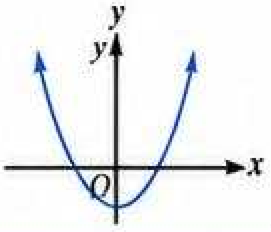
# Lesson 1 Quadratic Function



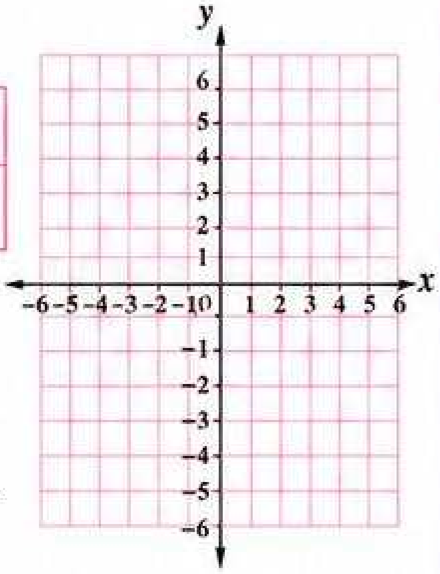

## D For each function, complete the table, then:

(a) Determine the vertex.
(b) Find the axis of symmetry.
(c) Find the $x$-intercepts.
(d) Sketch the graph of the function.

**1** $f(x) = x^2 - 4x + 3$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

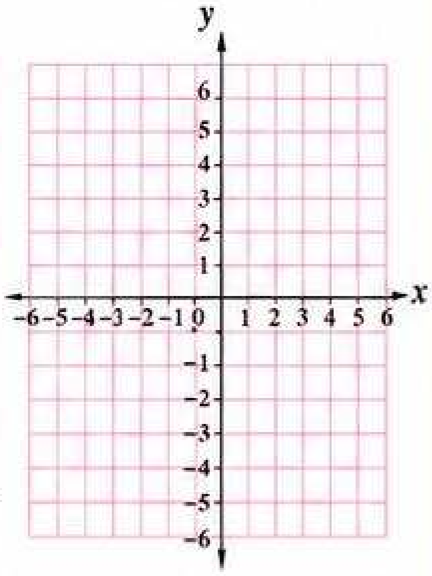
(a) Vertex = ........................
(b) Axis of symmetry: $x$ = ...........
(c) $x$-intercepts: ...........................
(d) Sketch the graph.



**2** $f(x) = x^2 + 2x - 8$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

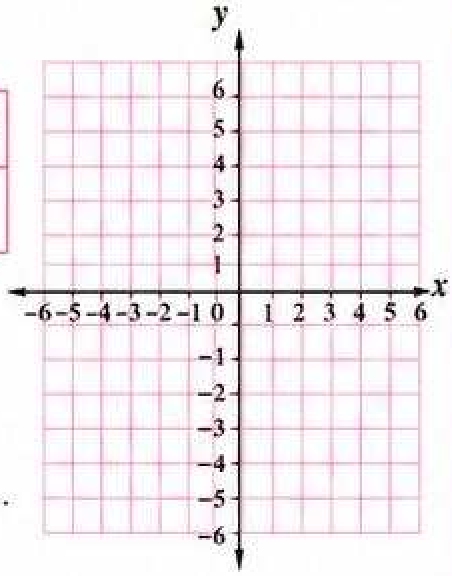
(a) Vertex = ........................
(b) Axis of symmetry: $x$ = ...........
(c) $x$-intercepts: ...........................
(d) Sketch the graph.



**3** $f(x) = -x^2 + 6x + 5$

| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

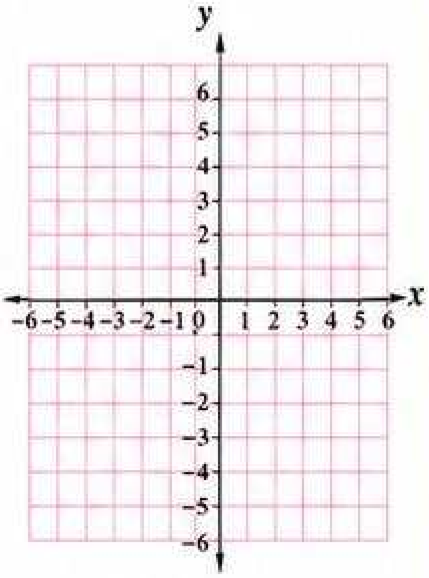
(a) Vertex = ........................
(b) Axis of symmetry: $x$ = ...........
(c) $x$-intercepts: ...........................
(d) Sketch the graph.



**4** $f(x) = 2x^2 - 4x - 6$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

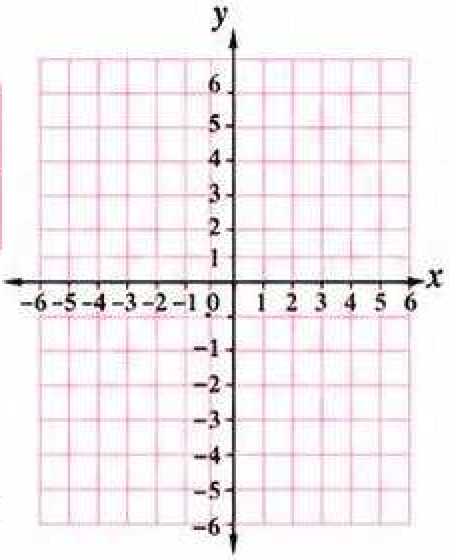
(a) Vertex = ........................
(b) Axis of symmetry: $x$ = ...........
(c) $x$-intercepts: ...........................
(d) Sketch the graph.



**5** $f(x) = x^2 - 2x - 3$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

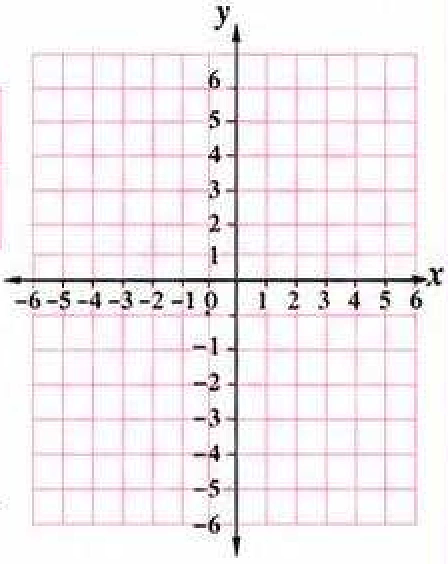
(a) Vertex = ........................
(b) Axis of symmetry: $x$ = ...........
(c) $x$-intercepts: ...........................
(d) Sketch the graph.



**6** $f(x) = -x^2 - 4x + 4$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | |

(a) Vertex = ........................
(b) Axis of symmetry: $x$ = ...........
(c) $x$-intercepts: ...........................
(d) Sketch the graph.





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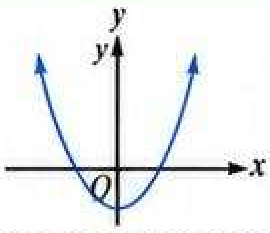


## D For each function, complete the table, then:

(a) Determine the vertex.

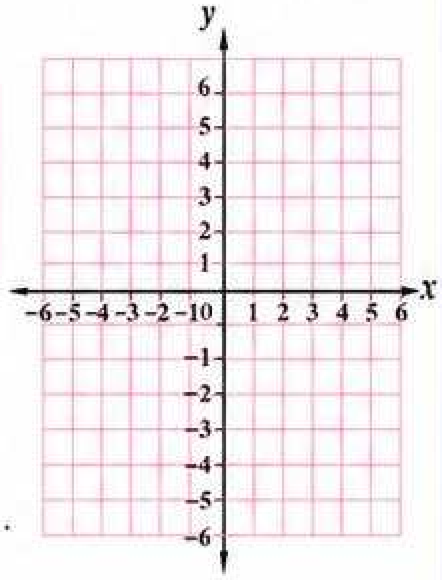
(b) Find the axis of symmetry.

(c) Find the $x$-intercepts.

(d) Sketch the graph of the function.

**1** $f(x) = x^2 + 4x - 5$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | |



(a) Vertex = ........................

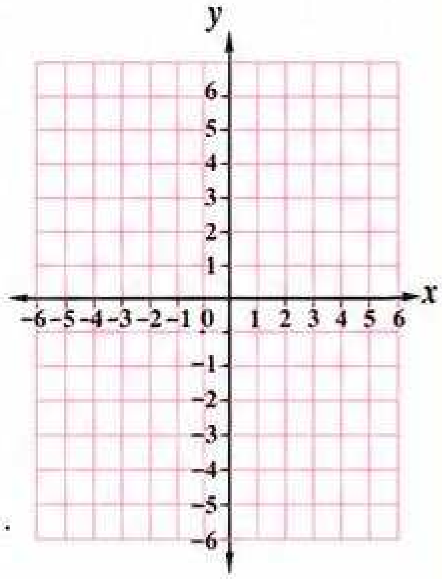
(b) Axis of symmetry: $x$ = ..........

(c) $x$-intercepts:...........................

(d) Sketch the graph.

**2** $f(x) = -x^2 + 2x + 3$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $3$ |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |



(a) Vertex = ........................

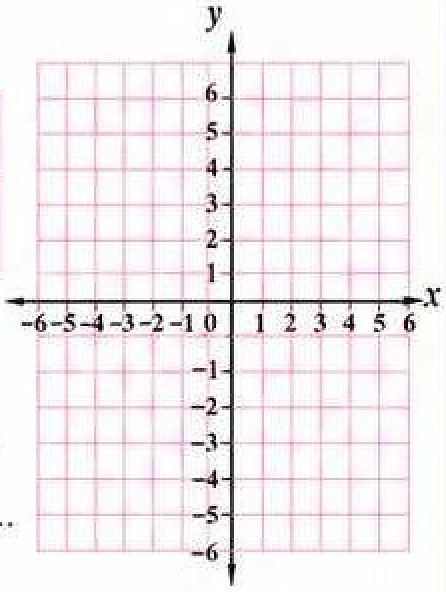
(b) Axis of symmetry: $x$ = ..........

(c) $x$-intercepts:...........................

(d) Sketch the graph.

**3** $f(x) = 2x^2 - 8x + 6$

| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $5$ |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |



(a) Vertex = ........................

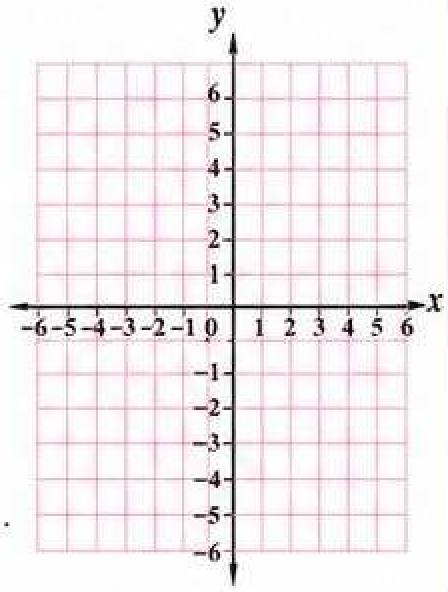
(b) Axis of symmetry: $x$ = ..........

(c) $x$-intercepts:...........................

(d) Sketch the graph.

**4** $f(x) = -x^2 - 6x + 8$

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $2$ |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |



(a) Vertex = ........................

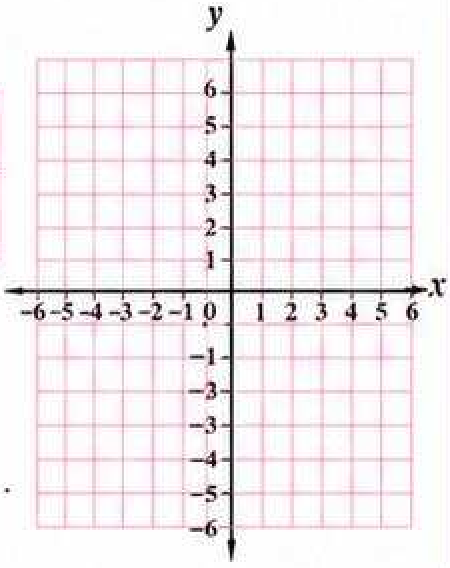
(b) Axis of symmetry: $x$ = ..........

(c) $x$-intercepts:...........................

(d) Sketch the graph.

**5** $f(x) = x^2 - 6x + 2$

| $x$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | |



(a) Vertex = ........................

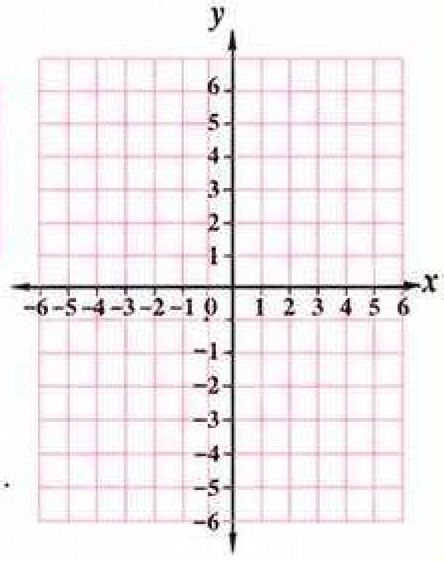
(b) Axis of symmetry: $x$ = ..........

(c) $x$-intercepts:...........................

(d) Sketch the graph.

**6** $f(x) = -2x^2 + 4x + 1$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | |



(a) Vertex = ........................

(b) Axis of symmetry: $x$ = ..........

(c) $x$-intercepts:...........................

(d) Sketch the graph.



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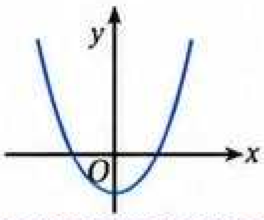
# Lesson 1 Quadratic Function




## Exercises

**A** For each quadratic function, find the axis of symmetry, the vertex, the value of $f(0)$, and the minimum or maximum value.

① $f(x) = x^2 - 6x + 8$

② $f(x) = 2x^2 + 4x - 3$

③ $f(x) = -x^2 + 8x - 5$

④ $f(x) = x^2 + 10x + 21$

⑤ $f(x) = -2x^2 + 12x - 7$

⑥ $f(x) = 3x^2 - 9x + 2$

**B** For each quadratic function, find the vertex, the axis of symmetry, and the minimum or maximum value.

① $f(x) = x^2 + 4x - 1$

② $f(x) = x^2 - 2x - 8$

③ $f(x) = 2x^2 - 8x + 6$

④ $f(x) = 3x^2 + 6x - 9$

⑤ $f(x) = -x^2 - 6x + 2$

⑥ $f(x) = -2x^2 + 4x + 5$

**C** Find the value of $k$ in each case so that the graph of $f(x)$ has the given property.

① $f(x) = x^2 + kx + 3$ has vertex at $(1, 2)$

② $f(x) = x^2 + kx - 5$ has vertex on the $y$ axis

③ $f(x) = 2x^2 + kx + 1$ has axis of symmetry $x = -2$

④ $f(x) = -x^2 + kx + 4$ has vertex at $(3, 7)$

⑤ $f(x) = 3x^2 + kx - 2$ its minimum value is $-5$

⑥ $f(x) = -2x^2 + kx + 1$ has maximum value $6$

**D** For each quadratic function, find the values of $x$ for which $f(x) = 0$.

① $f(x) = x^2 - 5x + 6$

② $f(x) = x^2 + x - 12$

③ $f(x) = 2x^2 - 9x + 4$

④ $f(x) = x^2 + 4x + 3$

⑤ $f(x) = 3x^2 - 7x + 2$

⑥ $f(x) = -x^2 + 6x - 5$

**E** If the quadratic function $f(x) = ax^2 + bx + c$ has the given vertex and passes through the given point, find the values of $a$, $b$ and $c$.

① Vertex $(2, -3)$ and passes through $(0, 5)$

② Vertex $(-1, 4)$ and passes through $(1, 1)$

③ Vertex $(3, 2)$ and passes through $(0, -7)$

④ Vertex $(-2, -1)$ and passes through $(2, 7)$

⑤ Vertex $(1, 0)$ and passes through $(0, 3)$

⑥ Vertex $(4, 5)$ and passes through $(2, 9)$

**F** Determine whether each quadratic function opens upward or downward.

① $f(x) = 2x^2 + 3x - 1$

② $f(x) = -4x^2 + x + 7$

③ $f(x) = \frac{1}{2}x^2 - 5x + 2$

④ $f(x) = -3x^2 - 6x + 1$

⑤ $f(x) = 5x^2 - 8x - 3$

⑥ $f(x) = -\frac{1}{3}x^2 + 2x + 4$

**G** For each pair of quadratic functions, write whether they represent the same parabola or not.

① $f(x) = x^2 - 4x + 7$ and $g(x) = (x-2)^2 + 3$

② $f(x) = 2x^2 + 4x - 1$ and $g(x) = 2(x+1)^2 - 3$

③ $f(x) = -x^2 + 6x - 2$ and $g(x) = -(x-3)^2 + 7$

④ $f(x) = 3x^2 - 12x + 9$ and $g(x) = 3(x-2)^2 - 3$

⑤ $f(x) = x^2 + 2x$ and $g(x) = (x+1)^2 - 1$

⑥ $f(x) = 2x^2 - 8x + 10$ and $g(x) = 2(x-2)^2 + 2$



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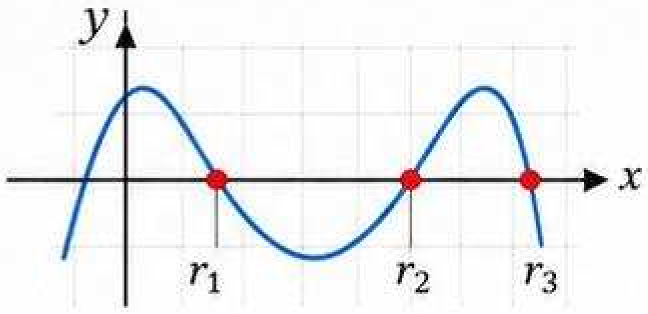
# Set of Real Zeros of Polynomial Functions




## Introduction

In mathematics and real life, many problems can be expressed by polynomial functions such as $f(x) = x^3 - 3x$, $f(x) = x^2 - 5$, … etc. The values of $x$ that make the function equal to zero are called the **real zeros** of the polynomial function.

**Example:** In the function $f(x) = x^2 - 4$, the real zeros are the values of $x$ that make $f(x) = 0$.

## Learning Outcomes

**After studying this lesson, you will be able to:**

- Define the set of real zeros of a polynomial function.
- Determine the real zeros of a polynomial function.
- Find the number of real zeros of a polynomial function up to degree 4.
- Find the sum and product of the real zeros.
- Verify real zeros by substitution.
- Solve polynomial equations.

## Key Concept

The real zeros of a polynomial function $f(x)$ are the real numbers $r$ that make $f(r) = 0$.
The set of real zeros of $f(x)$ is denoted by $Z(f)$.
If $r$ is a real zero, then $(x - r)$ is a factor of $f(x)$.
The **degree** of $f(x)$ is the highest power of $x$.
A polynomial of degree $n$ has at most $n$ real zeros.

**Notation**

| | | |
|---|---|---|
| $f(x) = 0$ | → | Equation form |
| $Z(f)$ | → | Set of real zeros of $f(x)$ |

## Think!

If $f(x) = x^2 - 9$, what are the real zeros?

Try it before looking at the examples.

## EXAMPLES

### Example 1

Find the set of real zeros of $f(x) = x - 5$.

Solution:

$f(x) = 0 \Rightarrow x - 5 = 0$
$\Rightarrow x = 5$

$\therefore Z(f) = \{5\}$ ✓

### Example 2

Find the set of real zeros of $f(x) = x^2 - 9$.

Solution:

$f(x) = 0 \Rightarrow x^2 - 9 = 0$
$\Rightarrow (x - 3)(x + 3) = 0$
$\Rightarrow x = 3$ or $x = -3$

$\therefore Z(f) = \{-3, 3\}$ ✓

### Example 3

Find the set of real zeros of $f(x) = x^2 + 4$.

Solution:

$f(x) = 0 \Rightarrow x^2 + 4 = 0$
$\Rightarrow x^2 = -4$

No real solution

$\therefore Z(f) = \varnothing$ ✓

### Example 4

Find the set of real zeros of $f(x) = 2x^2 - 8$.

Solution:

$f(x) = 0 \Rightarrow 2x^2 - 8 = 0$
$\Rightarrow 2(x^2 - 4) = 0$
$\Rightarrow 2(x - 2)(x + 2) = 0$
$\Rightarrow x = 2$ or $x = -2$

$\therefore Z(f) = \{-2, 2\}$ ✓

## Attention

- A zero of $f(x)$ must satisfy $f(r) = 0$.
- Do not confuse zeros with factors.
- Check your solutions by substitution.

## Quick Comparison

| Function $f(x)$ | Equation $f(x) = 0$ | Set of real zeros $Z(f)$ |
|---|---|---|
| $x - 5$ | $x - 5 = 0$ | $\{5\}$ |
| $x^2 - 9$ | $x^2 - 9 = 0$ | $\{-3, 3\}$ |
| $x^2 + 4$ | $x^2 + 4 = 0$ | $\varnothing$ |
| $2x^2 - 8$ | $2x^2 - 8 = 0$ | $\{-2, 2\}$ |

## Key Notes

1. The equation $f(x) = 0$ is called a polynomial equation.
2. The real zeros are the solutions of the polynomial equation.
3. The number of real zeros is at most the degree of $f(x)$.



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## Properties of Real Zeros

If $r_1, r_2, \ldots, r_n$ are the real zeros of a polynomial $f(x)$ of degree $n$, then:

1. The number of real zeros (counting multiplicity) is at most $n$.
2. If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with $a_n \neq 0$, then the sum of the real zeros is $-\frac{a_{n-1}}{a_n}$.
3. The product of the real zeros is $(-1)^n \frac{a_0}{a_n}$.
4. If $r$ is a real zero, then $(x - r)$ is a factor of $f(x)$.

**Note!**

Some polynomials of degree $n$ may have fewer than $n$ real zeros. The remaining zeros are complex (non-real).

**Study Tip**

To check your answers, use substitution: replace $x$ by the zero in the original function. If $f(r) = 0$, your answer is correct.

## Using Sum and Product

**Example 5**

The sum of the real zeros of $f(x) = x^2 - 7x + 10$ is 7. If one of the real zeros is 2, find the other zero.

Solution:

- For $f(x) = x^2 - 7x + 10$, $a = 1$, $b = -7$, $c = 10$
- Sum of zeros $= -\frac{b}{a} = -\frac{-7}{1} = 7$

Let the other zero be $r$.

Then $2 + r = 7 \Rightarrow r = 5$.

**Example 6**

The product of the real zeros of $f(x) = x^2 + 3x - 10$ is $-10$. If one of the real zeros is $-2$, find the other zero.

Solution:

- For $f(x) = x^2 + 3x - 10$, $a = 1$, $c = -10$
- Product of zeros $= (-1)^2 \frac{c}{a} = \frac{-10}{1} = -10$

Let the other zero be $r$.

Then $(-2)\cdot r = -10 \Rightarrow r = 5$.

## More Solved Examples

**Example 7**

Find the set of real zeros of $f(x) = x^3 - 6x^2 + 11x - 6$.

Solution:

$f(x) = x^3 - 6x^2 + 11x - 6$
$= (x-1)(x-2)(x-3)$

$x - 1 = 0 \Rightarrow x = 1$
$x - 2 = 0 \Rightarrow x = 2$
$x - 3 = 0 \Rightarrow x = 3$

$\therefore Z(f) = \{1, 2, 3\}$

**Example 8**

Find the set of real zeros of $f(x) = x^3 + x^2 - 4x - 4$.

Solution:

$f(x) = x^3 + x^2 - 4x - 4$
$= (x^2(x+1) - 4(x+1)$
$= (x^2 - 4)(x+1)$
$= (x-2)(x+2)(x+1)$

$x - 2 = 0 \Rightarrow x = 2$
$x + 2 = 0 \Rightarrow x = -2$
$x + 1 = 0 \Rightarrow x = -1$

$\therefore Z(f) = \{-2, -1, 2\}$

**Example 9**

Find the set of real zeros of $f(x) = 2x^3 - 2x$.

Solution:

$f(x) = 2x^3 - 2x$
$= 2x(x^2 - 1)$
$= 2x(x-1)(x+1)$

$2x = 0 \Rightarrow x = 0$
$x - 1 = 0 \Rightarrow x = 1$
$x + 1 = 0 \Rightarrow x = -1$

$\therefore Z(f) = \{-1, 0, 1\}$

**Example 10**

Find the set of real zeros of $f(x) = x^4 - 5x^2 + 4$.

Solution:

Let $y = x^2$.

$y^2 - 5y + 4 = 0$
$(y-1)(y-4) = 0$
$y = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$
$y = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$\therefore Z(f) = \{-2, -1, 1, 2\}$

## Try It Yourself

1. Find the set of real zeros of $f(x) = x - 4$.

.......................

2. Find the set of real zeros of $f(x) = x^2 - 1$.

.......................

3. Find the set of real zeros of $f(x) = x^2 + 6x + 8$.

.......................

4. Find the set of real zeros of $f(x) = x^3 - 3x^2$.

.......................

5. If the sum of the real zeros of $f(x) = x^2 - 5x + 6$ is 5 and one zero is 2, find the other.

.......................



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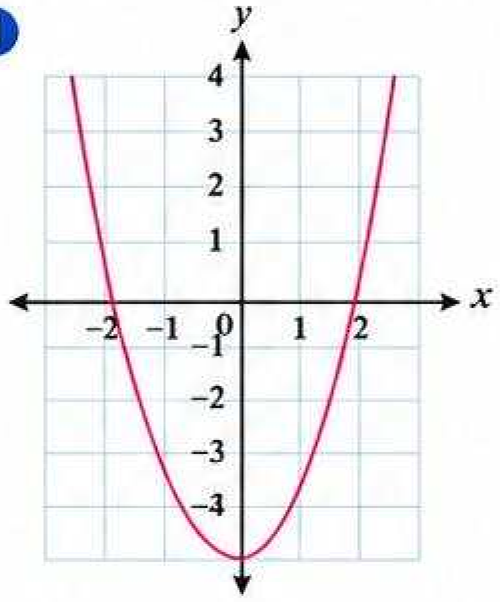
## Real Zeros and Graphs

If a curve cuts the $x$-axis at the point $(a,\ 0)$, then $a$ is a real zero of the function $f$.

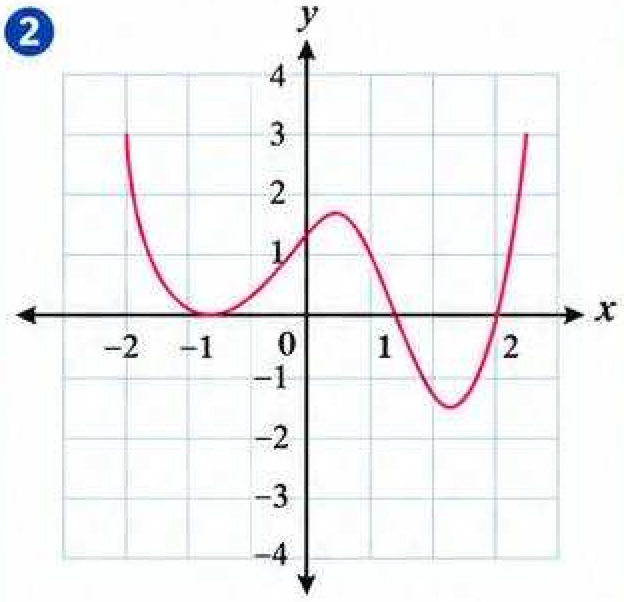
### Example 2

**Find the set of real zeros for each of the following graphs :**

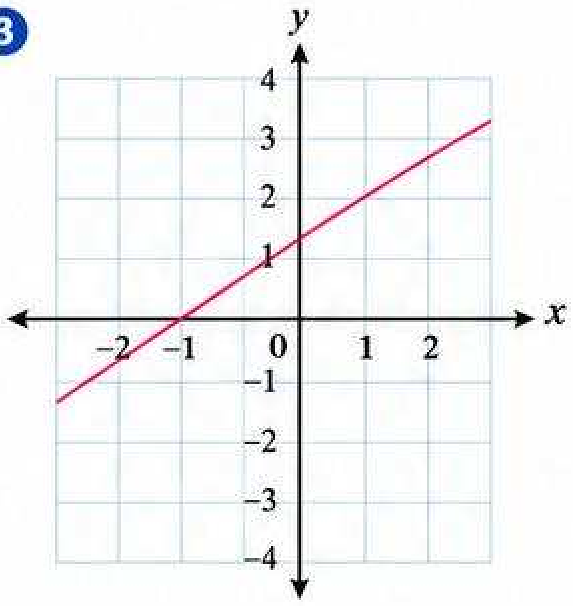
1



2



3



1. The curve cuts the $x$-axis at $x = -2$ and $x = 2$

$\therefore\ Z(f) = \{-2,\ 2\}$

2. The curve cuts the $x$-axis at $x = -1,\ 1,\ 2$

$\therefore\ Z(f) = \{-1,\ 1,\ 2\}$

3. The curve cuts the $x$-axis at $x = -1$

$\therefore\ Z(f) = \{-1\}$

### Example 3

If the degree of a polynomial function is 3 and $f(x) = 2x^2 + bx + 3$.

Find the value of $b$ if $x = -3$ is a real zero of $f$.

**Solution:**

$\therefore\ x = -3$ is a real zero $\quad \therefore f(-3) = 0$

$\therefore 2(-3)^2 + b(-3) + 3 = 0 \quad \therefore\ 18 - 3b + 3 = 0$

$\therefore 21 - 3b = 0 \quad \therefore\ -3b = -21 \quad \therefore b = 7$

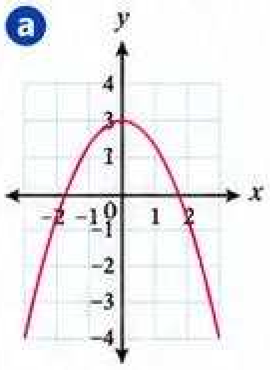
$\therefore f(x) = 2x^2 + 7x + 3$

$\therefore f(4) = 2(4)^2 + 7(4) + 3 = 63$

## Try It Yourself

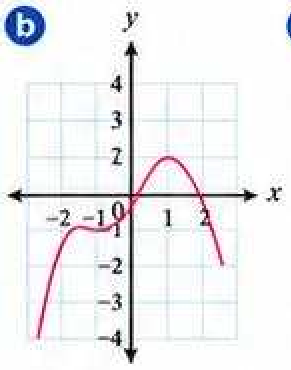
1. Find the set of real zeros for each graph.

a



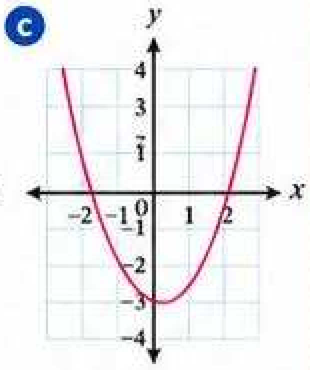
$Z(f) = \{\qquad\}$

b



$Z(f) = \{\qquad\}$

c



$Z(f) = \{\qquad\}$

2. If the degree of $f(x)$ is 3, find the value of $b$.

a) $f(x) = x^3 + 2x^2 + bx - 5$ if $x = 1$ is a real zero.

$b = \dots\dots\dots\dots$

b) $f(x) = 2x^3 - 3x^2 + bx + 1$ if $x = -2$ is a real zero.

$b = \dots\dots\dots\dots$

3. Find $f(4)$ if $f(x) = x^2 + ax + b$ and $f(-2) = 0$.

a) If $a = 3,\ b = 2$

$f(4) = \dots\dots\dots\dots$

b) If $a = -1,\ b = 5$

$f(4) = \dots\dots\dots\dots$

### Key Takeaways

- Real zeros are the solutions of $f(x) = 0$.
- A curve cuts the $x$-axis at the real zeros.
- A polynomial of degree $n$ has at most $n$ real zeros.
- Use substitution to verify zeros.

### Note

A polynomial can have less than $n$ real zeros.

The remaining zeros (if any) are complex (non-real) numbers.



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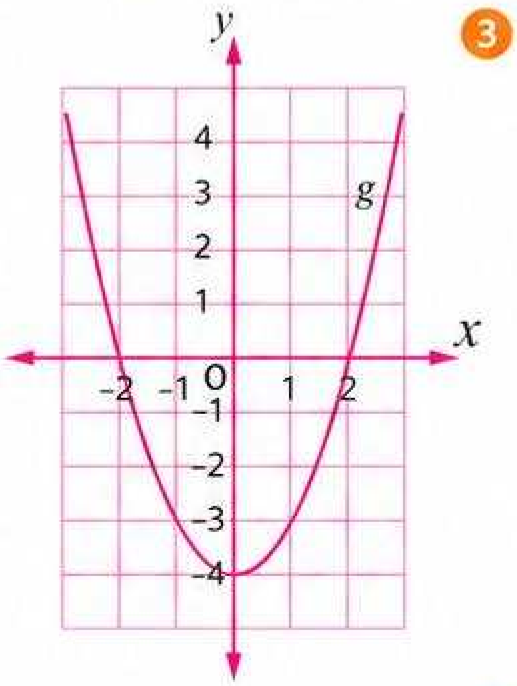
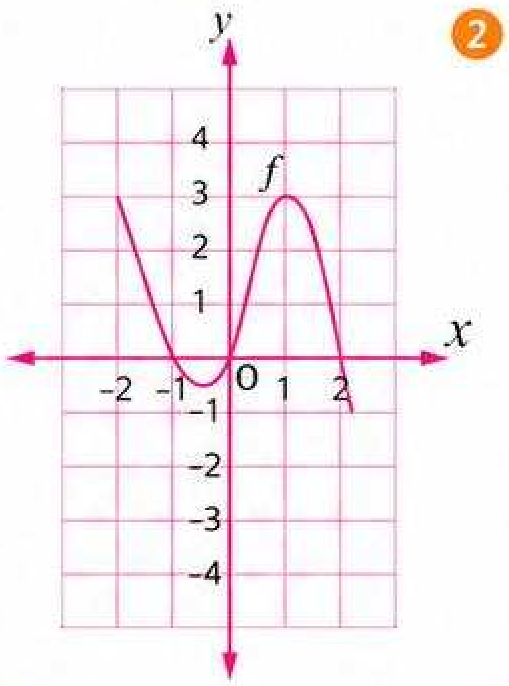
## Real Zeros and Graphs

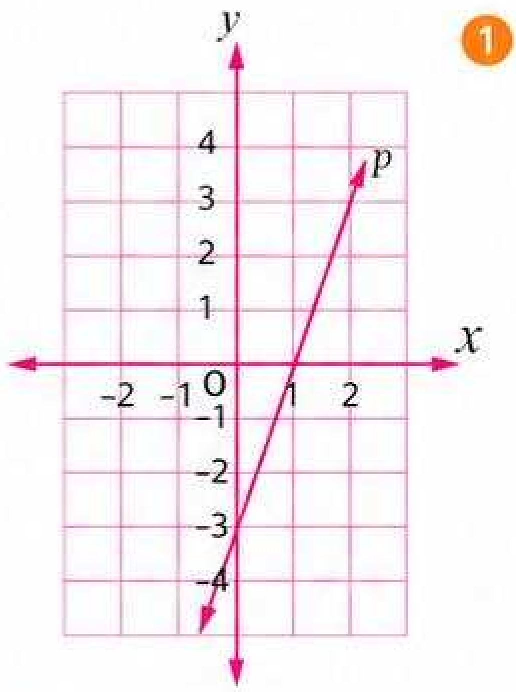
If a curve cuts the $x$-axis at the point $(a,\ 0)$, then $a$ is a real zero of the function $f$.

## Example 2

Find the set of real zeros for each of the following graphs :







③ : The curve $g$ cuts the $x$-axis at $x = -2,\ x = 2$

$\therefore\ Z(g) = \{-2,\ 2\}$

② : The curve $f$ cuts the $x$-axis at $x = -1,\ 0,\ 2$

$\therefore\ Z(f) = \{-1,\ 0,\ 2\}$

① : The curve $p$ cuts the $x$-axis at $x = 1$

$\therefore\ Z(p) = \{1\}$

## Example 3

If the degree of a polynomial function is 3 and $f(x) = 2x^2 + bx + 3$.

Find the value of $b$ if $x = -3$ is a real zero of $f$.

**Solution:**

$\therefore\ x = -3$ is a real zero $\qquad \because\ f(-3) = 0$

$\therefore\ 2(-3)^2 + b(-3) + 3 = 0 \qquad \therefore\ 18 - 3b + 3 = 0$

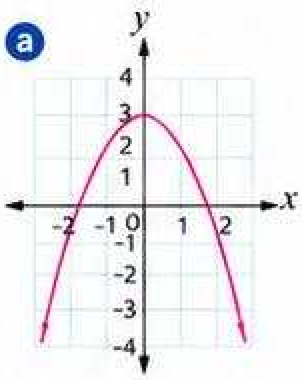
$\therefore\ 21 - 3b = 0 \qquad \therefore\ -3b = -21 \qquad \therefore\ b = 7$

$\therefore\ f(x) = 2x^2 + 7x + 3$

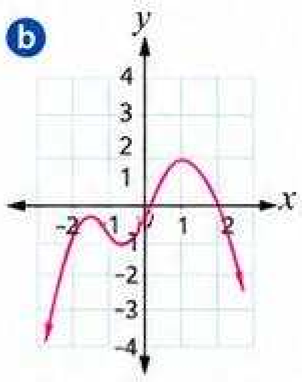
$\therefore\ f(4) = 2(4)^2 + 7(4) + 3 = 63$

## Try It Yourself

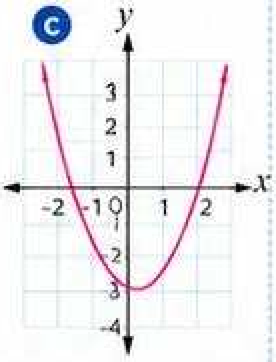
**1** Find the set of real zeros for each of the following graphs.

a 

$Z(f) = \{..........\}$

b 

$Z(f) = \{..........\}$

c 

$Z(f) = \{..........\}$

**2** If the degree of $f(x)$ is 3, find the value of $b$.

a) $f(x) = x^3 + 2x^2 + bx - 5$ if $x = 1$ is a real zero.

$b = ..................$

b) $f(x) = 2x^3 - 3x^2 + bx + 1$ if $x = -2$ is a real zero.

$b = ..................$

**3** Find $f(4)$ if $f(x) = x^2 + ax + b$ and $f(-2) = 0$.

a) If $a = 3,\ b = 2$

$f(4) = ....................$

b) If $a = -1,\ b = 5$

$f(4) = ....................$

### Key Takeaways

- A real zero is a value of $x$ that makes $f(x) = 0$.
- A curve cuts the $x$-axis at the real zeros.
- A polynomial of degree $n$ has at most $n$ real zeros.
- Verify your zeros by substitution.

### Note

- Real zeros may be positive, negative, or zero.
- Some polynomials have less than $n$ real zeros.
- The remaining zeros (if any) are complex numbers.



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## Using Substitution to Find Real Zeros

To check if a number $r$ is a real zero of $f(x)$, substitute $x = r$ in $f(x)$.
If $f(r) = 0$, then $r$ is a real zero.

### Example 4

Check whether $x = 2$ is a real zero of $f(x) = x^3 - 4x + 6$.

**Solution:**

Substitute $x = 2$ in $f(x)$.

$f(2) = (2)^3 - 4(2) + 6$

$= 8 - 8 + 6$

$= 6 \neq 0$

$\therefore x = 2$ is **not** a real zero of $f(x)$. ✗

### Example 5

Check whether $x = -1$ is a real zero of $f(x) = x^3 + 2x^2 - x - 2$.

**Solution:**

Substitute $x = -1$ in $f(x)$.

$f(-1) = (-1)^3 + 2(-1)^2 - (-1) - 2$

$= -1 + 2 + 1 - 2$

$= 0$

$\therefore x = -1$ is a real zero of $f(x)$. ✓

**Think!**

If $x - 3$ is a factor of $f(x)$, what is one real zero of $f(x)$?

**Attention**

- Always substitute in the original function.
- A zero makes the function equal to 0, not the factors.
- Double-check your answers.

## More Solved Examples

### Example 6

Find the set of real zeros of $f(x) = x^2 - 5x + 6$.

**Solution:**

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ or $x = 3$

$\therefore Z(f) = \{2, 3\}$ ✓

### Example 7

Find the set of real zeros of $f(x) = x^3 - x^2 - 6x$.

**Solution:**

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x(x - 3)(x + 2) = 0$

$x = 0$ or $x - 3 = 0$ or $x + 2 = 0$

$x = 0$ or $x = 3$ or $x = -2$

$\therefore Z(f) = \{-2, 0, 3\}$ ✓

### Example 8

Find the set of real zeros of $f(x) = x^4 - 5x^2 + 4$.

**Solution:**

Let $y = x^2$.

$y^2 - 5y + 4 = 0$

$(y - 1)(y - 4) = 0$

$y = 1$ or $y = 4$

$x^2 = 1$ or $x^2 = 4$

$x = \pm 1$ or $x = \pm 2$

$\therefore Z(f) = \{-2, -1, 1, 2\}$ ✓

### Example 9

Find the set of real zeros of $f(x) = 3x^3 + 9x^2 - 12x$.

**Solution:**

$3x^3 + 9x^2 - 12x = 0$

$3x(x^2 + 3x - 4) = 0$

$3x(x + 4)(x - 1) = 0$

$x = 0$ or $x = -4$ or $x = 1$

$\therefore Z(f) = \{-4, 0, 1\}$ ✓

## Try It Yourself

**1** Find the set of real zeros of each polynomial.

a) $f(x) = x^2 - 7x + 12$ ..................

b) $f(x) = x^3 + 3x^2 - 4x$ ..................

c) $f(x) = x^4 - 2x^2 - 3$ ..................

d) $f(x) = x^3 - 6x^2 + 11x - 6$ ..................

**2** Check whether each given number is a real zero of the function $f(x)$.

a) $f(x) = x^2 - 4x + 3$, $r = 1$

b) $f(x) = x^3 - x^2 - 2x$, $r = 2$

c) $f(x) = x^2 + 2x + 1$, $r = -1$

d) $f(x) = x^3 + x - 6$, $r = 2$

**3** If the following numbers are real zeros of $f(x)$, find the remaining zeros.

a) The zeros are 2 and 5, and $f(x) = x^3 - 7x^2 + 10x$.
The remaining zero: ..........................

b) The zeros are 0 and $-3$, and $f(x) = x^3 + 3x^2 - 9x$.
The remaining zero: ..........................

c) The zeros are 1 and $-2$, and $f(x) = x^3 - x^2 - 2x$.
The remaining zero: ..........................

## Key Summary

- Real zeros are the solutions of $f(x) = 0$.
- A curve cuts the $x$-axis at the real zeros.
- A polynomial of degree $n$ has at *most* $n$ real zeros.
- Use factoring, substitution, or graphs to find zeros.

## Homework

Solve the exercises 1 to 3 in the "Try It Yourself" section in your notebook.



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# Lesson Review

## First Measure Concepts

**Choose the correct answer from the given choices :**

1. What is the set of real zeros of the function $f(x) = -3x$ ?

   (a) $\{3\}$ (b) $\{-3\}$ (c) $\{0\}$ (d) $R$

2. What is the set of real zeros of the function $p(x) = x^2 + 25$ ?

   (a) $\{5 - 5\}$ (b) $\{5\}$ (c) $\{0\}$ (d) $\varnothing$

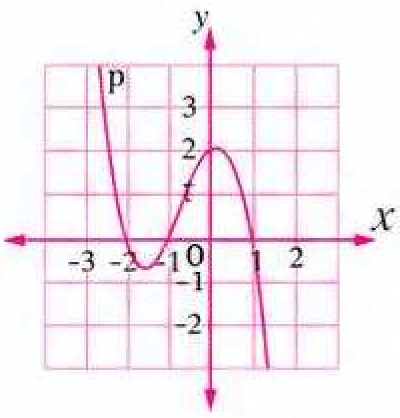
3. What are the real zeros of the function $f$ shown in the figure?

   (a) $2,\ 0$

   (b) $-1,\ 0,\ 1$

   (c) $-2,\ 2,\ 1$

   (d) $-2,\ -1,\ 1$




4. Which of the following could be the set of real zeros?

   (a) $\{-2, 0\}$ (b) $x^2 - 2x$

   (c) $x^3 + 8$ (d) $x^2 + 2x$

5. What is the set of real zeros of $g(x) = 3$ ?

   (a) $\{3\}$ (b) $\{0\}$ (c) $\varnothing$ (d) $R$

6. If one factor of $p(x)$ is $(x + 3)$, which of the following values of $p$ is correct?

   (a) $p(0) = 3$ (b) $p(0) = 1$ (c) $p(-3) = 0$

   (d) $p(3) = 0$

7. If 1 is one real zero of $p(x)$, which of the following is correct?

   (a) $p(0) = 1$ (b) $p(0) = -1$

   (c) $p(1) = 0$ (d) $p(-1) = 0$

## Second Apply Concepts

**Find the set of real zeros for each polynomial :**

8. $g(x) = 4x^2 + 9$

9. $f(x) = x^2 - 36$

10. $f(x) = 2x - 16$

11. $p(x) = x^2 - 3x + 2$

12. $p(x) = x^3 - x$

13. $p(x) = 3x^2 - 12x$

14. If the degree of $f(x) = x^2 + bx - 14$ is 2, find the value of $b$. ..............................

## Third Analyze and Integrate

Travel helps the economy because it promotes our beautiful country.

13. **Tourism:**

The number of daily short film views in a tourism channel is modeled by $p(t) = -3t^2 + 18t + 165$, where $t$ is the number of days since the channel was launched. During which days was the number of views equal to zero?

## Creative Thinking

14. Find the set of real zeros for each polynomial :

   (1) $f(x) = x^3 - 5x^2 - 4x + 20$ (2) $p(x) = 2x^3 - x^2 - 8x + 4$ (3) $g(x) = x^2 - 2x - 6$

**Self-evaluation**

How do you rate your understanding of the set of real zeros?

☐ ☐ ☐ ☐

**Homework**

Answer the exercises from 1 to 7 in the "Measure Concepts" section, and from 8 to 14 in the "Apply Concepts" section in your notebook.



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# Lesson Practice

## 1 Practice and Apply

### A Choose the correct answer :

1. The set of real zeros of $f(x) = 2x - 8$ is :
(a) $\{2\}$ (b) $\{-2\}$ (c) $\{0\}$ (d) $R$

2. The set of real zeros of $p(x) = x^2 - 9$ is :
(a) $\{9\}$ (b) $\{-3, 3\}$ (c) $\{-3\}$ (d) $\varnothing$

3. If $x - 2$ is a factor of $f(x)$, then one real zero of $f(x)$ is :
(a) $0$ (b) $1$ (c) $2$ (d) $-2$

4. The set of real zeros of $g(x) = 0$ is :
(a) $\{0\}$ (b) $\varnothing$ (c) $R$ (d) $\{1\}$

5. If $f(x) = (x-1)(x+2)$, then the set of real zeros is :
(a) $\{1, 2\}$ (b) $\{-2\}$ (c) $\{-2, 1\}$ (d) $R$

6. If $f(-1) = 0$, then $-1$ is a real zero of $f(x)$ because :
(a) $-1$ is in the domain (b) $-1$ makes $f(-1) = 0$
(c) $-1$ is a factor of $f(x)$ (d) $-1$ is positive

7. Which set could be the set of real zeros of a cubic function?
(a) $\{2\}$ (b) $\{-1, 0\}$ (c) $\{-2, 1, 3\}$ (d) $\varnothing$

8. If the graph of a function does not cut the $x$-axis, then :
(a) it has one real zero (b) it has two real zeros
(c) it has no real zeros (d) its set of real zeros is $R$

## 2 Complete the following :

1. If $f(3) = 0$, then ........... is one real zero of $f(x)$.
2. The set of real zeros of $f(x) = x^2 + 4$ is ............
3. If $x + 1$ is a factor of $p(x)$, then $p(..........) = 0$.
4. The set of real zeros of $g(x) = x(x-5)$ is ............
5. If the degree of $f(x)$ is 2, the maximum number of real zeros is ............
6. The set of real zeros of $f(x) = 0$ is ............
7. If $f(x) = x^3 - 2x$, then one real zero is ............

## 3 Solve the following equations :

1. $x^2 - 5x + 6 = 0$
2. $x^2 - 9 = 0$
3. $2x^2 + x - 3 = 0$
4. $x^3 - x^2 - 6x = 0$
5. $x^3 + 2x^2 - x - 2 = 0$
6. $2x^3 - x = 0$
7. $x^2 + 3x + 2 = 0$
8. $x^3 - 4x^2 - x + 4 = 0$
9. $x^2 - 2x - 3 = 0$

## 4 Word Problems

1. A rectangle has length $x + 5$ and width $x - 3$. The area of the rectangle is $x^2 + 2x - 15$. Find the possible values of $x$ that make the area equal to zero.
2. The profit (in dollars) made by a company in $x$ months is given by $P(x) = x^2 - 9$. In which months was there no profit?
3. A square park has an area of $x^2 - 16$ square meters. Find the possible values of $x$ for which the area of the park is zero.

## 5 Challenge Yourself

1. If the degree of $f(x) = ax^3 - bx + 4$ is 3 and $x = 2$ is a real zero of $f(x)$, find the value of $a$ in terms of $b$.
2. If the set of real zeros of $f(x) = x^3 - 6x^2 + 11x - 6$ is $\{1, 2, 3\}$, proove that $f(4) = 24$.
3. Find the set of real zeros of $f(x) = x^4 - 5x^2 + 4$ without factoring.

### Key Takeaways

- A real zero is a value of $x$ that makes $f(x) = 0$.
- A curve cuts the $x$-axis at the real zeros.
- A polynomial of degree $n$ has at most $n$ real zeros.
- Use substitution, factoring, or graphs to find zeros.

### Homework

Solve the exercises 1 to 4 in this page in your notebook.
Review the examples and complete the "Lesson Review" on page 41 again.



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# Mixed Practice

## A Choose the correct answer :

1. The set of real zeros of $f(x) = 3x$ is :
   (a) $\{0\}$ (b) $\{3\}$ (c) $\{-3\}$ (d) $R$
2. If $x + 2$ is a factor of $p(x)$, then one real zero of $p(x)$ is :
   (a) 2 (b) $-2$ (c) 0 (d) 1
3. The set of real zeros of $f(x) = x^2 + 4$ is :
   (a) $\{2, -2\}$ (b) $\{0\}$ (c) $\varnothing$ (d) $R$
4. If $f(2) = 0$, then 2 is a real zero of $f(x)$ because :
   (a) 2 is in the domain
   (b) $f(x)$ is a quadratic function
   (c) substituting 2 makes $f(x) = 0$
   (d) 2 is a factor of $f(x)$
5. The maximum number of real zeros of a polynomial of degree 4 is :
   (a) 2 (b) 3 (c) 4 (d) 5
6. If $f(x) = x^3 - 3x^2 + 1$ and $x = 1$, then $f(1) =$ :
   (a) $-1$ (b) 0 (c) 1 (d) 2
7. Which of the following could be the set of real zeros of $f(x) = x^3 - 4x$ ?
   (a) $\{0, 2\}$ (b) $\{-2, 2\}$ (c) $\{0\}$ (d) $\varnothing$
8. If the graph of a polynomial touches the $x$−axis at $x = -3$ and crosses it at $x = 1$, then the polynomial has how many real zeros?
   (a) 1 (b) 2 (c) 3 (d) 4

## B Complete the following :

1. If $f(-3) = 0$, then ............ is a real zero of $f(x)$.
2. The set of real zeros of $x^2 - 5x$ is .............
3. If $x - 4$ is a factor of $g(x)$, then $g(4) =$ ..........
4. The set of real zeros of $x(x + 1)(x - 2)$ is .............
5. The maximum number of real zeros of a polynomial of degree 5 is ..............
6. If $f(x) = 2x^3 + 3x$ and one real zero is 0, the other real zeros are ............. and .............. .

## C Solve the following equations :

1. $x^2 - 2x - 8 = 0$
2. $x^2 + 6x + 8 = 0$
3. $2x^2 - x - 3 = 0$
4. $x^3 - x^2 - 4x = 0$
5. $x^3 + 3x^2 - 4x = 0$
6. $x^3 - 5x^2 + 6x = 0$
7. $x^4 - 5x^2 + 4 = 0$
8. $2x^3 + 7x^2 + 3x = 0$
9. $x^3 + 8x = 0$

## D Word Problems :

1. A rectangular plot has length $x + 4$ meters and width $x - 1$ meters. If its area is zero, find the possible values of $x$.
2. The area of a square park is $x^2 - 25$ square meters. Find the possible values of $x$ for which the area of the park is zero.
3. The profit (in thousands of dollars) of a small business after $x$ months is $P(x) = x^2 - 9$. For which values of $x$ is the profit zero?

## E Challenge Yourself :

1. If the degree of $f(x) = ax^3 + bx^2 - 5x + 2$ is 3 $x = -1$ is a real zero of $f(x)$, find the value of $a$ in terms of $b$.
2. If the set of real zeros of $f(x) = x^3 - 2x^2 + kx$ is $\{0, 1, 3\}$, find the value of $k$.
3. Find the set of real zeros of $f(x) = x^4 - 3x^2 - 4$ without factoring.

## F Thinking and Reasoning :

1. If $f(x) = x^2 + (m + 2)x + m - 3$ has equal real zeros, find the value of $m$.
2. A polynomial $p(x) = x^3 + px^2 + qx + r$ has $x = 1$ and $x = -2$ as two real zeros. If $p(0) = -6$, find $p$, $q$, and $r$.

### Key Takeaways

- A real zero is a value of $x$ that makes $f(x) = 0$.
- A curve cuts the $x$−axis at the real zeros.
- A polynomial of degree $n$ has at most $n$ real zeros.
- Use substitution, factoring, or graphs to find zeros.

### Note for You

- Always check your answers by substitution.
- Look for patterns and use suitable methods.
- Practice more to improve accuracy and speed.



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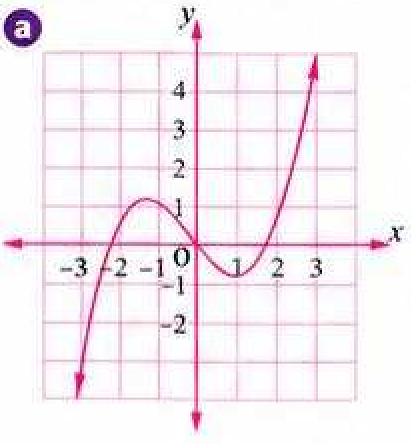
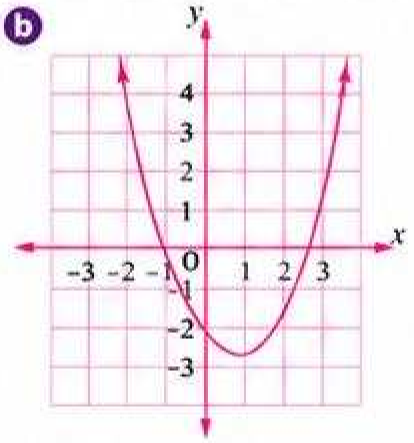
# More Practice

## 1 From the graph :

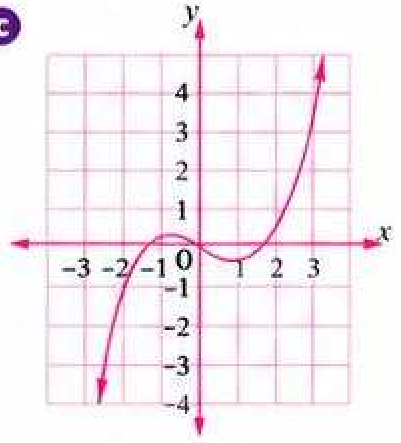
For each of the following graphs, find the set of real zeros of the function.

a


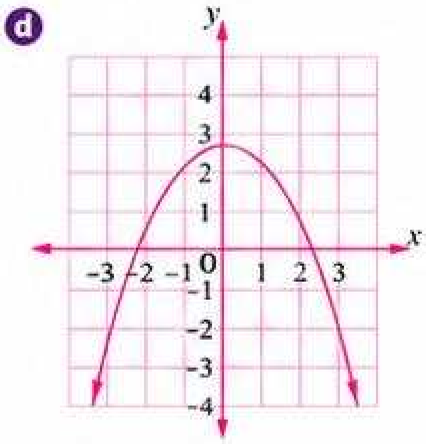
Set of real zeros = { __________ }

b


Set of real zeros = { __________ }

c


Set of real zeros = { __________ }

d


Set of real zeros = { __________ }

## 2 Fill in the blanks :

1. If 0 is a real zero of $f(x) = 2x^3 - 5x^2 + x$, then $f(0) =$ ............
2. If $x + 3$ is a factor of $f(x)$, then one real zero of $f(x)$ is ............
3. The set of real zeros of $x^2 - 9$ is ............
4. If a cubic polynomial has real zeros $-2, 0, 3$, then the polynomial is ............
5. The maximum number of real zeros of a polynomial of degree 6 is ............
6. If $f(x) = x^3 - 6x^2 + 11x - 6$, then the sum of its real zeros is ............

## 3 Match the columns :

| Column (A) | Column (B) |
|---|---|
| 1. $x^2 - 1$ | a. $\{0\}$ |
| 2. $x^2 + 4x$ | b. $\{-1\}$ |
| 3. $x^2 - 4x + 4$ | c. $\{-2, 0\}$ |
| 4. $x^3 - x$ | d. $\{1, -1\}$ |
| 5. $x^2 + 2x + 1$ | e. $\{-1, 0\}$ |

## 4 Multiple Choice Questions :

1. The polynomial whose set of real zeros is $\{-3, 2\}$ is :
   (a) $x^2 + x - 6$ (b) $x^2 - 5x - 6$ (c) $x^2 + x + 6$ (d) $x^2 - 6$
2. If $f(x) = x^3 + x^2 - x$, then 0 is a real zero because :
   (a) 0 is in the domain (b) substituting 0 makes $f(x) = 0$
   (c) $x + 1$ is a factor (d) degree of $f(x)$ is 3
3. Which of the following has exactly two real zeros ?
   (a) $x^2 - 4$ (b) $x^2 + 1$ (c) $x^3 - 1$ (d) $x^2(x - 1)$
4. The set of real zeros of $(x - 2)^3$ is :
   (a) $\{2\}$ (b) $\{0, 2\}$ (c) $\{-2, 2\}$ (d) $\varnothing$
5. If $x = 3$ is a real zero of $f(x) = x^2 + bx - 3$, then $b =$ :
   (a) $-3$ (b) 0 (c) 3 (d) 6
6. The maximum number of real zeros of a polynomial of degree 7 is :
   (a) 6 (b) 7 (c) 8 (d) 3
7. If the graph of a quadratic function cuts the $x$-axis at two distinct points, then the function has :
   (a) no real zeros (b) one real zero
   (c) two real zeros (d) three real zeros
8. If $f(x) = 2x^3 - 5x$ and $x = 0$, then $f(0) =$ :
   (a) $-5$ (b) 0 (c) 2 (d) 5

## 5 Word Problems :

1. A rectangular field has length $x + 3$ meters and width $x - 2$ meters. If the area of the field is zero, find the possible values of $x$.
2. A company's monthly profit (in thousand dollars) is given by
   $$P(x) = x^2 - 5x,$$
   where $x$ is the number of months.
   In which months is the profit zero?
3. The height (in meters) of a ball thrown vertically upward is
   $$h(t) = -5t^2 + 20t,$$
   where $t$ is time in seconds.
   When does the ball hit the ground?

## 6 Thinking :

1. If the polynomial $f(x) = x^3 + ax^2 + bx$ has 0 and 2 as two of its zeros, find the value of $a$ and the value of $b$.
2. The set of real zeros of a polynomial is $\{-2, a, 3\}$, and the sum of the zeros is 1. Find the value of $a$.

**Self-check**

How confident are you in finding the set of real zeros of a polynomial?

☐ ☐ ☐ ☐

**Homework**

Answer the exercises from 1 to 4 in this page in your notebook.
Review the Lesson Review on page 41 again.



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UNIT 2 **Algebra**

Lesson 3

# Rational Algebraic Functions

## Getting Started

Engineers use algebraic functions to describe many real-life situations such as speed, cost, and growth rate. In this lesson, we will learn rational algebraic functions and how to find their domain, values, and simplify expressions.

## What will I learn?

- Define rational algebraic function.
- Find the domain of a rational function.
- Find the value of a rational function.
- Simplify rational expressions.
- Perform the basic operations on rational functions.
- Solve real-life problems using rational algebraic functions.

## Let's Learn

**By the end of this lesson, you will be able to:**

- Identify rational algebraic functions.
- Determine the domain of a rational function.
- Find the value of a rational function for given inputs.
- Simplify rational expressions.
- Add, subtract, multiply and divide rational functions.
- Solve problems involving rational functions.

## The Key Rule

A rational algebraic function is a function that can be written in the form:

$$f(x) = \frac{P(x)}{Q(x)} \quad \text{where } Q(x) \neq 0$$

**where:**

$P(x), Q(x)$ are algebraic expressions.

$Q(x) \neq 0$ to keep the function defined.

## Understanding the Notation

| | |
|---|---|
| $f(x)$ | The value of the function at $x$ |
| $P(x)$ | The numerator (top expression) |
| $Q(x)$ | The denominator (bottom expression) |
| $Q(x) \neq 0$ | The denominator must not be zero |

## Important Note

A rational function is undefined when the denominator equals zero. Always check the values that make $Q(x) = 0$.

**Example:**

If $f(x) = \frac{x+2}{x-3}$, then $x - 3 = 0 \Rightarrow x = 3$

So, $x = 3$ is not allowed in the domain.

## Solved Examples

**Example 1:** Find the domain of the function.

$$f(x) = \frac{x+4}{x^2+16}$$

**Solution:**

- Set the denominator $\neq 0$.
- $x^2 + 16 \neq 0$
- $x^2 = -16$
- This has no real solutions.
- So, the domain is all real numbers.

Domain: $\mathbb{R}$

**Tip:** If the denominator is always positive (for all real $x$), then the domain is $\mathbb{R}$.

**Example 2:** Find the domain of the function.

$$f(x) = \frac{x-3}{x^2-3x+2}$$

**Solution:**

- Set the denominator $\neq 0$.
- $x^2 - 3x + 2 \neq 0$
- Factorize the denominator: $(x-1)(x-2) \neq 0$
- So, $x - 1 \neq 0 \Rightarrow x \neq 1$ and $x - 2 \neq 0 \Rightarrow x \neq 2$

Domain: $\mathbb{R} \setminus \{1, 2\}$

**Tip:** Factorizing the denominator helps to find the values that make it zero.



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## Solved Examples

**Example 3:** Find the domain of the function.

$$f(x) = \frac{2x-6}{x^2-x-2}$$

**Solution:**

- Set the denominator $\neq 0$.
- $x^2 - x - 2 \neq 0$
- Factorize the denominator: $(x-2)(x+1) \neq 0$
- $x - 2 \neq 0 \Rightarrow x \neq 2$ and $x + 1 \neq 0 \Rightarrow x \neq -1$
- The domain is all real numbers except $x = 2$ and $x = -1$.

Domain: $\mathbb{R} \setminus \{-1, 2\}$

**Tip**

When the denominator is a product, make each factor equal to zero.

**Example 4:** Find the domain of the function.

$$f(x) = \frac{x^2-1}{x^2+4x+3}$$

**Solution:**

- Set the denominator $\neq 0$.
- $x^2 + 4x + 3 \neq 0$
- Factorize: $(x+1)(x+3) \neq 0$
- $x + 1 \neq 0 \Rightarrow x \neq -1$ and $x + 3 \neq 0 \Rightarrow x \neq -3$
- The domain is all real numbers except $x = -1$ and $x = -3$.

Domain: $\mathbb{R} \setminus \{-3, -1\}$

**Tip**

Always exclude the values that make the denominator zero.

## Find the Value of a Rational Function

**Example 5:** Find $f(-2)$ if $f(x) = \frac{x+4}{x^2+16}$

**Solution:**

- Substitute $x = -2$.

$$f(-2) = \frac{-2+4}{(-2)^2+16} = \frac{2}{4+16} = \frac{2}{20} = \frac{1}{10}$$

$f(-2) = \frac{1}{10}$

**Example 6:** Find $f(3)$ if $f(x) = \frac{2x-1}{x^2-5x+6}$

**Solution:**

- Substitute $x = 3$.

$$f(3) = \frac{2(3)-1}{(3)^2-5(3)+6} = \frac{6-1}{9-15+6} = \frac{5}{0}$$

- The denominator is zero.
- So, $f(3)$ is undefined.

$f(3)$ is undefined

**Example 7:** Find $f(x)$ when $f(x) = \frac{x^2+2x-3}{x-1}$ for $x = 2$.

**Solution:**

- Substitute $x = 2$.

$$f(2) = \frac{(2)^2+2(2)-3}{2-1} = \frac{4+4-3}{1} = 5$$

$f(2) = 5$

**Note**

If the result makes the denominator zero, the function is undefined.

## Common Mistakes

- Setting the numerator equal to zero when finding the domain.
  - ✗ Wrong: $x^2 + 1 = 0$
  - ✓ Correct: Set the denominator $\neq 0$.
- Forgetting to exclude all values that make the denominator zero.
  - ✗ Wrong: Excluding only one value.
  - ✓ Correct: Exclude all such values.
- Simplifying before finding the domain (may remove important restrictions).
  - ✗ Wrong: Canceling factors first.
  - ✓ Correct: Find the domain from the original function.

## Practice Time 1

**A) Find the domain of each function.**

1. $f(x) = \frac{x+1}{x^2-9}$ ..................
2. $f(x) = \frac{x-2}{x^2+5x+6}$ ..................
3. $f(x) = \frac{3x+2}{x^2-4x}$ ..................
4. $f(x) = \frac{x^2+1}{x-3}$ ..................
5. $f(x) = \frac{2x+5}{x^2+x-6}$ ..................

**B) Find the value of each function.**

1. $f(x) = \frac{x-1}{x^2+4}$, $f(2) =$ ..........
2. $f(x) = \frac{2x+3}{x-2}$, $f(5) =$ ..........
3. $f(x) = \frac{x^2-1}{x+3}$, $f(-4) =$ ..........
4. $f(x) = \frac{3x-2}{x^2-1}$, $f(0) =$ ..........
5. $f(x) = \frac{x+7}{x^2+1}$, $f(-3) =$ ..........

**C) State whether the function is defined or undefined at the given value.**

1. $f(x) = \frac{x+2}{x-1}$, $x = 1$ ..................
2. $f(x) = \frac{x^2+1}{x+4}$, $x = -4$ ..................
3. $f(x) = \frac{x-3}{x^2-9}$, $x = -3$ ..................
4. $f(x) = \frac{x^2+5}{x-2}$, $x = 2$ ..................
5. $f(x) = \frac{2x+1}{x^2+x}$, $x = 0$ ..................



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## Finding Values That Make the Function Undefined

A rational function is undefined when the denominator equals zero.
Find the values of $x$ that make the function undefined.

**Example 8:**
Find the values of $x$ for which $f(x) = \frac{1}{x-4}$ is undefined.

**Solution:**
- The denominator is $x-4$.
- Set $x-4=0$
- $x=4$

> **Note**
> Only the value that makes the denominator zero causes the function to be undefined.

The function is undefined at $x = 4$.

**Example 9:**
Find the values of $x$ for which $g(x) = \frac{x+3}{x^2-9}$ is undefined.

**Solution:**
- The denominator is $x^2-9$.
- Set $x^2-9=0$
- Factor: $(x-3)(x+3)=0$
- $x-3=0$ or $x+3=0$
- $x=3$ or $x=-3$

The function is undefined at $x=3$ and $x=-3$.

**Example 10:**
Find the values of $x$ for which $h(x) = \frac{2x-5}{x^2+x}$ is undefined.

**Solution:**
- The denominator is $x^2+x$.
- Set $x^2+x=0$
- Factor: $x(x+1)=0$
- $x=0$ or $x+1=0$
- $x=0$ or $x=-1$

The function is undefined at $x=0$ and $x=-1$.

## Simplifying Rational Expressions

We can cancel common factors from the numerator and denominator, but we must remember any restrictions from the original denominator.

> **Attention!**
> - Cancel common factors only.
> - Never cancel a factor that equals zero in the domain.

**Example 11:** Simplify $\frac{x^2-9}{x-3}$

**Solution:**
- Factor the numerator: $x^2-9=(x-3)(x+3)$
- Cancel the common factor $(x-3)$:

$$\frac{(x-3)(x+3)}{x-3} = x+3$$

- Restriction: $x-3 \neq 0 \Rightarrow x \neq 3$

$$\frac{x^2-9}{x-3} = x+3,\ x \neq 3$$

> **Tip**
> Always write the restriction after simplifying.

**Example 12:** Simplify $\frac{x^2-4}{x+2}$

**Solution:**
- Factor the numerator: $x^2-4=(x-2)(x+2)$
- Cancel the common factor $(x+2)$:

$$\frac{(x-2)(x+2)}{x+2} = x-2$$

- Restriction: $x+2 \neq 0 \Rightarrow x \neq -2$

$$\frac{x^2-4}{x+2} = x-2,\ x \neq -2$$

**Example 13:** Simplify $\frac{2x^2+4x}{x}$

**Solution:**
- Factor the numerator: $2x^2+4x=2x(x+2)$
- Cancel the common factor $x$:

$$\frac{2x(x+2)}{x} = 2(x+2)$$

- Restriction: $x \neq 0$

$$\frac{2x^2+4x}{x} = 2(x+2),\ x \neq 0$$

## Practice Time 2

**A) Find the values of $x$ that make each function undefined.**

1. $f(x) = \frac{5}{x-7}$ ..................
2. $g(x) = \frac{2x+1}{x^2-4}$ ..................
3. $h(x) = \frac{x-2}{x^2+3x}$ ..................
4. $p(x) = \frac{1}{x^2-1}$ ..................
5. $q(x) = \frac{3x}{x^2-5x}$ ..................

**B) Simplify each rational expression.**

1. $\frac{x^2-16}{x-4}$ ..................
2. $\frac{x^2+5x}{x}$ ..................
3. $\frac{x^2-x-6}{x-3}$ ..................
4. $\frac{x^2+2x+1}{x+1}$ ..................
5. $\frac{3x^2-12x}{3x}$ ..................

**C) Write each expression in simplest form and state the restriction on $x$.**

1. $\frac{x^2-9}{x+3}$ ..................
2. $\frac{x^2-2x}{x-2}$ ..................
3. $\frac{x^2+7x+10}{x+5}$ ..................
4. $\frac{2x^2-8x}{x-4}$ ..................
5. $\frac{x^2-1}{x^2+x}$ ..................


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## Zero of a Rational Function

A rational function is given by $f(x)=\dfrac{P(x)}{Q(x)}$, where $Q(x)\neq 0$.

The zero(s) of $f(x)$ are the values of $x$ that make the numerator $P(x)$ equal zero.

**Note:** The denominator must not be zero at the same time.

Zero(s) of $f(x)=\dfrac{P(x)}{Q(x)}$ are the solutions of $P(x)=0$ with $Q(x)\neq 0$.

**Example 4** Find the zero(s) of the function

$$f(x)=\frac{x^2-7x+12}{x^2-x-6}$$

**Solution:**

- Set the numerator = 0:

$x^2-7x+12=0$

$(x-3)(x-4)=0$

$x-3=0$ or $x-4=0$

$x=3$ or $x=4$

- Check the denominator:

$x^2-x-6=(x-3)(x+2)$

So, $x=3$ or $x=-2$ make the denominator zero (not allowed).

- Therefore, the zero(s) are:

$\{3,4\}-\{3,-2\}=\{4\}$

Zero of $f(x)=\{4\}$

**Check Your Work**

If $x=4$

in $f(x)=\dfrac{x^2-7x+12}{x^2-x-6}$

$f(4)=\dfrac{0}{2}=0$

So, $x=4$ is a zero.

**Example 5** ① If $f(x)=\dfrac{x^2-4}{x-2}=5$, find the solution set of the equation in $\mathbb{R}$.

② If $g(x)=\dfrac{x^2+2x-3}{x-1}=4$, find the solution set of the equation in $\mathbb{R}$.

**Solution:**

❶ $\dfrac{x^2-4}{x-2}=5$

- $x^2-4=5(x-2)$
- $x^2-4=5x-10$
- $x^2-5x+6=0$
- $(x-2)(x-3)=0$
- $x-2=0$ or $x-3=0$
- $x=2$ or $x=3$
- But $x=2$ makes the denominator zero.

Solution set = $\{3\}$

❷ $\dfrac{x^2+2x-3}{x-1}=4$

- $x^2+2x-3=4(x-1)$
- $x^2+2x-3=4x-4$
- $x^2-2x+1=0$
- $(x-1)^2=0$
- $x-1=0$
- $x=1$
- But $x=1$ makes the denominator zero.

Solution set = $\varnothing$ (no solution)

## Try It Yourself

**A) Find the zero(s) of each function.**

❶ $f(x)=\dfrac{x^2-5x+6}{x^2-x-6}$ ....................

❷ $g(x)=\dfrac{x^2+3x-4}{x^2-4}$ ....................

❸ $h(x)=\dfrac{x^2-9}{x^2+5x+6}$ ....................

❹ $p(x)=\dfrac{x^2+2x}{x^2-3x}$ ....................

❺ $q(x)=\dfrac{2x^2-8x+6}{x^2-1}$ ....................

**B) Solve for $x$ in $\mathbb{R}$.**

❶ $\dfrac{x^2-1}{x-1}=3$ ....................

❷ $\dfrac{x^2+3x+2}{x+1}=4$ ....................

❸ $\dfrac{x^2-4x}{x-2}=2$ ....................

❹ $\dfrac{x^2+2x-3}{x-3}=-1$ ....................

❺ $\dfrac{x^2-5x+6}{x-2}=0$ ....................

### Remember

The zero(s) of a rational function are the values of $x$ that make $f(x)=0$ with $Q(x)\neq 0$.

### ④ Zero of a Rational Function

Find the zero of $f(x)$ where

$$f(x)=\frac{x^3+x^2-2x}{x^3+2x^2}$$

### Evaluate Yourself ④

Find the zero(s) of $f(x)$ where

$$f(x)=\frac{x^3+2x^2-2x}{x^3+2x^2}$$

### Evaluate Yourself ⑤

If:

❶ $f(x)=\dfrac{x^2-3x}{x-3}$

Find the solution set in $\mathbb{R}$.

❷ $g(x)=\dfrac{x^2+2x-8}{x^2+4x}$

Find the solution set in $\mathbb{R}$.

### Thinking Challenge

Find $k$ so that the function

$$f(x)=\frac{x^2+kx+8}{x-2}$$

has no zeros.

....................

....................

....................

....................



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# Lesson 3

Exercises

## Rational Algebraic Functions

### First Choose the correct answer :

**1** The value of $x$ for which $\dfrac{x-1}{x+2}$ is undefined is :

(a) $-2$ (b) $2$ (c) $-1$ (d) $0$

**2** The value of $x$ for which $\dfrac{x+3}{x-2}$ is undefined is :

(a) $2$ (b) $-2$ (c) $-3$ (d) $0$

**3** The set of values of $x$ for which $f(x)=\dfrac{x^2-x-2}{x^2+4}$ is undefined is :

(a) R (b) $\{0\}$ (c) $R-\{-2\}$ (d) $R-\{-4\}$

**4** The set of values of $x$ for which $f(x)=\dfrac{x^2-25}{7}$ is undefined is :

(a) R (b) $\{7\}$ (c) $\{-5, 5\}$ (d) $R-\{7\}$

**5** If $f(x)=\dfrac{2x}{x-a}$ and the set $R-\{-2\}$ equals the set of values of $x$ for which $f$ is defined, then $a=$ :

(a) $2$ (b) $-2$ (c) $0$ (d) $-1$

**6** The value of $x$ for which $f(x)=\dfrac{x^2-2x}{x^2+2x}$ is undefined is :

(a) $0$ (b) $-2$ (c) $\{0, -2\}$ (d) $\{2, 0\}$

**7** If the set of zeros of the function $f(x)=\dfrac{x^2-x+a}{x+2}$ is $\{3\}$, then $a=$ :

(a) $6$ (b) $3$ (c) $-2$ (d) $-6$

**Discover the zeros :**

Omar tried to simplify the function $f(x)=\dfrac{4x^2-12x}{x^2-4}$ as follows :

$$f(x)=\frac{4x^2-12x}{x^2-4}=\frac{4x(x-3)}{(x-2)(x+2)}$$

$$=\frac{4x}{x+2}$$

He concluded that the function has a zero at $x=0$ only.

Find the mistake he made and determine the correct zeros of the function.

**Correct solution**

$$f(x)=\frac{4x^2-12x}{x^2-4}$$

$$=\frac{4x(x-3)}{(x-2)(x+2)}$$

(There is no common factor.)

The zeros are the values that make the numerator equal to zero and do not make the denominator zero.

$4x(x-3)=0$

$x=0$ or $x=3$

(But $x\neq 2$ and $x\neq -2$)

**8** The simplest form of the function $f(x)=\dfrac{4x+12}{x^2-x-12}$ is :

(a) $\dfrac{4}{x-4}$ (b) $\dfrac{x-4}{4}$ (c) $\dfrac{-4}{x+3}$ (d) $\dfrac{-4}{x-3}$

### Second Find the set of zeros (values of $x$) of each function :

**9** $f(x)=\dfrac{x-4}{x^2-16}$ Set of zeros = { ..................... }

**10** $f(x)=\dfrac{2x-5}{5-2x}$ Set of zeros = { ..................... }

**11** $f(x)=\dfrac{6x+12}{2x^2+7x+6}$ Set of zeros = { ..................... }

**12** $f(x)=\dfrac{x^2-3x+2}{3x-3}$ Set of zeros = { ............ }

**13** $f(x)=\dfrac{4x^2-8x-6}{15-x-2x^2}$ Set of zeros = { ............ }

**14** $f(x)=\dfrac{16+16x+3x^2}{x^2-3x-28}$ Set of zeros = { ............ }

### Extra Find the set of values of $x$ for which each of the following functions is defined :

**15** $f(x)=\dfrac{x^2-9}{x+3}$ $R-\{$ ..................... $\}$

**16** $f(x)=\dfrac{2x^2+5x-3}{x^2+x-6}$ $R-\{$ ..................... $\}$

**17** $f(x)=\dfrac{x+2}{x-2}$ $R-\{$ ..................... $\}$

**18** $f(x)=\dfrac{x^2-3x+2}{x^2-7x+12}$ $R-\{$ ..................... $\}$



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## Exercises

### 1 Choose the correct answer from the given choices:

**1** The value(s) of $x$ that make the rational expression $\frac{x-1}{x+2}$ undefined is/are:

(a) $1$ (b) $2$ (c) $-1$ (d) $-2$

**2** If $f(x)=\frac{x+3}{x-2}$, the value of $f(-2)$ equals:

(a) $2$ (b) $-2$ (c) $3$ (d) Not defined

**3** If $g(x)=\frac{x^2-x-2}{x^2+4}$, the domain of $g$ is:

(a) $\mathbb{R}-\{-2\}$ (b) $\mathbb{R}-\{2\}$

(c) $\mathbb{R}-\{-4\}$ (d) $\mathbb{R}$

**4** The value of $f(7)$ if $f(x)=\frac{x^2-25}{7}$ is:

(a) $7$ (b) $\mathbb{R}$ (c) $\mathbb{R}-\{7\}$ (d) $\{-5,5\}$

**5** The set of zeros of $f(x)=\frac{x^2-2x}{x^2+2x}$ is:

(a) $\{0\}$ (b) $\{2\}$ (c) $\{2,0\}$ (d) $\{-2\}$

**6** If $f(x)=\frac{2x}{x-a}$ and $f(a)=6$, then the value of $a$ in $\mathbb{R}-\{-2\}$ is:

(a) $1$ (b) $-1$ (c) $2$ (d) $-2$

**7** If $f(3)=3$ where $f(x)=\frac{x^2-x+a}{x+2}$, then the value of $a$ is:

(a) $-6$ (b) $-2$ (c) $3$ (d) $6$

### 2 Find the error and correct it:

**8** A student simplifies $f(x)=\frac{4x+12}{x^2-x-12}$ as follows:

$$f(x)=\frac{4x+12}{x^2-x-12}=\frac{4(x+3)}{(x+3)(x-4)}=\frac{4}{x-4}$$

Domain: $\mathbb{R}-\{-4\}$

Find the error and write the correct solution.

**9** A student simplifies $f(x)=\frac{4x+12}{x^2-x-12}$ as follows:

$$f(x)=\frac{4x+12}{x^2-x-12}=\frac{4(x+3)}{(x+3)(x-4)}=\frac{4}{x-4}$$

Domain: $\mathbb{R}-\{-3,4\}$

Find the error and write the correct solution.

### 3 Find the simplified form of each rational expression:

**10** $\frac{x-4}{x^2-16}$ ..............................

**11** $\frac{2x-5}{5-2x}$ ..............................

**12** $\frac{6x+12}{2x^2+7x+6}$ ..............................

**13** $\frac{x^2-3x+2}{3x-3}$ ..............................

**14** $\frac{4x^2-8x-5}{15-x-2x^2}$ ..............................

### 4 Find the zero(s) of the function in each case:

**15** $f(x)=\frac{x^2-9}{x+3}$ ..............................

**16** $f(x)=\frac{x+2}{x-2}$ ..............................

**17** $f(x)=\frac{2x^2+5x-3}{x^2+x-6}$ ..............................

**18** $f(x)=\frac{2x-6}{x^2-x-6}$ ..............................

**19** $f(x)=\frac{4x^2-4x-3}{9-3x-2x^2}$ ..............................

**20** $f(x)=\frac{x^2-3x+2}{x^2-7x+12}$ ..............................

### 5 In each of the following, find the solution set of the equation in $\mathbb{R}$:

**21** $\frac{x^2-4}{x-2}=5$ ..............................

**22** $\frac{x^2+2x-3}{x-1}=4$ ..............................

**23** $\frac{2x^2+7x-15}{2x-3}=10$ ..............................

**24** $\frac{x+6}{x^2-2x+36}=a$ , where $a\in\mathbb{R}$ ..............................

**25** $\frac{x+a}{x-b}=2$ , where $b\in\mathbb{R}$ and $b\neq 0$ ..............................

### 6 Find the domain of each function:

**26** $f(x)=\frac{x^3-2x}{x^2-4}$ ..............................

**27** $f(x)=\frac{x^2+3x-2}{x^2-9}$ ..............................

**28** $f(x)=\frac{x^2+x-2}{x^2-5x+6}$ ..............................

**29** $f(x)=\frac{x^3+1}{x^2-x}$ ..............................

### 7 A flower bed has the shape of a rectangle with dimensions as shown in the figure. Its area is the same as a square with side length $k$ cm.

① Find the value of $k$ in terms of $x$. ..............................

② Find the value of $k$ when $x=2$ and when $x=4$. ..............................

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Unit 2 Algebra

Lesson 4

# Common Domain and Equivelant Rational Algebraic Functions

## Let's Learn

- Determine the domain of a rational algebraic function.
- Find the common domain of two or more rational functions.
- Decide whether a value belongs to the domain or not.
- Define equivalent rational algebraic functions.
- Find equivalent rational functions.

## Key Rule

A rational algebraic function is in the form:

$$f(x) = \frac{P(x)}{Q(x)}, \quad Q(x) \neq 0$$

where $P(x)$ and $Q(x)$ are polynomials.

**Domain**

The domain of $f(x)$ is all real numbers except the values of $x$ that make $Q(x) = 0$.

**Equivalent Functions**

Two rational functions $f(x)$ and $g(x)$ are equivalent if they have the same domain and $f(x) = g(x)$ for all $x$ in the domain.

## Common Mistakes

- Forgetting to exclude values that make the denominator equal to zero.
- Thinking the numerators of two functions must be the same to be equivalent.
- Not checking equality for all values in the common domain.

## 2 Common Domain of Two or More Functions

The common domain of two or more rational functions is the set of all real numbers that belong to the domains of all of them.

**Steps:**

1. Find the domain of each function.
2. Exclude all values that are not allowed in any of them.
3. The remaining values form the common domain.

**Note**

When solving problems, always write the domain clearly using set notation.

## 1 Domain of a Rational Function

To find the domain of $f(x) = \frac{P(x)}{Q(x)}$ :

1. Find the values of $x$ that make $Q(x) = 0$.
2. Exclude those values from $\mathbb{R}$.
3. The remaining values are the domain.

### Solved Examples

**Example 1** Find the domain of $f(x) = \frac{x^2 - 3x + 2}{x - 2}$

**Solution:**

Set $x - 2 = 0 \rightarrow x = 2$

The domain is $\mathbb{R} - \{2\}$.

**Example 2** Find the domain of $g(x) = \frac{x + 4}{x^2 - 9}$

**Solution:**

Set $x^2 - 9 = 0 \rightarrow (x - 3)(x + 3) = 0 \rightarrow x = 3$ or $x = -3$

The domain is $\mathbb{R} - \{-3, 3\}$.

**Example 3** Find the domain of $h(x) = \frac{2x - 1}{x^2 - x - 6}$

**Solution:**

Set $x^2 - x - 6 = 0 \rightarrow (x - 3)(x + 2) = 0 \rightarrow x = 3$ or $x = -2$

The domain is $\mathbb{R} - \{-2, 3\}$.

### Solved Examples

**Example 4** Find the common domain of $f(x) = \frac{x + 1}{x - 2}$ and $g(x) = \frac{x - 3}{x + 2}$

**Solution:**

Domain of $f$: $\mathbb{R} - \{2\}$

Domain of g: $\mathbb{R} - \{-2\}$

Common domain: $\mathbb{R} - \{-2, 2\}$

**Example 5** Find the common domain of $f(x) = \frac{x^2 - 1}{x - 1}$ , $g(x) = \frac{2x + 5}{x + 3}$

**Solution:**

Domain of $f$: $x - 1 = 0 \rightarrow x = 1 \rightarrow \mathbb{R} - \{1\}$

Domain of $g$: $x + 3 = 0 \rightarrow x = -3 \rightarrow \mathbb{R} - \{-3\}$

Common domain: $\mathbb{R} - \{-3, 1\}$



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# Common Domain and Equivelant Rational Algebraic Functions

## 3 Equivalent Rational Algebraic Functions

Two rational algebraic functions $f(x) = \frac{P(x)}{Q(x)}$ and $g(x) = \frac{R(x)}{S(x)}$ are equivalent if:

1. $D_f = D_g$
2. $f(x) = g(x)$ for all $x \in D_f$

### Key Idea

- Simplify each function.
- Find the domain of each function.
- Compare the domains.
- If domains are the same and simplified functions are equal, then they are equivalent.

### Solved Examples

**Example 6** Determine whether $f(x) = \frac{x^2-4}{x-2}$ and $g(x) = x+2$ are equivalent or not.

**Solution:**

- Simplify $f(x)$: $\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2 \quad , x \neq 2$
- Domain of $f$: $\mathbb{R} - \{2\}$
- Domain of $g$: $\mathbb{R}$

The domains are not the same, so $f$ and $g$ are not equivalent.

**Example 7** Determine whether $f(x) = \frac{x^2+2x}{x}$ and $g(x) = x+2$ are equivalent or not.

**Solution:**

- Simplify $f(x)$: $\frac{x^2+2x}{x} = \frac{x(x+2)}{x} = x+2 \quad , x \neq 0$
- Domain of $f$: $\mathbb{R} - \{0\}$
- Domain of $g$: $\mathbb{R}$

The domains are not the same, so $f$ and $g$ are not equivalent.

**Example 8** Determine whether $f(x) = \frac{x^2-x}{x(x-1)}$ and $g(x) = \frac{1}{x-1}$ are equivalent or not.

**Solution:**

- Simplify $f(x)$: $\frac{x^2-x}{x(x-1)} = \frac{x(x-1)}{x(x-1)} = \frac{1}{x-1} \quad , x \neq 0, 1$
- Domain of $f$: $\mathbb{R} - \{0, 1\}$
- Domain of $g$: $\mathbb{R} - \{1\}$

The domains are not the same, so $f$ and $g$ are not equivalent.

**Note**

If two functions have the same domain and become identical after simplification, then they are equivalent rational algebraic functions.

## Practice 1

Find the domain of each of the following functions.

1. $f(x) = \frac{3x+1}{x+5}$

   $D_f = \{__________\}$

2. $f(x) = \frac{x^2-5}{x^2+2x}$

   $D_f = \{__________\}$

3. $f(x) = \frac{x-7}{x^2-1}$

   $D_f = \{__________\}$

4. $f(x) = \frac{2x}{x-4}$

   $D_f = \{__________\}$

5. $f(x) = \frac{x^2+x-6}{x^2-x}$

   $D_f = \{__________\}$

6. $f(x) = \frac{5}{x^2+9}$

   $D_f = \{__________\}$

## Practice 2

Find the common domain of each pair of functions.

1. $f(x) = \frac{x+1}{x-3}$ , $g(x) = \frac{2}{x+2}$

   Common domain = {__________}

2. $f(x) = \frac{x^2-4}{x}$ , $g(x) = \frac{x+3}{x-1}$

   Common domain = {__________}

3. $f(x) = \frac{x-5}{x^2-9}$ , $g(x) = \frac{2x}{x+3}$

   Common domain = {__________}

4. $f(x) = \frac{x^2+1}{x-2}$ , $g(x) = \frac{x-1}{x(x+2)}$

   Common domain = {__________}

5. $f(x) = \frac{3}{x-1}$ , $g(x) = \frac{x^2-1}{x+1}$

   Common domain = {__________}

6. $f(x) = \frac{2x+1}{x^2-4}$ , $g(x) = \frac{x+4}{x-2}$

   Common domain = {__________}

## Practice 3

Determine whether the following pairs of functions are equivalent or not. Justify your answer.

1. $f(x) = \frac{x^2-9}{x-3}$ , $g(x) = x+3$

   Equivalent / Not equivalent

   Reason : __________

2. $f(x) = \frac{x^2+3x}{x}$ , $g(x) = x+3$

   Equivalent / Not equivalent

   Reason : __________

3. $f(x) = \frac{x^2-1}{x-1}$ , $g(x) = x+1$

   Equivalent / Not equivalent

   Reason : __________

4. $f(x) = \frac{x^2+2x+1}{x+1}$ , $g(x) = x+1$

   Equivalent / Not equivalent

   Reason : __________

5. $f(x) = \frac{x^2-4x}{x(x-4)}$ , $g(x) = \frac{1}{x}$

   Equivalent / Not equivalent

   Reason : __________

6. $f(x) = \frac{x^2-2x+1}{x-1}$ , $g(x) = x-1$

   Equivalent / Not equivalent

   Reason : __________


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# Common Domain and Equivelant Rational Algebraic Functions

## Solved Examples

**Example 9** Find the value(s) of $x$ for which $f(x) = \frac{x+2}{x-1}$ is defined.

**Solution:**

Set $x - 1 = 0 \rightarrow x = 1$

$f(x)$ is defined for all real numbers except $x = 1$.

Domain: $\mathbb{R} - \{1\}$

**Example 10** Find the value(s) of $x$ for which $g(x) = \frac{2x-5}{x^2-4}$ is defined.

**Solution:**

Set $x^2 - 4 = 0 \rightarrow (x-2)(x+2) = 0 \rightarrow x = 2$ or $x = -2$

$g(x)$ is defined for all real numbers except $x = 2$ and $x = -2$.

Domain: $\mathbb{R} - \{-2, 2\}$

**Example 11** Determine whether $f(x) = \frac{x^2+4x+4}{x+2}$ and $g(x) = x+2$ are equivalent.

**Solution:**

- Simplify $f(x)$: $\frac{x^2+4x+4}{x+2} = \frac{(x+2)^2}{x+2} = x+2$ , $x \neq -2$
- Domain of $f$: $\mathbb{R} - \{-2\}$
- Domain of $g$: $\mathbb{R}$

They are not equivalent because their domains are not the same.

### Key Tips

- The domain of a rational function excludes the values that make the denominator zero.
- The common domain excludes all values that are not allowed in any of the functions.
- To check equivalence, simplify first, compare domains, then compare the functions.
- Two functions can be identical after simplification but not equivalent if their domains differ.

## Practice 4

Find the values of $x$ that are NOT in the domain of each function.

1. $f(x) = \frac{x+3}{x-5}$
   Not in domain: {________}
2. $f(x) = \frac{2x-1}{x^2-1}$
   Not in domain: {________}
3. $f(x) = \frac{x^2+1}{x}$
   Not in domain: {________}
4. $f(x) = \frac{5}{x+7}$
   Not in domain: {________}
5. $f(x) = \frac{x-4}{x^2-9}$
   Not in domain: {________}
6. $f(x) = \frac{3x+2}{x^2-16}$
   Not in domain: {________}

## Practice 5

Determine whether each statement is True (T) or False (F).

1. The domain of $f(x) = \frac{x+1}{x-3}$ is $\mathbb{R} - \{3\}$. (  )
2. The domain of $g(x) = \frac{2x}{x^2+4}$ is $\mathbb{R} - \{-2, 2\}$. (  )
3. The common domain of $f(x) = \frac{x+2}{x-1}$ and $g(x) = \frac{x}{x+1}$ is $\mathbb{R} - \{1, -1\}$. (  )
4. $f(x) = \frac{x^2-1}{x-1}$ and $g(x) = x+1$ are equivalent. (  )
5. The function $h(x) = \frac{x^2+3x+2}{x+2}$ is not defined at $x = -2$. (  )
6. $f(x) = \frac{x}{x^2-4}$ is defined for all real numbers. (  )
7. If two functions have the same domain and different expressions, they cannot be equivalent. (  )
8. The domain of $f(x) = \frac{x^2+5}{x-10}$ is $\mathbb{R} - \{10\}$. (  )

## Mixed Exercises

Solve each of the following.

1. Find the domain of $f(x) = \frac{x^2-3x+2}{x^2-4}$.
   $D_f = \{$________________________$\}$
2. Find the domain of $g(x) = \frac{3x+4}{x+2}$.
   $D_g = \{$________________________$\}$
3. Find the common domain of $f(x) = \frac{x+1}{x-2}$ and $g(x) = \frac{x}{x+3}$.
   Common domain = {________________________}
4. Determine whether $f(x) = \frac{x^2-9}{x-3}$ and $g(x) = x+3$ are equivalent.
   ☐ Equivalent ☐ Not equivalent
   Reason : ________________________
   ________________________
5. Find the values of $x$ for which $h(x) = \frac{x+5}{x^2-25}$ is defined.
   Answer : ________________________
6. Find the values of $x$ for which $p(x) = \frac{2x-1}{x(x-3)}$ is not defined.
   Answer : ________________________


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# Exercises

## 1 Find the domain of each of the following functions.

1. $f(x) = \frac{x+4}{x-5}$ $D_f = \{__________\}$
2. $f(x) = \frac{2x-1}{x^2-4}$ $D_f = \{__________\}$
3. $f(x) = \frac{x^2+2x}{x+3}$ $D_f = \{__________\}$
4. $f(x) = \frac{5}{x^2-9}$ $D_f = \{__________\}$
5. $f(x) = \frac{x^2-3x+2}{x-1}$ $D_f = \{__________\}$
6. $f(x) = \frac{x-7}{x^2+1}$ $D_f = \{__________\}$
7. $f(x) = \frac{x^2+1}{x^2-2x}$ $D_f = \{__________\}$
8. $f(x) = \frac{3x}{x^2+4}$ $D_f = \{__________\}$
9. $f(x) = \frac{2x+5}{x^2-5x+6}$ $D_f = \{__________\}$
10. $f(x) = \frac{x^2-1}{x+1}$ $D_f = \{__________\}$

## 2 Find the common domain of each pair of functions.

1. $f(x) = \frac{x+1}{x-2}$, $g(x) = \frac{x-3}{x+4}$ Common domain $= \{__________\}$
2. $f(x) = \frac{x^2-4}{x-2}$, $g(x) = \frac{x+5}{x+2}$ Common domain $= \{__________\}$
3. $f(x) = \frac{2x+1}{x+3}$, $g(x) = \frac{x-1}{x^2-1}$ Common domain $= \{__________\}$
4. $f(x) = \frac{x-2}{x^2-4}$, $g(x) = \frac{x+2}{x-3}$ Common domain $= \{__________\}$
5. $f(x) = \frac{x^2+1}{x+1}$, $g(x) = \frac{3}{x-1}$ Common domain $= \{__________\}$
6. $f(x) = \frac{x+4}{x^2-9}$, $g(x) = \frac{2x}{x+3}$ Common domain $= \{__________\}$

## 3 Determine whether the following pairs of functions are equivalent or not.

1. $f(x) = \frac{x^2-1}{x-1}$, $g(x) = x+1$
   ☐ Equivalent ☐ Not equivalent
   Reason : __________
2. $f(x) = \frac{x^2+2x}{x}$, $g(x) = x+2$
   ☐ Equivalent ☐ Not equivalent
   Reason : __________
3. $f(x) = \frac{x^2-4}{x-2}$, $g(x) = x+2$
   ☐ Equivalent ☐ Not equivalent
   Reason : __________
4. $f(x) = \frac{x^2+3x}{x}$, $g(x) = x+3$
   ☐ Equivalent ☐ Not equivalent
   Reason : __________
5. $f(x) = \frac{x^2-9}{x+3}$, $g(x) = x-3$
   ☐ Equivalent ☐ Not equivalent
   Reason : __________
6. $f(x) = \frac{x^2-2x+1}{x-1}$, $g(x) = x-1$
   ☐ Equivalent ☐ Not equivalent
   Reason : __________

## 4 Determine whether each statement is True (T) or False (F).

1. The domain of $f(x) = \frac{x+2}{x-3}$ is $\mathbb{R} - \{3\}$. ( )
2. The function $g(x) = \frac{x^2+1}{x^2+4}$ is defined for all real numbers. ( )
3. The common domain of $f(x) = \frac{x}{x-1}$ and $g(x) = \frac{x+2}{x-1}$ is $\mathbb{R} - \{1\}$. ( )
4. If $f(x) = \frac{x^2-1}{x-1}$ and $g(x) = x+1$, then $f = g$. ( )
5. The domain of $h(x) = \frac{1}{x^2-9}$ is $\mathbb{R} - \{-3, 3\}$. ( )
6. If two functions have the same domain, then they are equivalent. ( )
7. The domain of $p(x) = \frac{2x}{x^2-4}$ is $\mathbb{R} - \{2, -2\}$. ( )
8. The functions $f(x) = \frac{x^2+x}{x}$ and $g(x) = x+1$ are equivalent. ( )
9. If $f(x) = \frac{x^2-4}{x+2}$, then $f$ is defined at $x = -2$. ( )
10. If $f(x) = g(x)$ for all $x$ in their common domain, then $f$ and $g$ are equivalent. ( )

## 5 Find the values of $x$ for which each function is not defined.

1. $f(x) = \frac{x+1}{x-6}$ $x = ________$
2. $f(x) = \frac{x^2-9}{x+3}$ $x = ________$
3. $f(x) = \frac{x^2+2x}{x}$ $x = ________$
4. $f(x) = \frac{5x}{x^2-25}$ $x = ________$
5. $f(x) = \frac{x^2-2}{x^2-4}$ $x = ________$
6. $f(x) = \frac{2}{x-7}$ $x = ________$
7. $f(x) = \frac{x^2-x-12}{x+4}$ $x = ________$
8. $f(x) = \frac{x+3}{x^2-1}$ $x = ________$



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# Exercises

## 1 Find the domain of each of the following functions.

1. $f(x) = \frac{x+2}{x-7}$ 
$D_f = \{$______________$\}$

2. $f(x) = \frac{3x-5}{x^2-16}$ 
$D_f = \{$______________$\}$

3. $f(x) = \frac{x^2+4}{x+1}$ 
$D_f = \{$______________$\}$

4. $f(x) = \frac{x-1}{x^2-5x}$ 
$D_f = \{$______________$\}$

5. $f(x) = \frac{2x+3}{x^2+2x-8}$ 
$D_f = \{$______________$\}$

6. $f(x) = \frac{x^2-6}{2x+1}$ 
$D_f = \{$______________$\}$

7. $f(x) = \frac{x+5}{x^2-9}$ 
$D_f = \{$______________$\}$

8. $f(x) = \frac{4x}{x^2-3x+2}$ 
$D_f = \{$______________$\}$

9. $f(x) = \frac{x^2-1}{x^2+4x+3}$ 
$D_f = \{$______________$\}$

10. $f(x) = \frac{2x-7}{x^2-2x-3}$ 
$D_f = \{$______________$\}$

## 2 Find the common domain of each pair of functions.

1. $f(x) = \frac{x+1}{x-2}$ , $g(x) = \frac{x-5}{x+3}$ 
Common domain = $\{$______________$\}$

2. $f(x) = \frac{x^2-9}{x+3}$ , $g(x) = \frac{2x}{x-4}$ 
Common domain = $\{$______________$\}$

3. $f(x) = \frac{2x+3}{x^2-1}$ , $g(x) = \frac{x-1}{x+2}$ 
Common domain = $\{$______________$\}$

4. $f(x) = \frac{x-3}{x+1}$ , $g(x) = \frac{x+4}{x^2-4}$ 
Common domain = $\{$______________$\}$

5. $f(x) = \frac{x^2+1}{x-5}$ , $g(x) = \frac{3}{x^2-9}$ 
Common domain = $\{$______________$\}$

6. $f(x) = \frac{x+6}{x^2+3x}$ , $g(x) = \frac{2x-1}{x-6}$ 
Common domain = $\{$______________$\}$

## 3 Determine whether the following pairs of functions are equivalent or not.

1. $f(x) = \frac{x^2-4}{x-2}$ , $g(x) = x+2$ 
☐ Equivalent ☐ Not equivalent 
Reason : ____________________

2. $f(x) = \frac{x^2+3x}{x}$ , $g(x) = x+3$ 
☐ Equivalent ☐ Not equivalent 
Reason : ____________________

3. $f(x) = \frac{x^2-1}{x^2-1}$ , $g(x) = 1$ 
☐ Equivalent ☐ Not equivalent 
Reason : ____________________

4. $f(x) = \frac{x^2+2x+1}{x+1}$ , $g(x) = x+1$ 
☐ Equivalent ☐ Not equivalent 
Reason : ____________________

5. $f(x) = \frac{x^2-9}{x^2+3x}$ , $g(x) = \frac{x-3}{x}$ 
☐ Equivalent ☐ Not equivalent 
Reason : ____________________

6. $f(x) = \frac{x^2+4}{x-2}$ , $g(x) = \frac{x+2}{1}$ 
☐ Equivalent ☐ Not equivalent 
Reason : ____________________

## 4 Determine whether each statement is True (T) or False (F).

1. The domain of $f(x) = \frac{x+3}{x-1}$ is $\mathbb{R} - \{1\}$. ( )
2. The function $g(x) = \frac{x^2+2}{x^2-4}$ is defined for all real numbers. ( )
3. The common domain of $f(x) = \frac{x}{x-2}$ and $g(x) = \frac{x+1}{x-2}$ is $\mathbb{R} - \{2\}$. ( )
4. If $f(x) = \frac{x^2-1}{x+1}$ and $g(x) = x-1$, then $f = g$. ( )
5. The domain of $h(x) = \frac{1}{(x-2)(x+3)}$ is $\mathbb{R} - \{-3, 2\}$. ( )
6. If two functions have different expressions but the same domain, they cannot be equivalent. ( )
7. The domain of $p(x) = \frac{2x-1}{x^2+9}$ is $\mathbb{R}$. ( )
8. If $f(x) = \frac{x^2+2x}{x}$ and $g(x) = x+2$, then $f = g$. ( )
9. If $f(x) = \frac{x^2-5x+6}{x-2}$, then $f$ is not defined at $x = 2$. ( )
10. If $f(x) = g(x)$ for all $x$ in their common domain, then $f$ and $g$ are equivalent. ( )

## 5 Find the values of $x$ for which each function is not defined.

1. $f(x) = \frac{x+2}{x-4}$ 
$x =$ ____________

2. $f(x) = \frac{x^2-4}{x+2}$ 
$x =$ ____________

3. $f(x) = \frac{x+1}{x^2-5}$ 
$x =$ ____________

4. $f(x) = \frac{6x}{x^2-36}$ 
$x =$ ____________

5. $f(x) = \frac{x^2+3}{x-3}$ 
$x =$ ____________

6. $f(x) = \frac{2}{x+7}$ 
$x =$ ____________

7. $f(x) = \frac{x^2-1}{x(x-2)}$ 
$x =$ ____________

8. $f(x) = \frac{3x-2}{x^2-x-6}$ 
$x =$ ____________

9. $f(x) = \frac{x-5}{x^2+4x+4}$ 
$x =$ ____________

10. $f(x) = \frac{x^2-9}{x+3}$ 
$x =$ ____________



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# Exercises

## 1 Find the domain of each of the following functions.

1. $f(x) = \dfrac{x+3}{x-2}$ $D_f = \{$______$\}$
2. $f(x) = \dfrac{5x-1}{x^2-9}$ $D_f = \{$______$\}$
3. $f(x) = \dfrac{x^2-4}{x+4}$ $D_f = \{$______$\}$
4. $f(x) = \dfrac{2x}{x^2+3x}$ $D_f = \{$______$\}$
5. $f(x) = \dfrac{x^2+2x+1}{x-1}$ $D_f = \{$______$\}$
6. $f(x) = \dfrac{3x+7}{x^2-5x+6}$ $D_f = \{$______$\}$
7. $f(x) = \dfrac{x-5}{x^2+3x-10}$ $D_f = \{$______$\}$
8. $f(x) = \dfrac{4x+8}{2x^2-x-6}$ $D_f = \{$______$\}$
9. $f(x) = \dfrac{x^2-9}{x^2+2x+1}$ $D_f = \{$______$\}$
10. $f(x) = \dfrac{6}{x^2-7x+10}$ $D_f = \{$______$\}$

## 2 Find the common domain of each pair of functions.

1. $f(x) = \dfrac{x+2}{x-3}$, $g(x) = \dfrac{x-1}{x+4}$ Common domain = {______}
2. $f(x) = \dfrac{x^2-1}{x-1}$, $g(x) = \dfrac{2x}{x-2}$ Common domain = {______}
3. $f(x) = \dfrac{2x-3}{x+1}$, $g(x) = \dfrac{x+5}{x^2-4}$ Common domain = {______}
4. $f(x) = \dfrac{x+1}{x^2-4}$, $g(x) = \dfrac{5}{x+2}$ Common domain = {______}
5. $f(x) = \dfrac{x^2+4}{x^2+1}$, $g(x) = \dfrac{3x-9}{x-3}$ Common domain = {______}
6. $f(x) = \dfrac{4x+1}{x-5}$, $g(x) = \dfrac{x^2-25}{x+5}$ Common domain = {______}

## 3 Determine whether the following pairs of functions are equivalent or not.

1. $f(x) = \dfrac{x^2-1}{x-1}$ , $g(x) = x+1$
   ☐ Equivalent ☐ Not equivalent
   Reason : ______
2. $f(x) = \dfrac{x^2+5x}{x}$ , $g(x) = x+5$
   ☐ Equivalent ☐ Not equivalent
   Reason : ______
3. $f(x) = \dfrac{x^2-2x+1}{x-1}$ , $g(x) = x-1$
   ☐ Equivalent ☐ Not equivalent
   Reason : ______
4. $f(x) = \dfrac{x^2+3x}{x+3}$ , $g(x) = x$
   ☐ Equivalent ☐ Not equivalent
   Reason : ______
5. $f(x) = \dfrac{x^2-4}{x+2}$ , $g(x) = x-2$
   ☐ Equivalent ☐ Not equivalent
   Reason : ______
6. $f(x) = \dfrac{2x^2+4x}{2x}$ , $g(x) = x+2$
   ☐ Equivalent ☐ Not equivalent
   Reason : ______

## 4 Determine whether each statement is True (T) or False (F).

1. The domain of $f(x) = \dfrac{x+4}{x-2}$ is $\mathbb{R} - \{2\}$. ( )
2. The function $g(x) = \dfrac{x^2+1}{x^2+4}$ is defined for all real numbers. ( )
3. The common domain of $f(x) = \dfrac{x-1}{x+1}$ and $g(x) = \dfrac{2}{x-1}$ is $\mathbb{R} - \{-1, 1\}$. ( )
4. If $f(x) = \dfrac{x^2-9}{x-3}$ and $g(x) = x+3$, then $f = g$. ( )
5. The domain of $h(x) = \dfrac{1}{x^2-4}$ is $\mathbb{R} - \{-2, 2\}$. ( )
6. If two functions have the same domain and $f(x) = g(x)$ for all $x$, then $f$ and $g$ are equivalent. ( )
7. The domain of $p(x) = \dfrac{3x-2}{x^2+1}$ is $\mathbb{R}$. ( )
8. If $f(x) = \dfrac{x^2+2}{x}$ and $g(x) = x + \dfrac{2}{x}$, then $f = g$. ( )
9. If $f(x) = \dfrac{x}{x+2}$, then $f$ is not defined at $x = 2$. ( )
10. If $f(x) = g(x)$ for all $x$ in their common domain, then $f$ and $g$ are not equivalent. ( )

## 5 Find the values of $x$ for which each function is not defined.

1. $f(x) = \dfrac{x+1}{x-5}$ $x =$ ______
2. $f(x) = \dfrac{x^2-16}{x+4}$ $x =$ ______
3. $f(x) = \dfrac{x^2+2x}{x}$ $x =$ ______
4. $f(x) = \dfrac{5x}{x^2-25}$ $x =$ ______
5. $f(x) = \dfrac{x^2-3x}{x-3}$ $x =$ ______
6. $f(x) = \dfrac{2}{x+6}$ $x =$ ______
7. $f(x) = \dfrac{x^2-x-6}{x-3}$ $x =$ ______
8. $f(x) = \dfrac{3x-9}{x^2-1}$ $x =$ ______
9. $f(x) = \dfrac{x+7}{x^2+7x}$ $x =$ ______
10. $f(x) = \dfrac{x^2+1}{x-1}$ $x =$ ______


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# Exercises

## 1 Find the common domain of each pair of functions.

1. $f(x)=\dfrac{x+1}{x-3}$ , $g(x)=\dfrac{x-1}{x-2}$

   Common domain = { ______________ }

2. $f(x)=\dfrac{x^2-x-20}{x^2-3x-10}$ , $g(x)=\dfrac{x^2-3x-10}{x^2-x-6}$

   Common domain = { ______________ }

3. $f(x)=\dfrac{x+2}{x-5}$ , $g(x)=\dfrac{x-4}{x-2}$

   Common domain = { ______________ }

4. $f(x)=\dfrac{2x+1}{x+1}$ , $g(x)=\dfrac{3}{x-1}$

   Common domain = { ______________ }

5. $f(x)=\dfrac{x^2-9}{x^2-4}$ , $g(x)=\dfrac{x}{x-2}$

   Common domain = { ______________ }

6. $f(x)=\dfrac{x+3}{x+2}$ , $g(x)=\dfrac{x-5}{x-1}$

   Common domain = { ______________ }

7. $f(x)=\dfrac{2x-7}{x^2-9}$ , $g(x)=\dfrac{5}{x-3}$

   Common domain = { ______________ }

8. $f(x)=\dfrac{x^2+x-12}{x^2-4}$ , $g(x)=\dfrac{x^2-1}{x+3}$

   Common domain = { ______________ }

9. $f(x)=\dfrac{4x}{x-6}$ , $g(x)=\dfrac{x^2-5x}{x^2-9}$

   Common domain = { ______________ }

10. $f(x)=\dfrac{x^2-2x-8}{x^2-1}$ , $g(x)=\dfrac{x+4}{x-1}$

   Common domain = { ______________ }

## 2 Determine whether $f=g$. Justify your answer.

1. $f(x)=\dfrac{x+1}{x-3}$ , $g(x)=\dfrac{x-1}{x-2}$

   ☐ Equal ☐ Not equal

   Reason : ______________

2. $f(x)=\dfrac{x^2-x-20}{x^2-3x-10}$ , $g(x)=\dfrac{x^2-3x-10}{x^2-x-6}$

   ☐ Equal ☐ Not equal

   Reason : ______________

3. $g(x)=\dfrac{3x}{3x^2-12}$ , $f(x)=\dfrac{x}{x^2-4}$

   ☐ Equal ☐ Not equal

   Reason : ______________

4. $g(x)=\dfrac{x}{x-2}$ , $f(x)=\dfrac{x^2-2x}{x^2-4x+4}$

   ☐ Equal ☐ Not equal

   Reason : ______________

5. $g(x)=\dfrac{6x}{6x+12}$ , $f(x)=\dfrac{x^2+2x}{x^2+4x+4}$

   ☐ Equal ☐ Not equal

   Reason : ______________

6. $f(x)=\dfrac{x}{x+1}$ , $g(x)=\dfrac{x-1}{x^2-1}$

   ☐ Equal ☐ Not equal

   Reason : ______________

7. $f(x)=\dfrac{x+2}{x-3}$ , $g(x)=\dfrac{x^2-4}{x^2-6x+9}$

   ☐ Equal ☐ Not equal

   Reason : ______________

8. $g(x)=\dfrac{3x}{3x^2-12}$ , $f(x)=\dfrac{x}{x-2}$

   ☐ Equal ☐ Not equal

   Reason : ______________

9. $g(x)=\dfrac{6x}{6x+12}$ , $f(x)=\dfrac{x^2+2x}{x^2+4x+4}$

   ☐ Equal ☐ Not equal

   Reason : ______________

10. $f(x)=\dfrac{x^2+2x-8}{x^2-4}$ , $g(x)=\dfrac{x+2}{x-2}$

   ☐ Equal ☐ Not equal

   Reason : ______________

11. $f(x)=\dfrac{x-3}{x^2-9}$ , $g(x)=\dfrac{1}{x+3}$

   ☐ Equal ☐ Not equal

   Reason : ______________

12. $f(x)=\dfrac{x^2+3x+2}{x+1}$ , $g(x)=x+2$

   ☐ Equal ☐ Not equal

   Reason : ______________



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# Operations on Rational Algebraic Fractions

## Get Started!

Ahmed received excellent marks in mathematics in the last four years of preparatory school as follows:

First year: $3x^2 + 18x + 21$

Second year: $x^2 + 5x + 6$

Third year: $5x^2 + 24x + 45$

Fourth year: $x^2 + 7x + 10$

If he succeeds in the final exam, what a great success!

## Learning Outcomes

By the end of this lesson, you will be able to:

- Perform the four operations (addition, subtraction, multiplication, and division) on rational algebraic fractions.
- Simplify rational algebraic fractions.

## Think!

Write the function $f(x) = \dfrac{x}{x+2} + \dfrac{3}{x+3}$.

If $f(x) = 1$, what is the value of $x$ so that the statement is true?
And what is the value of $x$ so that the statement is not true?

## Key Vocabulary

- Rational algebraic fractions
- Reciprocal of a function

## Prerequisite

Add, subtract, multiply and divide integers.

If a, b, c, d are integers and $b \neq 0,\ d \neq 0$, then:

| |
|---|
| $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$ |
| $\dfrac{a}{b} - \dfrac{c}{d} = \dfrac{ad - bc}{bd}$ |
| $\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{ac}{bd}$ |
| $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \times \dfrac{d}{c} = \dfrac{ad}{bc}$ |

## Let's Learn!

### Collecting and Simplifying Rational Algebraic Fractions — Part 1

**Rational algebraic fractions** are fractions in which the numerator and the denominator are polynomials in one variable.

**Steps:**

1. Factor each polynomial completely.
2. Cancel common factors between the numerator and denominator.
3. Write the result in simplest form.

**Example:** Simplify each of the following rational algebraic fractions:

1. $\dfrac{2x-1}{x+1} + \dfrac{x+3}{x+1} = \dfrac{2x-1+x+3}{x+1} = \dfrac{3x+2}{x+1}$

2. $\dfrac{x^2-x}{x^2+2x-3} + \dfrac{3}{x+3} = \dfrac{x(x-1)}{(x+3)(x-1)} + \dfrac{3}{x+3} = \dfrac{x+3}{x+3} = 1$

3. $\dfrac{2}{x} - \dfrac{2}{x+1} = \dfrac{2(x+1)}{x(x+1)} - \dfrac{2x}{x(x+1)} = \dfrac{2+2x-2x}{x(x+1)} = \dfrac{2}{x(x+1)}$

## Real Life Link

Rational algebraic fractions are used in many real-life situations such as speed, mixture problems, and rate of work.


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## Let's Learn!

### Adding Rational Algebraic Fractions — Part 2

To add or subtract rational algebraic fractions, they must have a **common denominator**.

**Steps:**

1. Factor each denominator completely.
2. Find the Least Common Denominator (LCD).
3. Write equivalent fractions with the LCD.
4. Add or subtract the numerators.
5. Simplify the result.

**Remember!**

The LCD is the product of each distinct factor raised to its highest power.

**Examples:** Add or subtract and simplify.

**1** $\frac{2}{x+2} + \frac{3}{x+3}$

**Solution:**

- The LCD of $(x+2)$ and $(x+3)$ is $(x+2)(x+3)$
- Write equivalent fractions.
- Add the numerators.
- Simplify.

$$\frac{2}{x+2} + \frac{3}{x+3} = \frac{2(x+3)}{(x+2)(x+3)} + \frac{3(x+2)}{(x+2)(x+3)}$$

$$= \frac{2x+6+3x+6}{(x+2)(x+3)} \quad \leftarrow \text{Add numerators}$$

$$= \frac{5x+12}{(x+2)(x+3)} \quad \leftarrow \text{Already in simplest form}$$

**2** $\frac{x}{x-1} - \frac{2}{x+1}$

**Solution:**

- The LCD of $(x-1)$ and $(x+1)$ is $(x-1)(x+1)$
- Write equivalent fractions.
- Subtract the numerators.
- Simplify.

$$\frac{x}{x-1} - \frac{2}{x+1} = \frac{x(x+1)}{(x-1)(x+1)} - \frac{2(x-1)}{(x-1)(x+1)}$$

$$= \frac{x^2+x-2x+2}{(x-1)(x+1)} \quad \leftarrow \text{Subtract numerators}$$

$$= \frac{x^2-x+2}{x^2-1} \quad \leftarrow \text{No common factor}$$

**3** $\frac{2}{x-2} + \frac{3}{x} - \frac{1}{x+2}$

**Solution:**

- The LCD of $(x-2)$, $x$, $(x+2)$ is $x(x-2)(x+2)$
- Write equivalent fractions.
- Add and subtract the numerators.
- Simplify.

$$\frac{2}{x-2} + \frac{3}{x} - \frac{1}{x+2} =$$

$$\frac{2x(x+2)}{x(x-2)(x+2)} + \frac{3(x-2)(x+2)}{x(x-2)(x+2)} - \frac{x(x-2)}{x(x-2)(x+2)}$$

$$= \frac{2x(x+2)+3(x-2)(x+2)-x(x-2)}{x(x-2)(x+2)}$$

$$= \frac{2x^2+4x+3(x^2-4)-(x^2-2x)}{x(x-2)(x+2)}$$

$$= \frac{2x^2+4x+3x^2-12-x^2+2x}{x(x-2)(x+2)}$$

$$= \frac{4x^2+6x-12}{x(x-2)(x+2)}$$

$$= \frac{2(2x^2+3x-6)}{x(x-2)(x+2)} \quad \leftarrow \text{2 is the common factor}$$

### Quick Summary

**To add or subtract rational algebraic fractions:**

1 Factor each denominator → 2 Find the LCD → 3 Write equivalent fractions → 4 Add or subtract the numerators → 5 Simplify the result

### Key Idea

Always work with a common denominator before adding or subtracting rational algebraic fractions.

### Common Mistakes

- Adding or subtracting numerators without having a common denominator.
- Forgetting to factor expressions before finding the LCD.
- Not simplifying the final answer.

### Tips

Factor first, then find the LCD. It makes the work easier and prevents mistakes.

### Check Yourself!

Why do we need a common denominator before adding or subtracting rational algebraic fractions?

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..............................................

..............................................



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## Let's Learn!

### Multiplying Rational Algebraic Fractions — Part 3

To multiply rational algebraic fractions, multiply the numerators together and multiply the denominators together. Then simplify the result.

**Steps:**

1. Factor each numerator and denominator completely.
2. Cancel common factors between numerators and denominators.
3. Multiply the remaining factors in the numerators.
4. Multiply the remaining factors in the denominators.
5. Write the result in simplest form.

**Remember!**

Simplify before multiplying if possible to make the work easier.

**Examples:** Multiply and simplify.

**1** $\frac{x+2}{x} \times \frac{x}{x+2}$

**Solution:**

- Factor each part.
- Cancel common factors.
- Multiply the remaining factors.

$= \frac{x+2}{x} \times \frac{x}{x+2} = \frac{\cancel{(x+2)}}{x} \times \frac{\cancel{x}}{\cancel{(x+2)}}$

$= \frac{1}{1} = 1$

**2** $\frac{2x-4}{3x} \times \frac{9x^2}{x-2}$

**Solution:**

- Factor each part.
- Cancel common factors.
- Multiply the remaining factors.

$= \frac{2(x-2)}{3x} \times \frac{9x^2}{\cancel{x-2}}$

$= \frac{2(x-2) \times 9x^2}{3x\ \cancel{(x-2)}}$

$= \frac{2}{3} \times 9x$

$= \frac{18x}{3}$

$= 6x$

**3** Evaluate the following: $\frac{x^2-1}{x+1} \times \frac{1}{x-1}$

**Solution:**

$= \frac{(x-1)(x+1)}{\cancel{x+1}} \times \frac{1}{\cancel{x-1}}$ $= \frac{1}{1}$

$= 1$

**4** $\frac{3x^2+3x}{x^2-1} \times \frac{x-1}{6x}$

**Solution:**

- Factor each part.
- Cancel common factors.
- Multiply the remaining factors.

$= \frac{3x(x+1)}{(x-1)\cancel{(x+1)}} \times \frac{\cancel{x-1}}{\cancel{6x}}$

$= \frac{3}{6}$

$= \frac{1}{2}$

### Quick Summary

To multiply rational algebraic fractions:

1. Factor the numerators and denominators. → 2. Cancel common factors. → 3. Multiply the remaining numerators. → 4. Multiply the remaining denominators. → 5. Write the result in simplest form.

### Previously Learned

- **Multiply integers:**

If $a$, $b$, $c$, $d$ are integers and $b \neq 0$, $d \neq 0$, then:

| |
|---|
| $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ |
| $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd}$ |
| $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ |
| $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ |

**Example (from integers)**

$\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5} = \frac{3}{10}$

### Common Mistakes

- Not factoring completely before canceling.
- Canceling terms that are added or subtracted.
- Forgetting to simplify the final answer.
- Canceling across addition or subtraction.

### Properties of Multiplication of Rational Algebraic Fractions

1. **Commutative Property:** $f(x) \times g(x) = g(x) \times f(x)$
2. **Associative Property:** $[f(x) \times g(x)] \times h(x) = f(x) \times [g(x) \times h(x)]$
3. **Identity Property:** $f(x) \times 1 = f(x)$
4. **Zero Property:** $f(x) \times 0 = 0$



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## Let's Learn!

### Dividing Rational Algebraic Fractions — Part 4

To divide rational algebraic fractions, multiply the first fraction by the reciprocal of the second fraction.

**Steps:**

1. Write the division as a multiplication by the reciprocal.
2. Factor each numerator and denominator completely.
3. Cancel common factors between numerators and denominators.
4. Multiply the remaining factors in the numerators.
5. Multiply the remaining factors in the denominators.
6. Write the result in simplest form.

**Remember!**

- Keep–Change–Flip: Change the division to multiplication and flip the second fraction.
- Simplify before multiplying if possible to make the work easier.

**Examples:** Divide and simplify.

**1** $f(x) = \dfrac{(x+1)(x+1)}{(1-x)(1+x)} \div \dfrac{(x+1)(x+4)}{(x-1)(x+4)}$ — ① Solve each part completely

$D_f = \mathbb{R} - \{1, -1, -4\}$

**Solution:**

$f(x) = \dfrac{(x+1)(x+1)}{(1-x)(1+x)} \times \dfrac{(x-1)(x+4)}{(x+1)(x+4)}$ — Convert division to multiplication by the reciprocal.

$= \dfrac{(x-1)(x+1)}{(1-x)(1+x)}$ — Cancel common factors.

$= \dfrac{x-1}{1-x} = \dfrac{-(x-1)}{x-1} = -1$ — Multiply and simplify.

---

**2** $f(x) = \dfrac{(2x+1)(x+2)}{(2x-1)(x+2)} \div \dfrac{(3x+1)(x+4)}{(2x-1)(x+4)}$ — ② Solve each part completely

$D_f = \mathbb{R} - \{\frac{1}{2}, -2, -4, \frac{-1}{3}\}$

**Solution:**

$f(x) = \dfrac{(2x+1)(x+2)}{(2x-1)(x+2)} \times \dfrac{(2x-1)(x+4)}{(3x+1)(x+4)}$ — Convert division to multiplication by the reciprocal.

$= \dfrac{(2x+1)(x+2)}{(3x+1)(x+4)}$ — Cancel common factors.

$= \dfrac{2x+1}{3x+1}$ — Multiply and simplify.

---

**3** $f(x) = \dfrac{x^2-2x-2}{x^2-4x-5} \div \dfrac{4-x^2}{x^2-3x-10}$ — ③ Solve each part completely

**Solution:**

$f(x) = \dfrac{x^2-2x-2}{x^2-4x-5} \times \dfrac{x^2-3x-10}{4-x^2}$ — Convert division to multiplication by the reciprocal.

$= \dfrac{(x^2-2x-2)(x^2-3x-10)}{(x^2-4x-5)(4-x^2)}$ — Multiply numerators and denominators.

$= \dfrac{(x^2-2x-2)(x-5)(x+2)}{(x-5)(x+1)(-1)(x-2)(x+2)}$ — Factor completely.

$= \dfrac{(x^2-2x-2)}{-(x+1)(x-2)}$ — Cancel common factors.

$= \dfrac{x^2-2x-2}{-x^2+x+2}$ — Simplify the signs.

$= \dfrac{-x^2+2x+2}{x^2-x-2}$ — Write in simplest form.

---

**4** $f(x) = \dfrac{x^2+5x+4}{x^2+3x-4} \div \dfrac{x^2+2x-15}{2x^2+7x-41}$ — ④ Solve each part completely

**Solution:**

$f(x) = \dfrac{x^2+5x+4}{x^2+3x-4} \times \dfrac{2x^2+7x-4}{x^2+2x-15}$ — Convert division to multiplication by the reciprocal.

$= \dfrac{(x+1)(x+4)}{(x-1)(x+4)} \times \dfrac{(2x-1)(x+4)}{(x+5)(x-3)}$ — Factor completely.

$= \dfrac{(x+1)(x+4)(2x-1)}{(x-1)(x+5)(x-2)}$ — Cancel common factors.

$= \dfrac{(x+1)(2x-1)(x+4)}{(x-1)(x+5)(x-3)}$ — Write in simplest form.

### Previously Learned

- **Divide integers:** If $a, b, c, d$ are integers and $b \neq 0, d \neq 0$, then:

| |
|---|
| $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ |
| $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ |
| $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$ |
| $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ |

**Example (from integers)**

$6 \div 3 = \frac{6}{1} \div \frac{3}{1} = \frac{6}{1} \times \frac{1}{3} = 2$

### Common Mistakes

- Forgetting to flip the second fraction.
- Not factoring completely before canceling.
- Canceling terms that are added or subtracted.
- Forgetting to simplify the final answer.
- Canceling across addition or subtraction.

### Properties of Division of Rational Algebraic Fractions

1. **Division Property:** $f(x) \div g(x) = f(x) \times \dfrac{1}{g(x)}$
2. **Reciprocal Property:** $g(x) \div f(x) = g(x) \times \dfrac{1}{f(x)}$
3. **Zero Property:** $f(x) \div 0 =$ Not defined
4. **Identity Property:** $f(x) \div f(x) = 1, \quad f(x) \neq 0$

### Quick Summary

To divide rational algebraic fractions:

1. Change division to multiplication by the reciprocal. ➜
2. Factor each numerator and denominator. ➜
3. Cancel common factors. ➜
4. Multiply the remaining numerators. ➜
5. Multiply the remaining denominators. ➜
6. Write the result in simplest form.



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## Let's Practice!

### A Multiply. Simplify your answer.

1. $\frac{x+3}{x-2} \times \frac{x-1}{x+4} =$ ________

2. $\frac{2x-5}{x+1} \times \frac{3x+6}{2x-3} =$ ________

3. $\frac{x^2-4}{x+2} \times \frac{x+5}{x-2} =$ ________

4. $\frac{3x+9}{x^2-1} \times \frac{x-1}{2x+6} =$ ________

5. $\frac{x^2+7x+10}{x+5} \times \frac{x+3}{x^2+4x-5} =$ ________

6. $\frac{2x^2-8}{x^2-1} \times \frac{x+1}{x-2} =$ ________

### B Divide. Simplify your answer.

1. $\frac{x+2}{x-1} \div \frac{x-3}{x+1} =$ ________

2. $\frac{2x-1}{x+4} \div \frac{x+2}{2x-3} =$ ________

3. $\frac{x^2-9}{x+3} \div \frac{x-2}{x^2+x-6} =$ ________

4. $\frac{3x+6}{x-2} \div \frac{x^2-4}{2x+4} =$ ________

5. $\frac{x^2+5x+6}{x+5} \div \frac{x+3}{x^2-1} =$ ________

6. $\frac{2x^2-3x-2}{x^2-4} \div \frac{x-1}{x+2} =$ ________

### C Solve the following word problems.

1. A factory can produce $\frac{x+4}{x}$ tons of paper per day. Another factory can produce $\frac{x-2}{x+5}$ tons of paper per day. If both factories work together, how many tons of paper can they produce per day?

**Solution:**

________________________________________

________________________________________

2. A recipe uses $\frac{2x+3}{x-1}$ cups of flour for a cake. How many cakes can be made from $\frac{6x}{x+2}$ cups of flour?

**Solution:**

________________________________________

________________________________________

3. A car travels $\frac{x^2-4}{x+2}$ kilometers per hour. How long will it take to travel $\frac{x^2-9}{x-3}$ kilometers?

**Solution:**

________________________________________

________________________________________

4. A painter can paint $\frac{3x+9}{x-1}$ square meters of wall in one day. How many days will he need to paint $\frac{6x^2+18x}{3x+3}$ square meters of wall?

**Solution:**

________________________________________

________________________________________

### Quick Tips

- **For multiplication:** Factor first, then multiply.
- **For division:** Change division to multiplication by the reciprocal, then simplify.
- Always simplify before moving to the next step.

### Common Mistakes

- Forgetting to factor completely.
- Not changing division to multiplication by the reciprocal.
- Cancelling terms that are added or subtracted.
- Forgetting to simplify the final answer.
- Cancelling across addition or subtraction.

### Properties of Rational Algebraic Fractions

1. **Commutative Property**
   $f(x) \cdot g(x) = g(x) \cdot f(x)$
2. **Associative Property**
   $[f(x) \cdot g(x)] \cdot h(x) = f(x) \cdot [g(x) \cdot h(x)]$
3. **Identity Property**
   $f(x) \cdot 1 = f(x)$
4. **Zero Property**
   $f(x) \cdot 0 = 0$

### Self Check

I can:

✓ Multiply rational algebraic fractions. ✓ Divide rational algebraic fractions. ✓ Simplify my answers correctly. ✓ Solve real-life problems.

Rate your understanding ☐ ☐ ☐



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## A Multiply. Simplify your answer.

1. $\frac{x+3}{x-2} \times \frac{x-1}{x+4} =$ ______
2. $\frac{2x-5}{x+1} \times \frac{3x+6}{2x-3} =$ ______
3. $\frac{x^2-4}{x+2} \times \frac{x+5}{x-2} =$ ______
4. $\frac{3x+9}{x^2-1} \times \frac{x-1}{2x+6} =$ ______
5. $\frac{x^2+7x+10}{x+5} \times \frac{x+3}{x^2+4x-5} =$ ______
6. $\frac{2x^2-8}{x^2-1} \times \frac{x+1}{x-2} =$ ______
7. $\frac{x^2}{x-1} \times \frac{x^2-1}{x^2} =$ ______
8. $\frac{x-1}{x} \times \frac{x}{x-1} =$ ______
9. $\frac{x+1}{x} \times \frac{x}{x+1} =$ ______
10. $\frac{x-5}{x+7} \times \frac{x+2}{x-5} =$ ______
11. $\frac{3x-15}{x+4} \times \frac{5-x}{6x+24} =$ ______
12. $\frac{6x^2+13x+6}{4x^2-9} \times \frac{6x^2+x-2}{4x^2-1} =$ ______

## B Divide. Simplify your answer.

1. $\frac{x+2}{x-1} \div \frac{x-3}{x+1} =$ ______
2. $\frac{2x-1}{x+4} \div \frac{x+2}{2x-3} =$ ______
3. $\frac{x^2-9}{x+3} \div \frac{x-2}{x^2+x-6} =$ ______
4. $\frac{3x+6}{x-2} \div \frac{x^2-4}{2x+4} =$ ______
5. $\frac{x^2+5x+6}{x+5} \div \frac{x+3}{x^2-1} =$ ______
6. $\frac{2x^2-3x-2}{x^2-4} \div \frac{x-1}{x+2} =$ ______

## C Solve the following word problems.

1. A factory can produce $\frac{x+4}{x}$ tons of paper per day. Another factory can produce $\frac{x-2}{x+5}$ tons of paper per day. If both factories work together, how many tons of paper can they produce per day?

______

2. A recipe uses $\frac{2x+3}{x-1}$ cups of flour for a cake. How many cakes can be made from $\frac{6x}{x+2}$ cups of flour?

______

3. A car travels $\frac{x^2-4}{x+2}$ kilometers per hour. How long will it take to travel $\frac{x^2-9}{x-3}$ kilometers?

______

4. A painter can paint $\frac{3x+9}{x-1}$ square meters of wall in one day. How many days will he need to paint $\frac{6x^2+18x}{3x+3}$ square meters of wall?

______


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## A Multiply. Simplify your answer.

1. $\frac{x^2+2x+4}{x^3-8} \times \frac{x^2-x-12}{x^2+x-6} =$ ________

7. $\frac{x^2-9x+20}{x^2+2x-16} \times \frac{x^2+4x-21}{x^2+3x-28} =$ ________

2. $\frac{x^2-11x+28}{x^2-13x+42} \times \frac{x^2+7x+10}{20-x-x^2} =$ ________

8. $\frac{x^2+11x+28}{x^2-13x+40} \times \frac{x^2+10x-21}{20-x-x^2} =$ ________

3. $\frac{3x^2+15}{x^2+7x+10} \times \frac{x+5}{x^2+6x+5} =$ ________

9. $\frac{2x^2+5x+2}{2x^2+7x+3} \times \frac{x^2-7x-30}{x^2-6x-40} =$ ________

4. $\frac{2x^2+5x+2}{2x^2+7x+3} \times \frac{x^2-7x-30}{x^2-6x-40} =$ ________

10. $\frac{3x-15}{x+4} \times \frac{5-x}{6x+24} =$ ________

5. $\frac{6x^2+13x+6}{4x^2-9} \times \frac{6x^2+x-2}{4x^2-1} =$ ________

11. $\frac{x^2+x-20}{x^2+2x-8} \times \frac{x^2+4x-21}{x^2+3x-28} =$ ________

6. $\frac{x^2-2x}{2x^2-x-6} \times \frac{2x^2-3x}{4x^2-9} =$ ________

12. $\frac{x^2-2x-8}{x^2+2x-3} \times \frac{2x^2+4x+2}{x^2-4} =$ ________

## B Divide. Simplify your answer.

1. $\frac{x^2-x-20}{x^2+2x-8} \div \frac{x^2+3x+2}{x^2+2x-3} =$ ________

4. $\frac{x^2+x-6}{x^2-9} \div \frac{2x^2-3x-2}{x^2-5x+6} =$ ________

2. $\frac{x^2-4}{x^2-x-2} \div \frac{x^2+5x+6}{x^2+2x-3} =$ ________

5. $\frac{x^2+6x+8}{x^2-7x+10} \div \frac{x^2-2x-3}{x^2+x-2} =$ ________

3. $\frac{2x^2+3x-2}{x^2-4} \div \frac{x-1}{x+2} =$ ________

6. $\frac{2x^2-x-6}{x^2+4x-12} \div \frac{x^2+3x}{x^2-1} =$ ________

## C Solve the following word problems.

1. A bakery can make $\frac{x^2+4x+4}{x^2-1}$ loaves of bread per hour. Another bakery can make $\frac{x^2-4}{x^2-x-2}$ loaves of bread per hour.
If both bakeries work together, how many loaves of bread can they make per hour?

________

2. A water tank can be filled by Pipe A in $\frac{x^2-9}{x^2+2x-3}$ hours. Pipe B can fill the same tank in $\frac{x^2+3x+2}{x^2+x-6}$ hours.
If both pipes are opened together, how long will it take to fill the tank?

________

3. A group of students can paint $\frac{2x^2+7x+3}{x^2+3x}$ square meters of a wall in one day. Another group can paint $\frac{x^2-4}{x^2-2x-3}$ square meters of the same wall in one day.
If both groups work together, how many days will it take to paint 1 square meter of the wall?

________

4. A machine produces $\frac{x^2+x-6}{x^2-4}$ items per minute. Another machine produces $\frac{x^2-5x+6}{x^2-2x-3}$ items per minute.
If both machines work together, how many items will they produce in one minute?

________



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## A Multiply. Simplify your answer.

1. $\frac{3x^2-8}{x^2+5x} \times \frac{x+2}{2x-3} =$ ______

7. $\frac{x^2+3x-4}{x^2+6x+9} \times \frac{x+3}{2x-5} =$ ______

2. $\frac{x^2-6x+9}{x^2-1} \times \frac{x-1}{x+3} =$ ______

8. $\frac{x^2-9}{x^2-5x+6} \times \frac{x+4}{x-3} =$ ______

3. $\frac{2x+5}{x^2-4} \times \frac{x^2-x-6}{x+1} =$ ______

9. $\frac{4x^2+4x-15}{x^2-7x+10} \times \frac{x-2}{2x+3} =$ ______

4. $\frac{5x-10}{x^2+3x} \times \frac{x^2+6x+9}{x-2} =$ ______

10. $\frac{3x^2-3}{x^2+2x} \times \frac{x^2-x-6}{x-1} =$ ______

5. $\frac{x^2-12}{x^2+4x+4} \times \frac{x+2}{x^2-3x-4} =$ ______

11. $\frac{x^2+7x+12}{x^2-1} \times \frac{x-1}{x+4} =$ ______

6. $\frac{2x^2+7x+3}{x^2-x-6} \times \frac{x+3}{x^2+2x-3} =$ ______

12. $\frac{2x^2+5x-3}{x^2+7x+6} \times \frac{x+2}{x^2-x-2} =$ ______

## B Divide. Simplify your answer.

1. $\frac{x^2+4x-5}{x-2} \div \frac{x+1}{x^2-1} =$ ______

4. $\frac{x^2+3x}{x-5} \div \frac{x^2-25}{2x+10} =$ ______

2. $\frac{x^2-7x+10}{x^2-4} \div \frac{x-5}{x^2+5x+6} =$ ______

5. $\frac{x^2-2x-15}{x^2+x-6} \div \frac{x-3}{x+5} =$ ______

3. $\frac{2x^2-3x-2}{x+1} \div \frac{x-2}{x^2-4} =$ ______

6. $\frac{3x^2+2x-8}{x^2-9} \div \frac{x+2}{x^2+2x} =$ ______

## C Solve the following word problems.

1. Two pumps can drain a pool. Pump A can drain $\frac{x^2-4}{x^2+2x-3}$ of the pool per hour, and Pump B can drain $\frac{x+3}{x-1}$ of the pool per hour. If both pumps work together, what fraction of the pool will they drain per hour?

______

2. A car can travel $\frac{x^2-9}{x^2-2x}$ kilometers on 1 liter of fuel. Another car can travel $\frac{x+2}{x-3}$ kilometers on 1 liter of fuel. If both cars travel together for 1 liter of fuel, what is the total distance they can travel?

______

3. A factory can complete $\frac{2x^2+5x+3}{x^2+3x}$ units of work in one day. Another factory can complete $\frac{x^2-1}{x^2-2x-3}$ units of work in one day. If both factories work together, how many units of work can they complete in one day?

______

______

4. A pipe fills a tank in $\frac{x^2+x-6}{x^2-9}$ hours. Another pipe fills the same tank in $\frac{x+3}{x-2}$ hours. If both pipes work together, how long will it take to fill the tank?

______

______



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# UNIT 2 ASSESSMENT

## Algebra

### A Choose the Correct Answer (6 Marks)

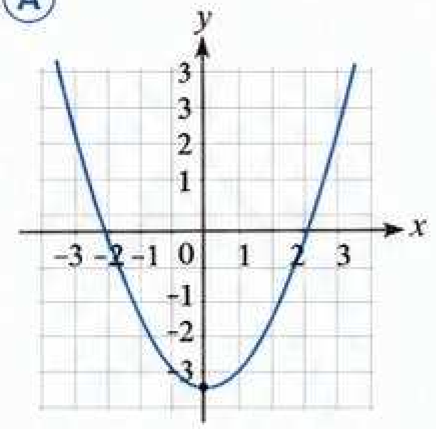
1. The domain of the function $f(x)=\dfrac{x+2}{x^2-4}$ is:

   Ⓐ $\mathbb{R}$ Ⓑ $\mathbb{R}-\{-2, 2\}$ Ⓒ $\mathbb{R}-\{2\}$ Ⓓ $\mathbb{R}-\{-2\}$

2. The range of the function $f(x)=x^2+4$ is:

   Ⓐ $\mathbb{R}$ Ⓑ $\mathbb{R}-\{4\}$ Ⓒ $[4, \infty)$ Ⓓ $(-\infty, 4]$

3. Which of the following graphs represents the function $f(x)=x^2-3$ ?

Ⓐ

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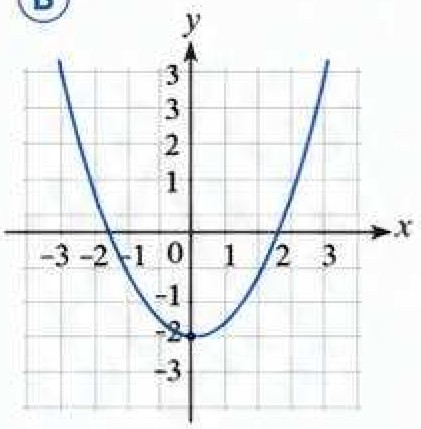
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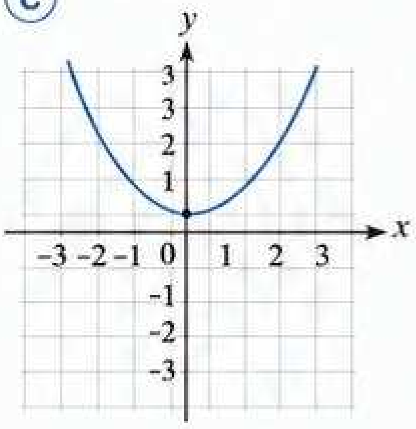
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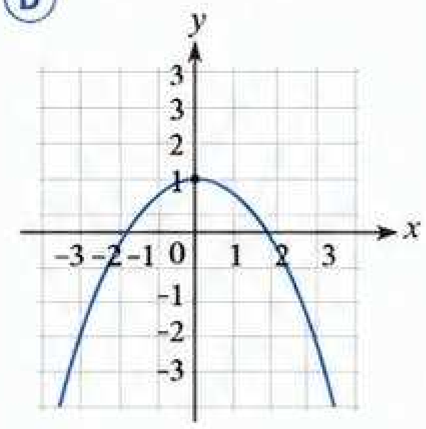

4. The domain of the function $f(x)=\dfrac{x-1}{x+3}$ is:

   Ⓐ $\mathbb{R}$ Ⓑ $\mathbb{R}-\{-3\}$ Ⓒ $\mathbb{R}-\{1\}$ Ⓓ $\mathbb{R}-\{-3, 1\}$

5. The simplified form of $\dfrac{x^2-4}{x+2}$ (where $x \neq -2$) is:

   Ⓐ $x+2$ Ⓑ $x-2$ Ⓒ $\dfrac{x^2}{x+2}$ Ⓓ $\dfrac{x-2}{x+2}$

6. If $f(x)=x^2-5$, then $f(3)$ equals:

   Ⓐ $-2$ Ⓑ $-4$ Ⓒ $4$ Ⓓ $9$

### B Complete (4 Marks)

1. The domain of the function $f(x)=\dfrac{x+5}{x^2-9}$ is ..............................................................
2. The vertex of the parabola $f(x)=x^2-6x+5$ is ( ...... , ...... ).
3. If $f(x)=\dfrac{x+1}{x-2}$ and $g(x)=\dfrac{x+3}{x-2}$, then the common domain of $f$ and $g$ is ....................................
4. The $x$-intercepts of the function $f(x)=x^2-4x$ are ..........................................................

### C Short Response (4 Marks)

1. Simplify: $\dfrac{x^2+3x}{x^2-1}+\dfrac{2x}{x+1}$ (where $x \neq 1, -1$).

..........................................................................................................................................

..........................................................................................................................................

2. Determine the domain of the function $f(x)=\dfrac{x^2-2x+1}{x^2+x-6}$.

..........................................................................................................................................

..........................................................................................................................................



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# UNIT 2 ASSESSMENT

## Algebra

### D Match Column A with Column B (4 Marks)

Match each item in Column A with its correct answer in Column B.

| Column A | Column B |
|---|---|
| 1 The domain of $f(x) = \dfrac{x+3}{x^2-4}$ • | Ⓐ $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ |
| 2 The range of $f(x) = x^2 + 1$ • | Ⓑ $(3, -1)$ |
| 3 The vertex of $f(x) = x^2 - 6x + 8$ • | Ⓒ $(-\infty, 1]$ |
| 4 The $x$-intercepts of $f(x) = x^2 - 5x$ • | Ⓓ $\{0, 5\}$ |

### E True or False (4 Marks)

Write **True** or **False** for each of the following statements.

1. The domain of $f(x) = \dfrac{x^2+1}{x+2}$ is $\mathbb{R} - \{-2\}$. ....................
2. The function $f(x) = x^2 - 3x + 2$ has minimum value $-\dfrac{9}{4}$. ....................
3. The graph of $f(x) = -(x-2)^2$ opens upward. ....................
4. If $f(x) = x^2 + 4$, then $f(x) \geq 4$ for all real values of $x$. ....................

### F Solve the Following (4 Marks)

Solve and show your steps.

1. Find the domain of $f(x) = \dfrac{x-1}{x^2+2x}$.

   ..........................................................................................

   ..........................................................................................

2. Find the range of $f(x) = x^2 - 4x + 5$.

   ..........................................................................................

   ..........................................................................................

3. Find the $x$-intercepts of $f(x) = x^2 + 2x - 8$.

   ..........................................................................................

   ..........................................................................................

4. Simplify: $\dfrac{x^2-1}{x+1} + \dfrac{2}{x-1}$ (where $x \neq -1, 1$).

   ..........................................................................................

   ..........................................................................................



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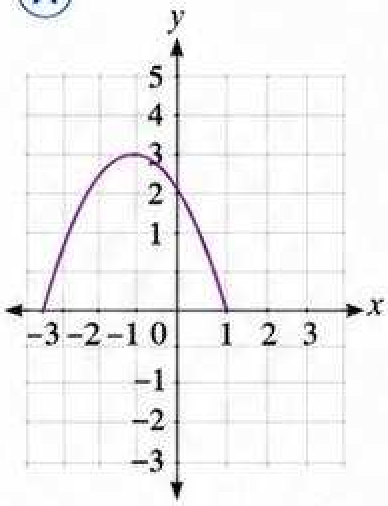
# UNIT 2 ASSESSMENT

## Algebra

### G Graphing (4 Marks)

Match each function with its correct graph.

1. $f(x) = x^2 + 2$
2. $f(x) = (x-1)^2 - 1$
3. $f(x) = -(x+1)^2 + 3$
4. $f(x) = x^2 - 4$

Ⓐ


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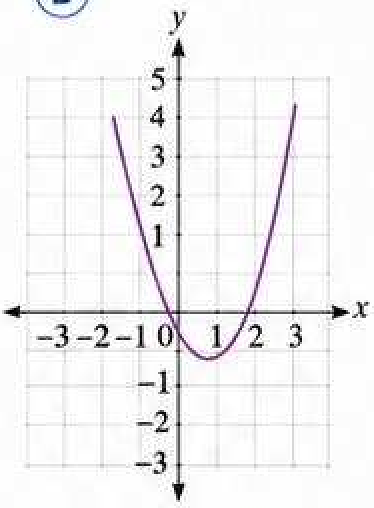

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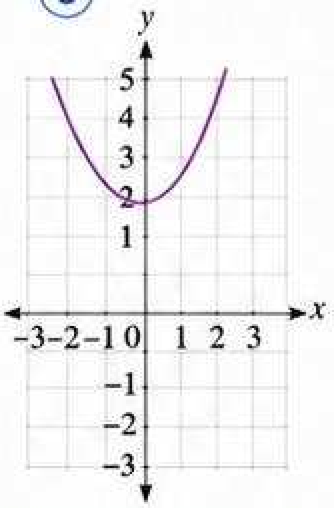

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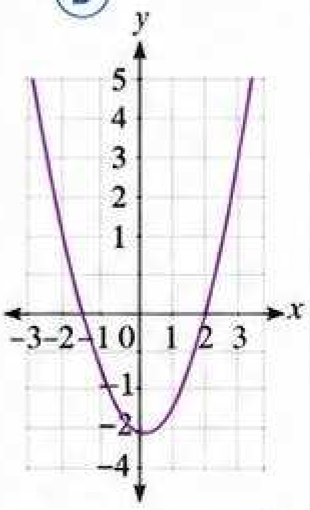

1 → .............. 2 → .............. 3 → .............. 4 → ..............

### H Application (4 Marks)

Solve each problem.

1. A rectangular park has an area given by $A(x) = x^2 + 6x + 5$ square meters.

   Find the possible values of $x$ for which the area equals 30 square meters.

   ..............................................................................

   ..............................................................................

2. The width of a garden is $(x+2)$ meters and its length is $(x-3)$ meters.

   If the area of the garden is 35 square meters, find the value(s) of $x$.

   ..............................................................................

   ..............................................................................

3. A ball is thrown upward and its height (in meters) after $t$ seconds is given by $h(t) = -5t^2 + 20t + 1$.

   After how many seconds will the ball reach its maximum height? What is that maximum height?

   ..............................................................................

   ..............................................................................

4. A function is given by $f(x) = \dfrac{x^2 - 9}{x - 3}$.

   Find the simplified form of $f(x)$ and state the domain of the original function.

   ..............................................................................

   ..............................................................................

### I Challenge (2 Marks)

For $f(x) = \dfrac{x^2 - 1}{x^2 - x}$, find all real values of $x$ for which $f(x)$ is undefined.

..............................................................................



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Lesson 4

# Similarity of Polygons

## Today I will learn:

- To identify similar polygons.
- To use the properties of similar polygons.
- To solve problems using similarity of polygons.

## Real Life Connection

Statue of King Menkheperre Thutmose III

Ancient artists used proportions to create beautiful statues. Today, we use the concept of similarity to compare shapes and solve real-life problems.

Can you find another example in real life where shapes are similar?

## Key Vocabulary

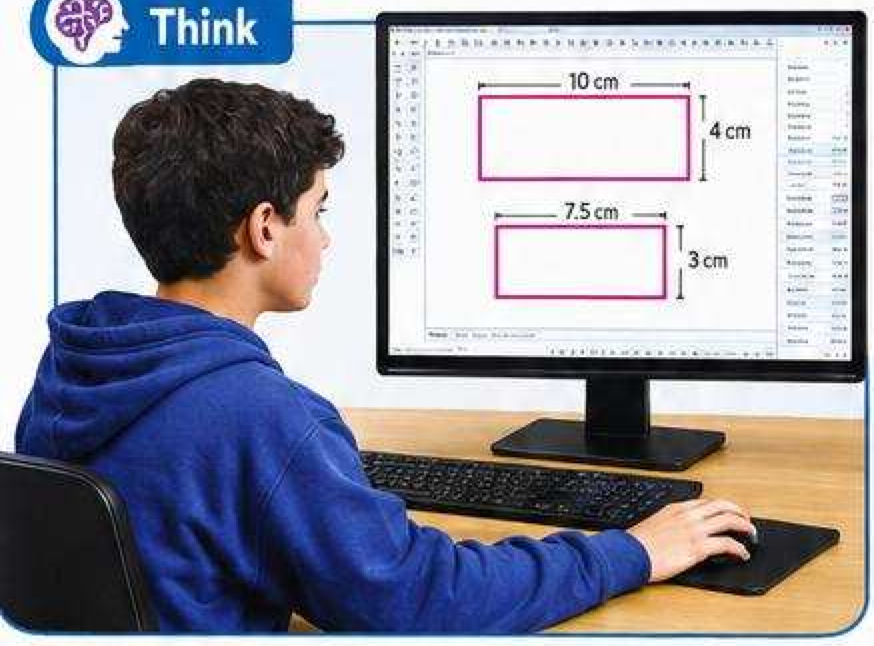
- Similarity
- Similar figures
- Similarity ratio
- Corresponding angles and sides
- Proportional

## Essential Ideas

When two polygons are similar:

- Their corresponding angles are equal.
- Their corresponding sides are proportional.

## Think




Marwan used a drawing program to design two rectangles as shown.

Do you think the two rectangles are similar? Why or why not?

## Before You Start

You should be able to:

- Measure angles.
- Find lengths of sides.
- Use ratios and proportions.
- Solve problems using perimeters and areas.

## Let's Learn!

### 1 Definition of Similar Polygons

Two polygons are similar if:

1. Their corresponding angles are equal.
2. Their corresponding sides are proportional.

### 2 Properties of Similar Polygons

If polygons *ABCD* and *EFGH* are similar, written as $ABCD \sim EFGH$, then:

- $m(\angle A) = m(\angle E)$, $m(\angle B) = m(\angle F)$, $m(\angle C) = m(\angle G)$, $m(\angle D) = m(\angle H)$
- $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = k$ (*k is the similarity ratio*)
- *The perimeter ratio* $= k$, *and the area ratio* $= k^2$.

## Math in Action

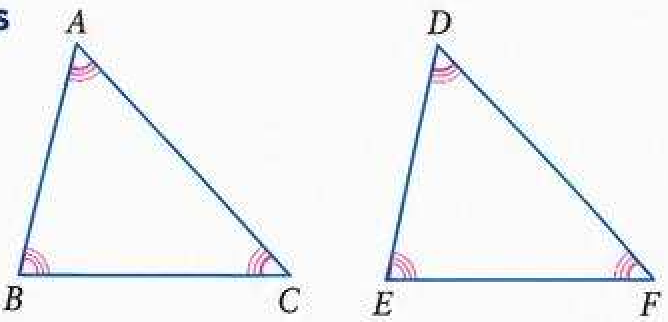
Architects make scale drawings of buildings.
Maps are similar to the real places.
Photos are resized using similarity.
Where else do we use similar shapes?

### 3 Naming Similar Polygons

*Write the vertices in order so that corresponding vertices match.*

*Example:* If $\triangle ABC \sim \triangle DEF$, then

$A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$.




## Symbols You Need

| Symbol | Meaning |
|---|---|
| $\sim$ | is similar to |
| $\angle$ | angle |
| $m(\angle A)$ | measure of angle A |
| $\frac{AB}{EF}$ | ratio of side AB to side EF |
| $k$ | similarity ratio |

**Common Mistakes**

- *Matching vertices in the wrong order.*
- *Thinking equal sides mean the figures are similar.*
- *Forgetting that all corresponding angles must be equal.*
- *Not keeping the same units.*



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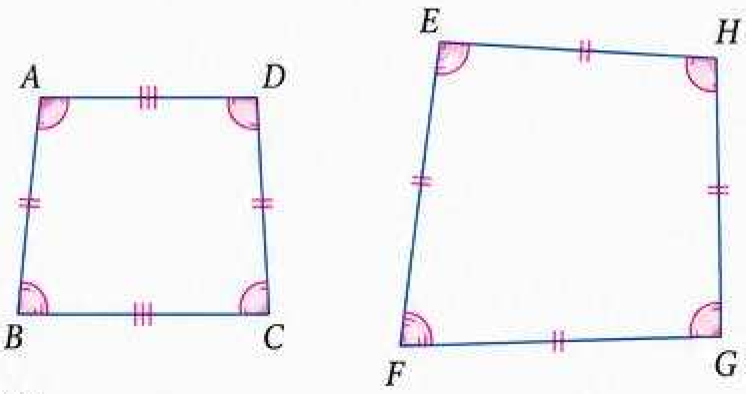
## 4 Corresponding Parts in Similar Polygons

In similar polygons, the vertices, sides and angles that are in the same position are called corresponding.

**Example**

If $ABCD \sim EFGH$, then:

- **Corresponding vertices:**
  $A \leftrightarrow E,\ B \leftrightarrow F,\ C \leftrightarrow G,\ D \leftrightarrow H$
- **Corresponding sides:**
  $\overline{AB} \leftrightarrow \overline{EF},\ \overline{BC} \leftrightarrow \overline{FG},\ \overline{CD} \leftrightarrow \overline{GH},\ \overline{DA} \leftrightarrow \overline{HE}$
- **Corresponding angles:**
  $\angle A \leftrightarrow \angle E,\ \angle B \leftrightarrow \angle F,\ \angle C \leftrightarrow \angle G,\ \angle D \leftrightarrow \angle H$




### Key Reminder

If two polygons are similar, all corresponding angles are equal and all corresponding sides are proportional.

### Similarity Ratios

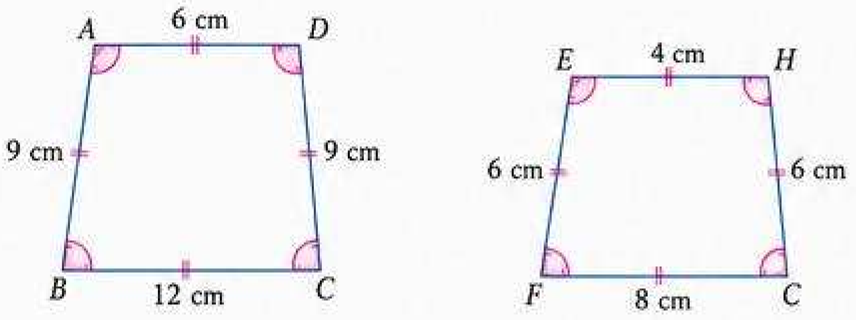
The ratio between any pair of corresponding sides is called the similarity ratio ($k$). It is the same for all pairs.

## 5 Finding the Similarity Ratio

The similarity ratio $k$ can be found by dividing the length of any side in one polygon by the length of its corresponding side in the other polygon.

**Example 1**

In the figure, $ABCD \sim EFGH$. Find the similarity ratio $k$ from $ABCD$ to $EFGH$.




**Solution**

The corresponding sides are:
$AB \leftrightarrow EF,\ BC \leftrightarrow FG,$
$CD \leftrightarrow GH,\ DA \leftrightarrow HE$

Use any pair to find $k$.

$$k = \frac{AB}{EF} = \frac{9}{6} = \frac{3}{2}$$

Check with another pair:

$$k = \frac{BC}{FG} = \frac{12}{8} = \frac{3}{2}$$

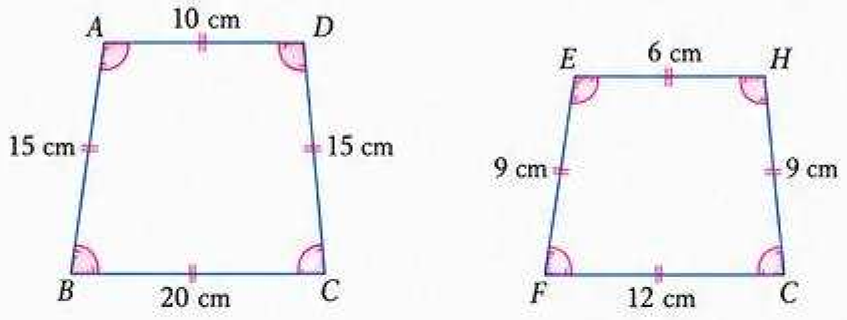
So, the similarity ratio from $ABCD$ to $EFGH$ is $\frac{3}{2}$.

### Quick Rule

If two polygons are similar, you can use any pair of corresponding sides to find the similarity ratio.

**Example 2**

In the figure, $ABCD \sim EFGH$. Find the similarity ratio from $EFGH$ to $ABCD$.




**Solution**

$$k = \frac{EF}{AB} = \frac{6}{10} = \frac{3}{5}$$

Check: $k = \frac{FG}{BC} = \frac{12}{20} = \frac{3}{5}$

So, the similarity ratio from $EFGH$ to $ABCD$ is $\frac{3}{5}$.

### Important Note

The similarity ratio depends on the order of the polygons.

If $ABCD \sim EFGH$ with $k = \frac{3}{2}$, then $EFGH \sim ABCD$ with $k = \frac{2}{3}$.

## 6 More Properties of Similar Polygons

**Property 1: Perimeter Ratio**

If two polygons are similar with similarity ratio $k$, then the ratio of their perimeters is $k$.

**Example**

If $ABCD \sim EFGH$ with $k = \frac{3}{2}$ and the perimeter of $ABCD$ is 36 cm, then the perimeter of $EFGH$ is:

Perimeter of $EFGH = k \times$ Perimeter of $ABCD$

$$= \frac{3}{2} \times 36 = 54 \text{ cm}$$

**Property 2: Area Ratio**

If two polygons are similar with similarity ratio $k$, then the ratio of their areas is $k^2$.

**Example**

If $ABCD \sim EFGH$ with $k = \frac{3}{2}$ and the area of $ABCD$ is 72 $\text{cm}^2$, then the area of $EFGH$ is:

Area of $EFGH = k^2 \times$ Area of $ABCD$

$$= \left(\frac{3}{2}\right)^2 \times 72 = \frac{9}{4} \times 72 = 162 \text{ cm}^2$$

### Think & Discuss

If two similar polygons have perimeters 30 cm and 45 cm, what is the similarity ratio between them?

What is the ratio of their areas?

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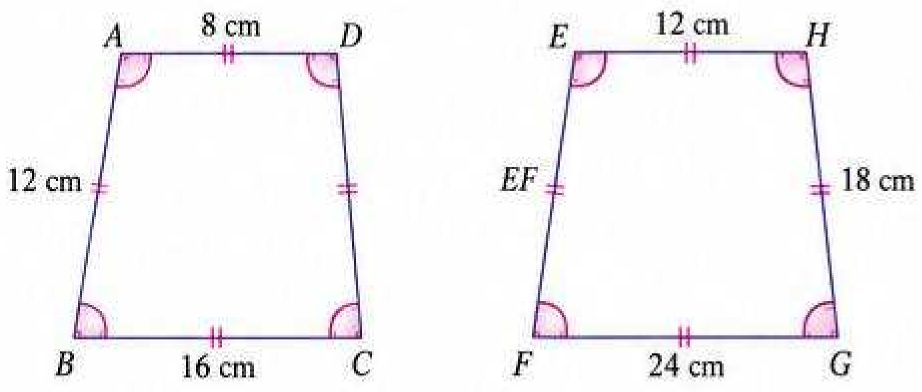
## 7 Solving Problems Using Similar Polygons

We can use similarity to find unknown side lengths, perimeters or areas.

**Example 1**

In the figure, $ABCD \sim EFGH$. Find the length of $EF$.




**Solution**

Since $ABCD \sim EFGH$, corresponding sides are proportional.

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{HE} = k$$

Use $\frac{BC}{FG}$ to find $k$.

$$k = \frac{BC}{FG} = \frac{16}{24} = \frac{2}{3}$$

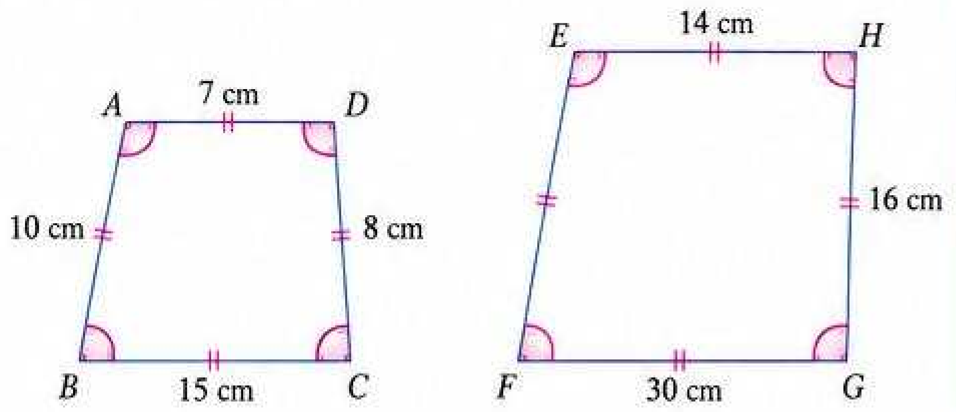
Use $\frac{AB}{EF} = k$

$$\frac{12}{EF} = \frac{2}{3} \Rightarrow EF = \frac{12 \times 3}{2} = 18 \text{ cm}$$

So, $EF = 18$ cm.

**Example 2**

In the figure, $ABCD \sim EFGH$ and the perimeter of $ABCD$ is 50 cm.
Find the perimeter of $EFGH$.




**Solution**

Find the similarity ratio $k$ using one pair:

$$k = \frac{EH}{AD} = \frac{14}{7} = 2$$

The ratio of perimeters $= k$

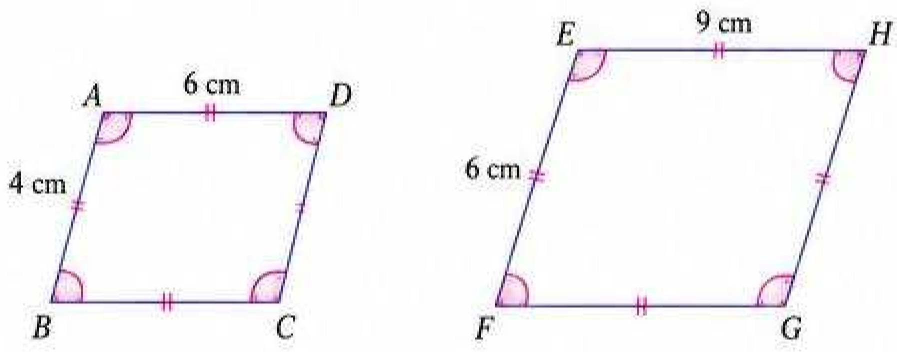
Perimeter of $EFGH = k \times$ Perimeter of $ABCD$

$= 2 \times 50 = 100$ cm

So, the perimeter of $EFGH$ is 100 cm.

**Example 3**

In the figure, $ABCD \sim EFGH$. The area of $ABCD$ is 36 cm$^2$. Find the area of $EFGH$.




**Solution**

Find similarity ratio $k$ using one pair:

$$k = \frac{EH}{AD} = \frac{9}{6} = \frac{3}{2}$$

The ratio of areas $= k^2$

Area of $EFGH = k^2 \times$ Area of $ABCD$

$$= \left(\frac{3}{2}\right)^2 \times 36 = \frac{9}{4} \times 36 = 81 \text{ cm}^2$$

So, the area of $EFGH$ is 81 cm$^2$.

### Helpful Tip

When you find any similarity ratio using one pair of corresponding sides, you can use it for all the other pairs.

### Remember!

| If... | Then... |
|---|---|
| Polygons are similar | Corresponding angles are equal and sides are proportional. |
| Perimeters | Ratio of perimeters $= k$ |
| Areas | Ratio of areas $= k^2$ |
| Same size | $k = 1$ |

### Real World

Maps, photos, blueprints and models of cars, planes and buildings are examples of similar figures.

### Think!

Two similar polygons have similarity ratio $k = \frac{2}{3}$.
If the area of the larger polygon is 54 cm$^2$, what is the area of the smaller polygon?

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## 8 Check Your Understanding

Complete the following:

1. If $ABCD \sim EFGH$ and $\frac{AB}{EF} = \frac{3}{5}$, then the similarity ratio $k =$ ____________ .
2. If two similar polygons have perimeters 24 cm and 40 cm, then the similarity ratio $k =$ ____________ .
3. If two similar polygons have similarity ratio $k = \frac{3}{4}$, then the ratio of their areas = ____________ .
4. If the area of one polygon is 64 cm$^2$ and the similarity ratio is $k = \frac{1}{2}$, then the area of the other polygon is ____________ cm$^2$.
5. If $ABCD \sim EFGH$ and $AB = 9$ cm corresponds to $EF = 15$ cm, then the similarity ratio from $ABCD$ to $EFGH$ is ______________ .

**Match the following:**

| | Column A | | Column B |
|---|---|---|---|
| 1 | Ratio of perimeters | a | $k$ |
| 2 | Ratio of areas | b | $k^2$ |
| 3 | Similarity ratio | c | 1 |
| 4 | Same size polygons | d | Equal |



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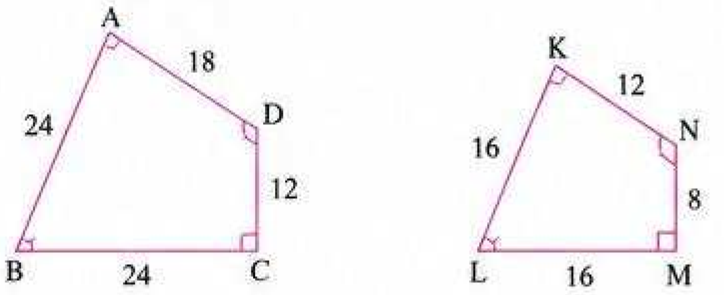
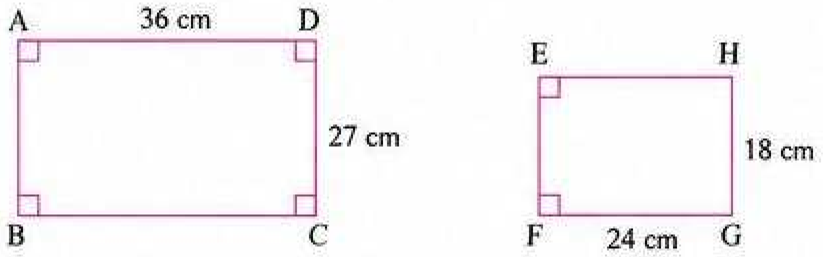
## A Choose the correct answer:

**1** In the figure, what is the similarity ratio of $KLMN$ to $ABCD$ ?



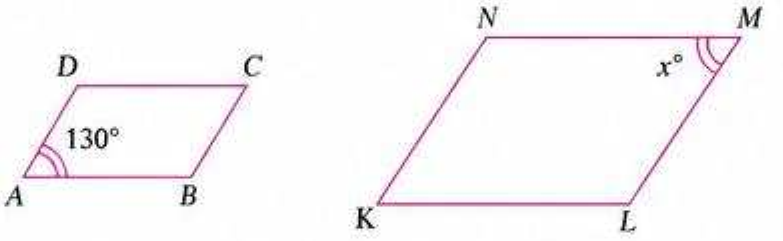

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{1}$

**2** In the figure, what is the similarity ratio of $ABCD$ to $EFGH$ ?



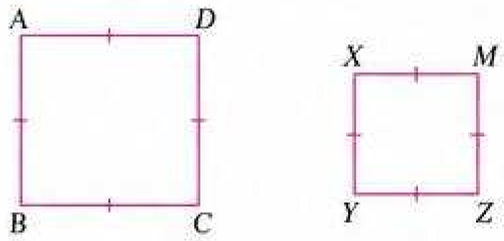

(a) 27 (b) 9 (c) 24 (d) 18

**3** Two parallelograms $ABCD$ and $KLMN$ are similar as shown. What is the value of $x$ ?



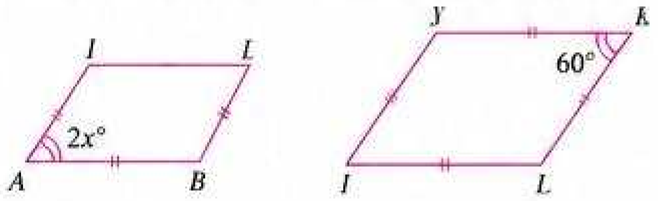

(a) 50° (b) 130° (c) 60° (d) 70°

**4** If the perimeters of similar squares $ABCD$ and $XYZM$ are in the ratio $\frac{3}{2}$. What is the ratio of their areas?



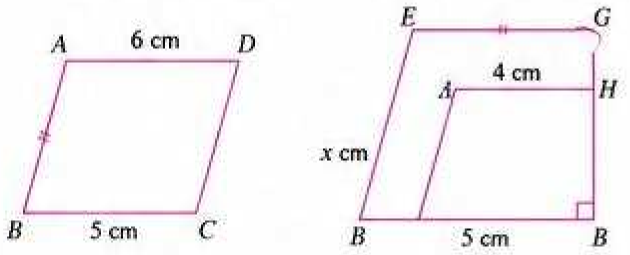

(a) $\frac{2}{3}$ (b) $2^2$ (c) $\frac{3}{2}$ (d) $\frac{4}{9}$

**5** If the measure of $\angle K = 60°$ in the parallelogram $IJKL$ similar to $ABCD$, what is the measure of $\angle A$ ?




(a) 120° (b) 60° (c) 80° (d) 100°

**6** If $ABCD \sim EBBHG$, what is the value of $x$?



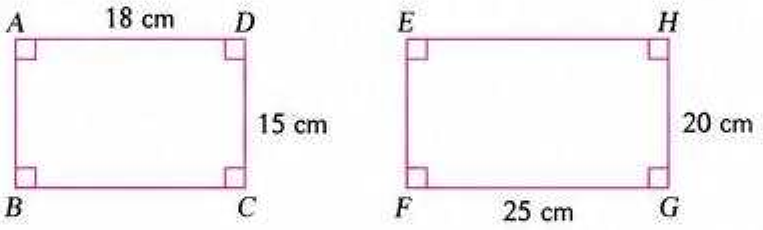

(a) 2.5 (b) 5 (c) 4 (d) 7.5

**7** Two similar rectangles have length 34 cm and width 17 cm. If the ratio of their lengths is 2 : 3, what is the width of the larger rectangle?

(a) 17 cm (b) 34 cm (c) 51 cm (d) 68 cm

## B Answer the following:

**8** Are the rectangles shown similar? Why or why not?

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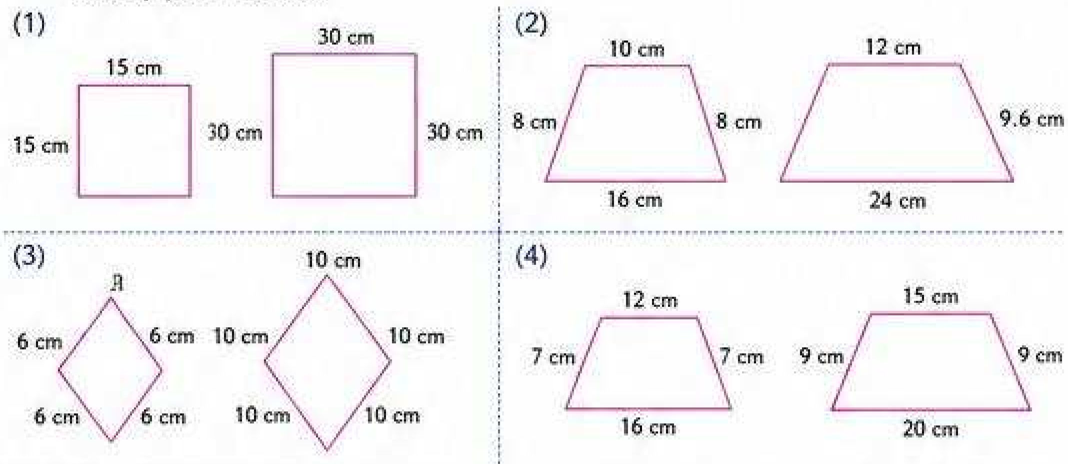
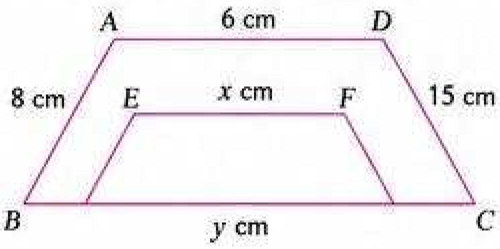
**9** Which of the following pairs of polygons are similar? Justify your answer.




**10** In the figure, $ABCD \sim EBGF$. Find the values of $x$ and $y$.




$x$ = .................... $y$ = ....................

**11** If the ratio of the areas of two similar polygons is 9 : 16 and the area of the smaller polygon is 36 $cm^2$, find the area of the larger polygon.

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**12** Two similar trapezoids have bases 5 cm and 8 cm, height 6 cm and 10 cm, and the perimeter of the smaller trapezoid is 80 cm. Find the perimeter of the larger trapezoid.

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" In geometry, similarity helps us see the same shape in different sizes. "



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# Challenge Time

## A Choose the correct answer:

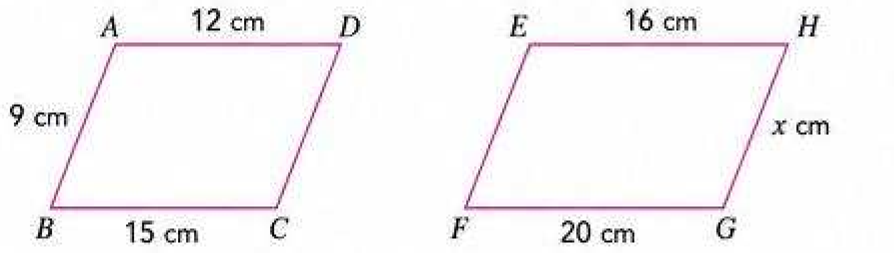
**1** If $ABCD \sim EFGH$ and $\frac{AB}{EF} = \frac{2}{5}$, then the similarity ratio from $ABCD$ to $EFGH$ is :

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

**2** Two similar polygons have perimeters 36 cm and 54 cm. The ratio of their areas is :

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

**3** In the figure, $ABCD \sim EFGH$. The value of $x$ is :

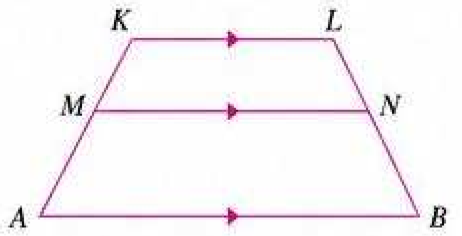



(a) 10 cm (b) 11 cm (c) 12 cm (d) 13 cm

**4** If the area of the larger square is 81 $cm^2$ and the similarity ratio from the smaller square to the larger square is $\frac{1}{3}$, then the area of the smaller square is :

(a) 9 $cm^2$ (b) 18 $cm^2$ (c) 27 $cm^2$ (d) 36 $cm^2$

**5** In the figure, $KL \parallel MN \parallel AB$.
If $LM = 6$ cm, $MN = 8$ cm, $AB = 20$ cm, then $KL = ?$

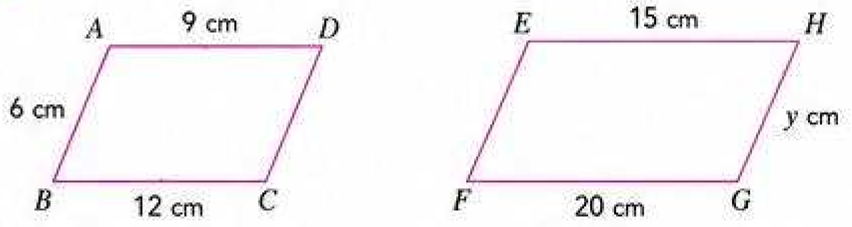



(a) 5 cm (b) 10 cm (c) 12 cm (d) 15 cm

**6** $ABCD \sim EFGH$. If the perimeter of $ABCD$ is 48 cm and the similarity ratio from $ABCD$ to $EFGH$ is $\frac{3}{4}$, then the perimeter of $EFGH$ is :

(a) 36 cm (b) 52 cm (c) 64 cm (d) 72 cm

**7** In the figure, $ABCD \sim EFGH$. Find $y$.




(a) 8 cm (b) 10 cm (c) 12 cm (d) 16 cm

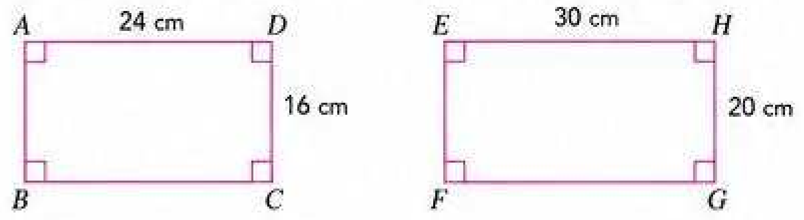
**8** If two similar polygons have areas 25 $cm^2$ and 100 $cm^2$, then the similarity ratio between them is :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

## B Answer the following:

**9** Are the following pairs of polygons similar? Why or why not?

(1)

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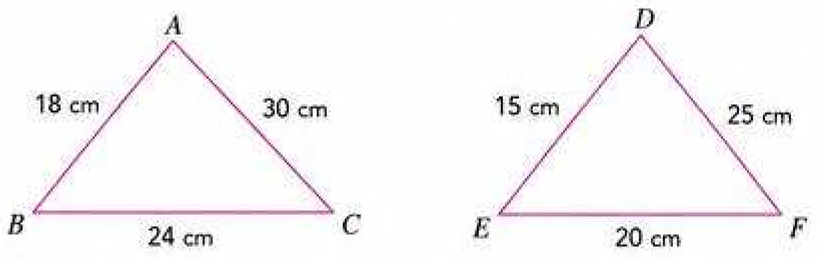
(2)

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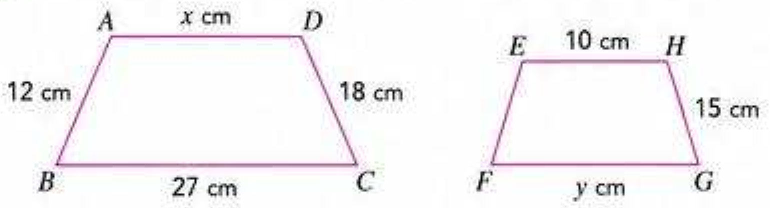
**10** In the figure, $ABCD \sim EFGH$. Find the values of $x$ and $y$.




$x =$ ..................................... $y =$ .....................................

**11** Two similar triangles have a similarity ratio (small to large) of $\frac{2}{3}$.
If the area of the larger triangle is 72 $cm^2$, find the area of the smaller triangle.

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## C Solve the following problems:

**12** The scale of a map is 1 : 250000.
If the distance between two cities on the map is 8 cm, what is the actual distance between them in kilometers?

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**13** An architect draws a plan of a garden using a scale of 1 : 500.
If the actual length of the garden is 45 m and its width is 30 m, find the length and width of the plan in centimeters.

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**14** Two similar parallelograms have bases 18 cm and 30 cm and heights 12 cm and 20 cm respectively.
Find the ratio of their areas.

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**15** The perimeter of a regular hexagon is 72 cm.
Another regular hexagon is similar to it with similarity ratio $\frac{2}{3}$ (from the second to the first).
Find the perimeter of the second hexagon.

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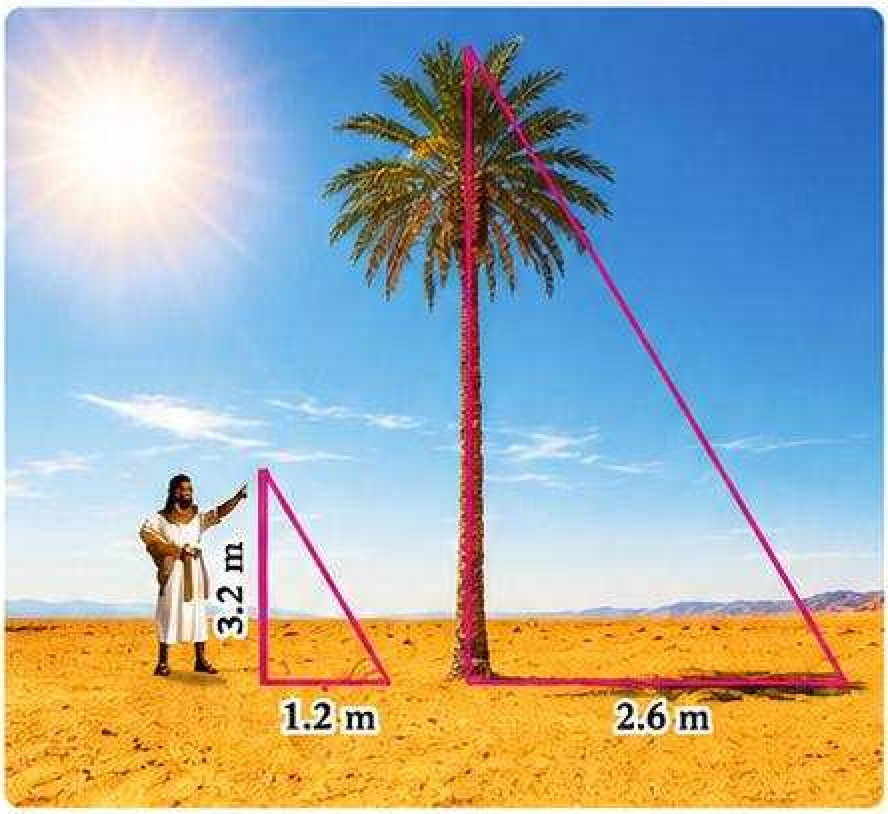
# Similarity of Triangles




We need to know the height of many things before using them.

We can use proportions to find unknown lengths.

For example, to know the height of a palm tree, we can use the height of a person and his shadow as a reference.

**In this lesson, we will learn about the similarity of triangles and some of its cases**, and how to use it to solve problems in real life.

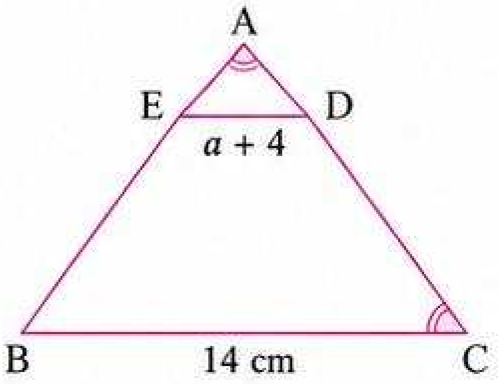
### Let's Learn

- Identify similar triangles.
- Use similarity to find unknown lengths.
- Find the perimeter of similar triangles.
- Find the area of similar triangles.
- Apply similarity in real-life problems.
- Solve real problems on similarity of triangles.

## Think & Discuss!

In the figure, if it is given that $\Delta ABC$ has perimeter 49 cm, what is the perimeter of $\Delta AED$?

Find the two correct answers from the following:




(a) Perimeter of $\Delta AED = 35$ cm

(b) Perimeter of $\Delta AED = 18$ cm

### Key Words

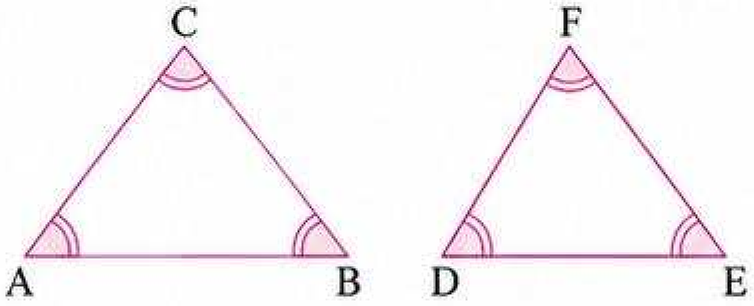
- Similar triangles
- Corresponding angles
- Corresponding sides
- Perimeter
- Area
- Scale factor

## We Will Learn !

### Cases of Similar Triangles

If two triangles have two pairs of equal angles, then they are **similar**.

If in $\Delta ABC$ and $\Delta DEF$:




- $m(\angle C) = m(\angle F)$
- $m(\angle A) = m(\angle D)$
- $m(\angle B) = m(\angle E)$

Then:

$\Delta ABC \sim \Delta DEF$

### Remember

In this lesson, you will need to know the following:

- $m(\angle C) = m(\angle F)$
- $\frac{AB}{DB} = \frac{BC}{BF} = \frac{AC}{DF}$



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Lesson 2

# Similarity of Triangles

Unit 3 Geometry

## Main Rule

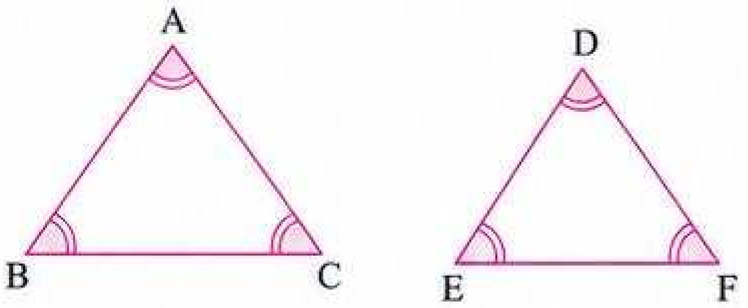
**If two triangles have two pairs of equal angles, then they are similar.**

If in $\Delta ABC$ and $\Delta DEF$:

- $m(\angle C) = m(\angle F)$
- $m(\angle A) = m(\angle D)$
- $m(\angle B) = m(\angle E)$

Then:

$\Delta ABC \sim \Delta DEF$




## Important Note

Similarity means that:

- Corresponding angles are equal.
- Corresponding sides are in proportion.
- The shape is the same, but the size may be different.

## Symbols Used

| | | |
|---|---|---|
| $m(\angle A)$ | : | measure of angle A |
| $AB$ | : | length of side AB |
| $\Delta ABC$ | : | triangle ABC |
| $\sim$ | : | is similar to |
| $\parallel$ | : | is parallel to |
| $\perp$ | : | is perpendicular to |

## Solved Example 1

Apply the definition of similar triangles

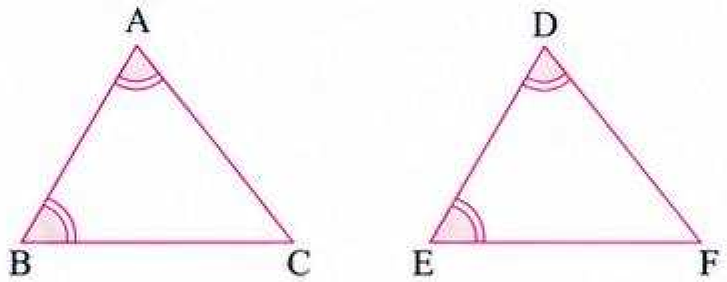
In the figure:

$m(\angle A) = m(\angle D)$

$m(\angle B) = m(\angle E)$

Prove that:

$\Delta ABC \sim \Delta DEF$




### Solution

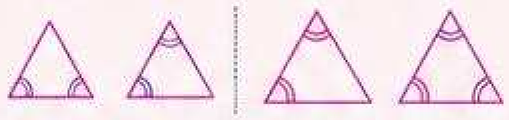
1. Given: $m(\angle A) = m(\angle D)$ , $m(\angle B) = m(\angle E)$
2. Then: $m(\angle C) = m(\angle F)$ (Angles in a triangle sum to 180°)
3. So: $m(\angle C) = m(\angle F)$ , $m(\angle A) = m(\angle D)$ , $m(\angle B) = m(\angle E)$
4. By the definition of similar triangles:

$\Delta ABC \sim \Delta DEF$

## Common Mistakes

1. Thinking that one pair of equal angles is enough for similarity.

✗ Wrong ✓ Correct



2. Not writing the vertices in the correct order.

✗ Wrong

$\Delta ABC \sim \Delta EDF$

✓ Correct

$\Delta ABC \sim \Delta DEF$

3. Not comparing corresponding sides.

✗ Wrong

$\frac{AB}{DF} = \frac{AC}{EF}$

✓ Correct

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

## Solved Example 2

Use similarity to find unknown sides

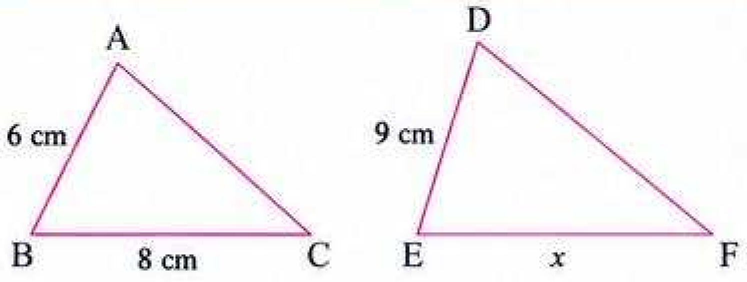
In the figure:

$\Delta ABC \sim \Delta DEF$

$AB = 6$ cm, $DE = 9$ cm,

$BC = 8$ cm

Find $EF$.




### Solution

1. Since $\Delta ABC \sim \Delta DEF$, the corresponding sides are in proportion:
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
2. Use: $\frac{AB}{DE} = \frac{BC}{EF}$
3. Substitute: $\frac{6}{9} = \frac{8}{x}$
4. Cross multiply: $6x = 9 \times 8 = 72$
5. $x = \frac{72}{6} = 12$
6. So, $EF = 12$ cm

**Quick Check**

Scale factor $k = \frac{DE}{AB} = \frac{9}{6} = \frac{3}{2}$

$EF = BC \times k = 8 \times \frac{3}{2} = 12$ cm ✓

## Comparison Table

| | Congruent Triangles | Similar Triangles |
|---|---|---|
| Corresponding angles | All equal | All equal |
| Corresponding sides | All equal | In proportion |
| Shape | Same | Same |
| Size | Same | May be different |



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# Similarity of Triangles

## ★ Solved Example 3
Use similarity to find an unknown length

In the figure:

$\Delta ABC \sim \Delta DEC$

$m(\angle ACB) = m(\angle DCE)$

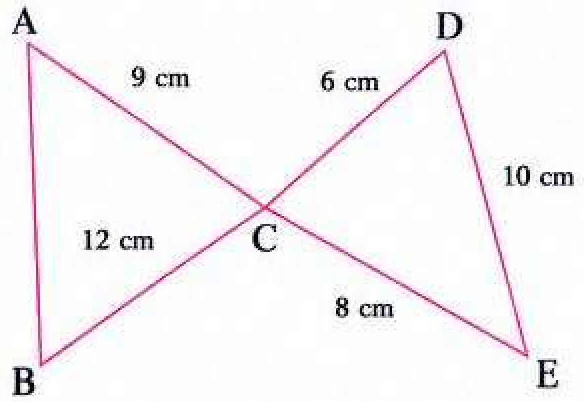
Given:

$AC = 9$ cm, $BC = 12$ cm,

$DC = 6$ cm, $CE = 8$ cm,

$DE = 10$ cm

Find $AB$.




### Solution

1. Check the ratios of corresponding sides:
   $\frac{AC}{DC} = \frac{9}{6} = \frac{3}{2}$ , $\frac{BC}{EC} = \frac{12}{8} = \frac{3}{2}$ $\therefore \frac{AC}{DC} = \frac{BC}{EC}$
2. Given that: $m(\angle ACB) = m(\angle DCE)$
3. $\therefore \Delta ABC \sim \Delta DEC$ (SAS similarity)
4. Corresponding sides are in proportion:
   $\frac{AB}{DE} = \frac{AC}{DC} = \frac{BC}{EC} = \frac{3}{2}$
5. $DE = 10$ cm $\therefore \frac{AB}{10} = \frac{3}{2}$
6. $AB = \frac{3}{2} \times 10 = 15$ cm

**Quick Check**

Scale factor from $\Delta DEC$ to $\Delta ABC = \frac{3}{2}$.

So each side of $\Delta DEC$ multiplied by $\frac{3}{2}$ gives the corresponding side in $\Delta ABC$.

$10 \times \frac{3}{2} = 15$ ✓

## ! Important Note

- When two triangles are similar:
  - Corresponding angles are equal.
  - Corresponding sides are in proportion.
- Write the vertices in the same order.

## How to Use Similarity?

1. Write the similarity statement.
2. Identify corresponding angles and sides.
3. Set up proportions.
4. Solve for the unknown.
5. Check your answer.

## ✕ Common Mistakes

1. Comparing non-corresponding sides.
   X Wrong
   $\frac{AB}{DE} = \frac{AC}{BC}$
   ✓ Correct
   $\frac{AB}{DE} = \frac{AC}{DC} = \frac{BC}{CE}$
2. Not writing the similarity statement.
   X Wrong
   Solve without writing $\Delta ABC \sim \Delta DEC$
   ✓ Correct
   Write $\Delta ABC \sim \Delta DEC$ first.
3. Using the wrong scale factor or mixing directions.
   X Wrong
   $\frac{AB}{DE} = \frac{2}{3}$ (should be $\frac{3}{2}$)
   ✓ Correct
   From $\Delta DEC$ to $\Delta ABC$, the scale factor is $\frac{3}{2}$.

## ★ Solved Example 4
Use similarity to find the area of a triangle

In the figure:

$\Delta ABC \sim \Delta DEF$

Given:

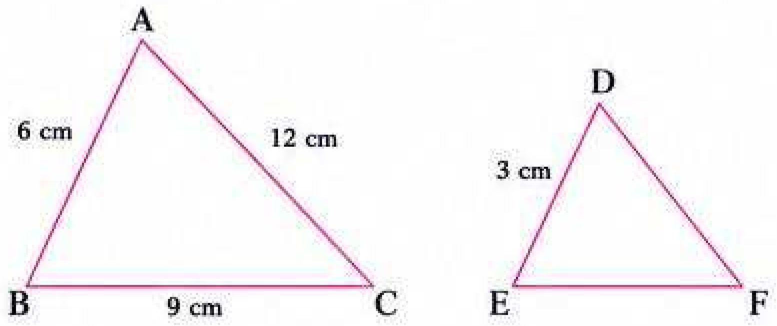
$AB = 6$ cm, $BC = 9$ cm,

$CA = 12$ cm

$DE = 3$ cm

The area of $\Delta DEF = 18\ cm^2$

Find the area of $\Delta ABC$.




### Solution

1. Since $\Delta ABC \sim \Delta DEF$, the ratio of corresponding sides is constant.
   $\frac{AB}{DE} = \frac{6}{3} = 2$ ⇒ Scale factor from $\Delta DEF$ to $\Delta ABC = 2$
2. The ratio of areas = (scale factor)$^2$ = $2^2$. = 4
3. Area of $\Delta ABC$ = Area of $\Delta DEF \times 4$
   $= 18 \times 4 = 72\ cm^2$

**Quick Check**

Check using another side:

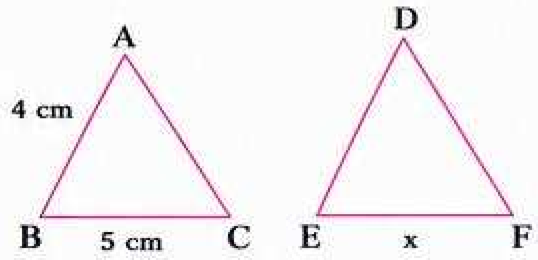
$\frac{BC}{EF} = 2$ or $\frac{CA}{DF} = 2$

So the scale factor is 2,

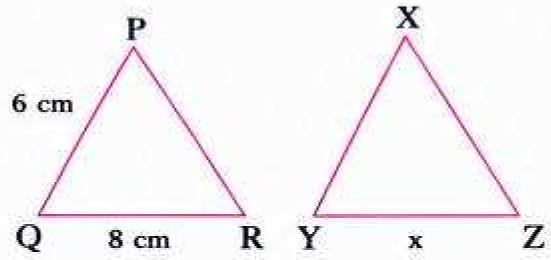
and areas multiply by $2^2 = 4$ ✓

## Practice Yourself
Answer the following:

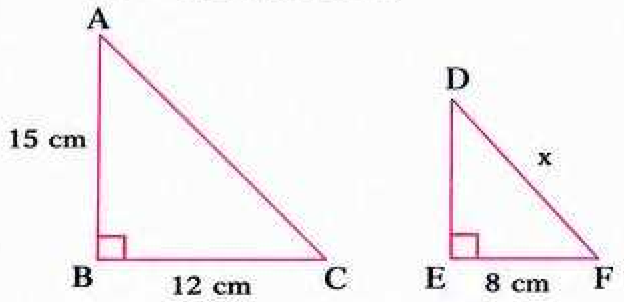
1. In the figure, $\Delta ABC \sim \Delta DEF$.
   If $AB = 4$ cm, $DE = 8$ cm and $BC = 5$ cm, find $EF$.




2. In the figure, $\Delta PQR \sim \Delta XYZ$.
   If $PR = 6$ cm, $XY = 9$ cm and $QR = 8$ cm, find $YZ$.




4. In the figure, $\Delta ABC \sim \Delta DEF$.
   If $BC = 12$ cm, $EF = 8$ cm and $AC = 15$ cm, find $DF$.



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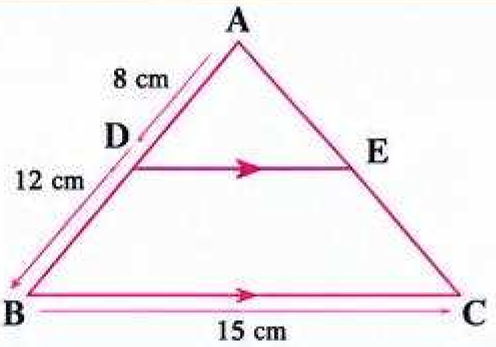
# Similarity of Triangles

## Solved Example 5
Use similarity with parallel lines

In the figure :
$DE \parallel BC$ , $D \in AB$ , $E \in AC$
$AB = 12$ cm , $AD = 8$ cm
$BC = 15$ cm
Find $DE$.



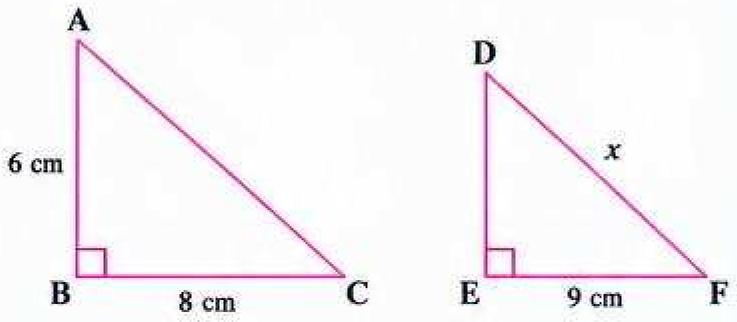

### Solution

1. Since $DE \parallel BC$ , then $\angle ADE = \angle ABC$ , $\angle AED = \angle ACB$ and $\angle A$ is common.
2. $\therefore \ \Delta ADE \sim \Delta ABC$ (AA similarity)
3. Corresponding sides are in proportion :
$$\frac{DE}{BC} = \frac{AD}{AB}$$
4. Substitute : $\frac{DE}{15} = \frac{8}{12} = \frac{2}{3}$
5. $DE = 15 \times \frac{2}{3} = 10$ cm
6. So , $DE = 10$ cm

**Quick Check**
If $DE \parallel BC$ , then
$\Delta ADE \sim \Delta ABC$ and
$\frac{DE}{BC} = \frac{AD}{AB} = \frac{AE}{AC}$ . ✓

## Solved Example 6
Find an unknown length in a right triangle

In the figure :
$\Delta ABC \sim \Delta DEF$
$\angle B = \angle E = 90°$
$AB = 6$ cm , $BC = 8$ cm
$EF = 9$ cm
Find $DF$.




### Solution

1. Since $\angle B = \angle E = 90°$ and one acute angle is equal,
$\Delta ABC \sim \Delta DEF$ (AA similarity).
2. Corresponding sides are in proportion : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
3. Find the scale factor $k = \frac{EF}{BC} = \frac{9}{8}$
4. $AC = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ cm
5. $DF = AC \times k = 10 \times \frac{9}{8} = 11.25$ cm
6. So , $DF = 11.25$ cm

**Quick Check**
In similar right triangles, the corresponding hypotenuses are in the same ratio as the corresponding legs. ✓

## Important Note

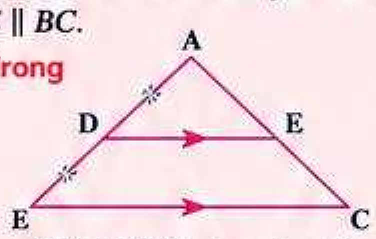
- A line parallel to one side of a triangle divides the other two sides proportionally and forms a smaller triangle similar to the original one.
- Always check the angle conditions (parallel lines → equal corresponding angles).

## Tips

When a problem has $DE \parallel BC$ :
1. Prove similarity using $AA$.
2. Use proportions to find unknown sides or areas.

## Common Mistakes

1. Thinking that $AD = BD$ just because $DE \parallel BC$.
✗ Wrong




$AD \neq BD$ (unless given)

2. Using the ratio in the wrong order.
✗ Wrong
$$\frac{DE}{AD} = \frac{BC}{AB}$$
✓ Correct
$$\frac{DE}{BC} = \frac{AD}{AB}$$

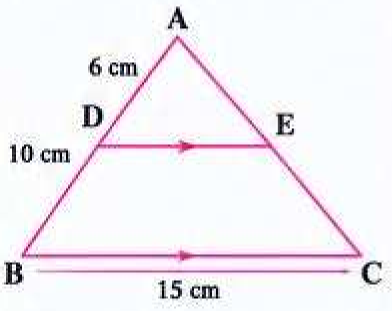
3. Forgetting to state the similarity.
✗ Wrong
$$\frac{DE}{BC} = \frac{AD}{AB}$$
✓ Correct
In $\Delta ADE \sim \Delta ABC$
$$\frac{DE}{BC} = \frac{AD}{AB}$$

## Summary

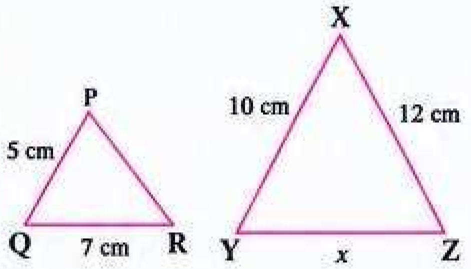
- Two triangles are similar if :
  - AA , SAS or SSS.
  - Corresponding angles are equal.
  - Corresponding sides are in proportion.
  - Similar triangles have the same shape.
  - Areas of similar triangles are in the square of the scale factor.

## Practice Yourself
Answer the following:

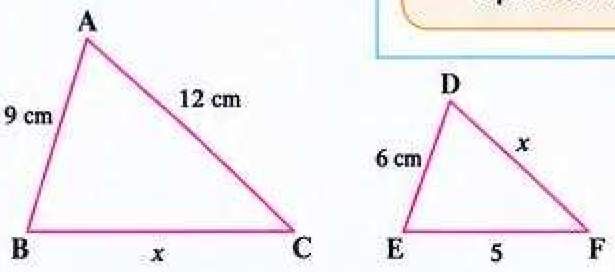
1. In the figure , $DE \parallel BC$.
If $AB = 10$ cm , $AD = 6$ cm and $BC = 15$ cm , find $DE$.




2. In the figure , $\Delta PQR \sim \Delta XYZ$.
If $PQ = 5$ cm , $QR = 7$ cm , $XY = 10$ cm and $XZ = 12$ cm , find $YZ$.




4. In the figure , $\Delta ABC \sim \Delta DEF$.
If $AB = 9$ cm , $AC = 12$ cm , $DE = 6$ cm and $DF = 8$ cm , find $BC$.



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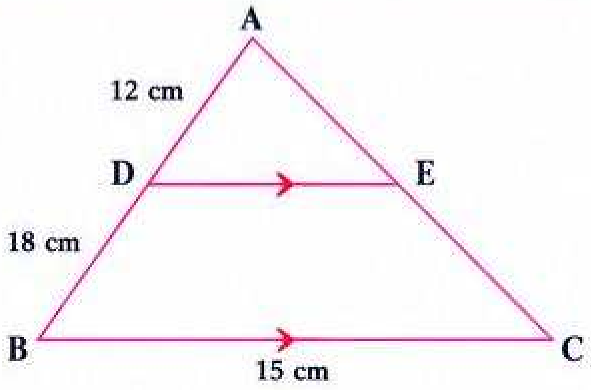
# Similarity of Triangles

## ★ Solved Example 7 — Use similarity to find an unknown length

In the figure:
$DE \parallel BC,\ D \in AB,\ E \in AC$
$AB = 18$ cm, $AD = 12$ cm,
$BC = 15$ cm
Find $DE$.



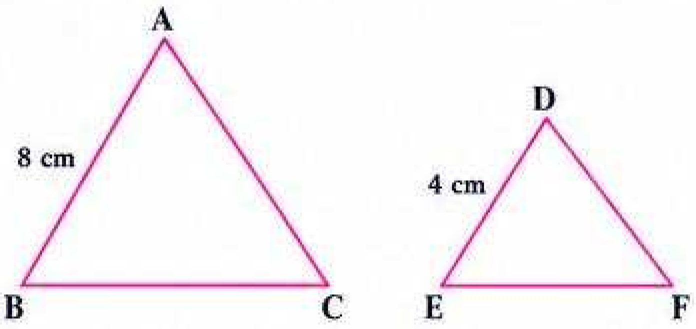

### Solution

1. Since $DE \parallel BC$, then $\Delta ADE \sim \Delta ABC$ (AA similarity).
2. Corresponding sides are in proportion:
$$\frac{AD}{AB} = \frac{DE}{BC}$$
3. Substitute the given values:
$$\frac{12}{18} = \frac{DE}{15}$$
4. Cross multiply: $DE = \frac{12 \times 15}{18} = \frac{180}{18} = 10$
5. So, $DE = 10$ cm.

**Quick Check**
Scale factor from $\Delta ABC$ to $\Delta ADE$:
$$k = \frac{AD}{AB} = \frac{12}{18} = \frac{2}{3}$$
$DE = BC \times k = 15 \times \frac{2}{3} = 10$ cm ✓

## ★ Solved Example 8 — Use similarity to find the area of a triangle

In the figure:
$\Delta ABC \sim \Delta DEF$
$AB = 8$ cm, $DE = 4$ cm
The area of $\Delta ABC = 48\ cm^2$
Find the area of $\Delta DEF$.




### Solution

1. Since $\Delta ABC \sim \Delta DEF$, the ratio of corresponding sides (from $\Delta DEF$ to $\Delta ABC$) is:
$$k = \frac{DE}{AB} = \frac{4}{8} = \frac{1}{2}$$
2. The ratio of areas $= k^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
3. Area of $\Delta DEF$ = Area of $\Delta ABC \times \frac{1}{4}$
$$= 48 \times \frac{1}{4} = 12\ cm^2$$
4. So, the area of $\Delta DEF$ is $12\ cm^2$.

**Quick Check**
If the scale factor is $\frac{1}{2}$, the smaller triangle's area is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ of the larger triangle's area.
$48 \div 4 = 12\ cm^2$ ✓

## Important Note

- If a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally.
- The small triangle is similar to the original triangle.
- Always check the parallel lines and the order of vertices.

## How to Solve Similarity Problems?

1. Read the given information carefully.
2. Identify the similar triangles.
3. Write the proportion of corresponding sides or the ratio of areas.
4. Substitute the given values.
5. Solve for the unknown.
6. Check your answer.

## Common Mistakes

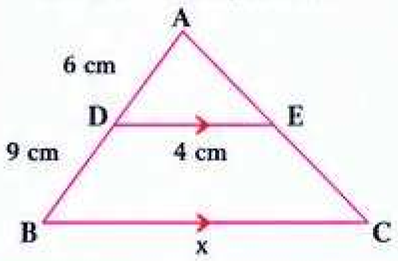
1. Assuming triangles are similar without a valid reason.
   ✗ Wrong
   $\Delta ABC \sim \Delta DEF$ (no reason)
   ✓ Correct
   Prove similarity using AA, SAS or SSS.
2. Mixing up corresponding sides.
   ✗ Wrong
   $\frac{AB}{DE} = \frac{AC}{BC}$
   ✓ Correct
   $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
3. Forgetting to square the scale factor when using areas.
   ✗ Wrong
   Area ratio $= k$
   ✓ Correct
   Area ratio $= k^2$

## Useful Table

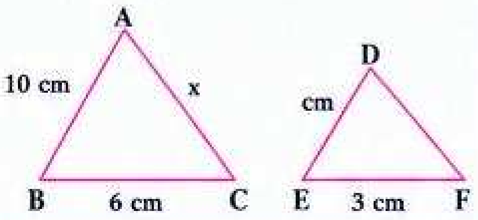
| Given | What to Do |
|---|---|
| Two angles are equal in two triangles | Use AA similarity |
| Two pairs of sides in proportion and included angle equal | Use SAS similarity |
| Three pairs of sides in proportion | Use SSS similarity |
| Triangles are similar | Use side ratios or area ratios to find unknowns |

## Practice Yourself — Answer the following:

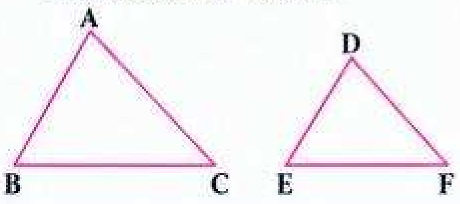
1. In the figure, $DE \parallel BC$. If $AD = 6$ cm, $DB = 9$ cm and $DE = 4$ cm, find $BC$.



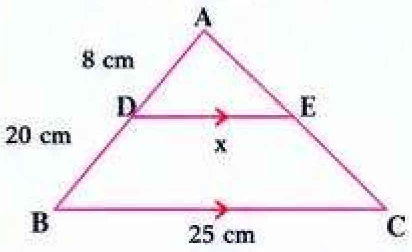

2. In the figure, $\Delta ABC \sim \Delta DEF$. If $AB = 10$ cm, $BC = 6$ cm, $DE = 5$ cm and $EF = 3$ cm, find $AC$.




3. In the figure, $\Delta ABC \sim \Delta DEF$. If the area of $\Delta ABC = 75\ cm^2$ and the ratio of corresponding sides ($ABC$ to $DEF$) $= 5 : 3$, find the area of $\Delta DEF$.




4. In the figure, $DE \parallel BC$. If $AB = 20$ cm, $AD = 8$ cm and $BC = 25$ cm, find $DE$.



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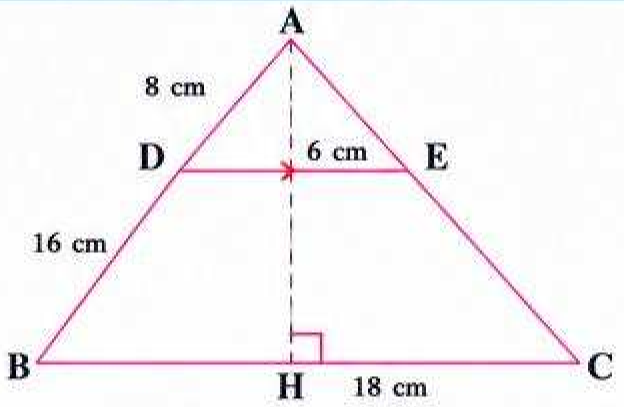
# Similarity of Triangles

## ★ Solved Example 9 — Use similarity to find a height

In the figure:
In $\triangle ABC$, $DE \parallel BC$.
$AD = 8$ cm, $DB = 16$ cm,
$DE = 6$ cm, $BC = 18$ cm
Find the height $AH$ of $\triangle ABC$.




### Solution

1. Since $DE \parallel BC$, then $\triangle ADE \sim \triangle ABC$ (AA similarity).
2. Corresponding sides are in proportion:
$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$
3. Find $AB$: $AB = AD + DB = 8 + 16 = 24$ cm
4. $\frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{8}{24} = \frac{6}{18} = \frac{1}{3}$
5. So, $\frac{AE}{AC} = \frac{1}{3} \Rightarrow AE = \frac{1}{3} AC$
6. Triangles $ADE$ and $ABC$ have the same altitude from $A$ to the parallel lines $DE$ and $BC$.
Hence, $\frac{AE}{AC} = \frac{AH}{AH} = \frac{1}{3} \Rightarrow AH = \frac{1}{3} AC$
7. In right $\triangle AHC$, $\frac{AH}{AC} = \frac{1}{3}$
Let $AH = x$, $AC = 3x$
8. Use Pythagoras in $\triangle AHC$: $AC^2 = AH^2 + HC^2$
$(3x)^2 = x^2 + (18/2)^2 = x^2 + 81$
$9x^2 = x^2 + 81 \Rightarrow 8x^2 = 81 \Rightarrow x = \frac{9}{2} = 4.5$
9. So, $AH = x =$ **4.5 cm** ✓

**Quick Check**

Scale factor from $\triangle ADE$ to $\triangle ABC = \frac{1}{3}$

Each length in $\triangle ABC$ is 3 times the corresponding length in $\triangle ADE$.

So, $BC = 3 \times DE = 18$ cm ✓

$AB = 3 \times AD = 24$ cm ✓

Everything is consistent.

## Important Note

- If a line is parallel to one side of a triangle and intersects the other two sides, it divides them in the same ratio.
- Heights from the same vertex to parallel lines are proportional to the sides on the same line.
- Always keep the order of vertices when writing similarity.

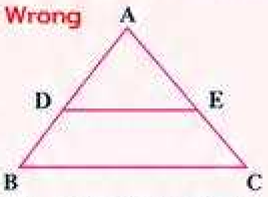
## How to Solve Similarity Problems?

1. Understand the figure and what is given.
2. Write the similarity statement.
3. Write the proportion of sides.
4. Substitute and solve for the unknown.
5. Check your answer and units.

## Common Mistakes

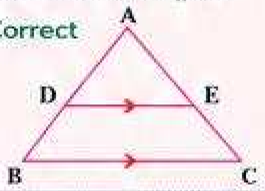
1. Not using parallel lines to prove similarity.

X Wrong




Must write $DE \parallel BC$

✓ Correct




2. Mixing up corresponding sides.

X Wrong $\frac{AD}{BC} = \frac{DE}{AB}$

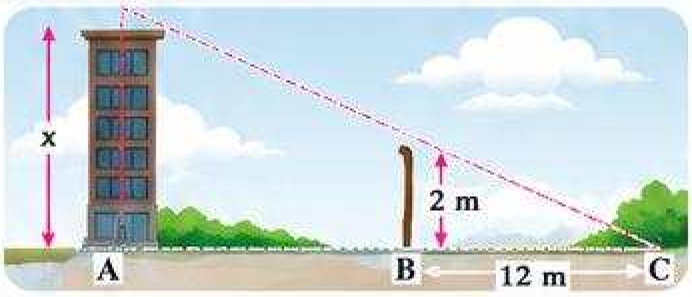
✓ Correct $\frac{AD}{AB} = \frac{DE}{BC}$

3. Forgetting units in the final answer. Always write the answer in cm.

## Key Points Table

| Situation | Use Similarity when: |
|---|---|
| Parallel line inside a triangle | It divides two sides in the same ratio. |
| Two triangles have two equal angles | (AA similarity) |
| Two triangles have two proportional sides and included angle equal | (SAS similarity) |
| All three pairs of sides are proportional | (SSS similarity) |

## ★ Solved Example 10 — Use similarity in a real-life problem




A vertical stick $BD$ of height 2 m casts a shadow $BC$ of length 12 m. At the same time, a building $AD$ casts a shadow $AC$ of length 36 m. Find the height $AD$ of the building.

☑ $\triangle ABD \sim \triangle ADC$
(Right angle and same angle of elevation of the sun)

### Solution

1. From $\triangle ABD \sim \triangle ADC$ : $\frac{AD}{BD} = \frac{AC}{BC}$
2. Substitute: $\frac{AD}{2} = \frac{36}{12}$
3. Cross multiply: $AD \times 12 = 2 \times 36 \Rightarrow AD = \frac{72}{12} = 6$ m
4. So, the height of the building $AD$ is **6 m** ✓

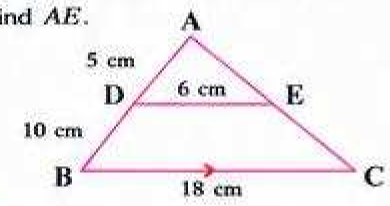
**Quick Check**

Scale factor from small triangle to large triangle $= \frac{AC}{BC} = \frac{36}{12} = 3$

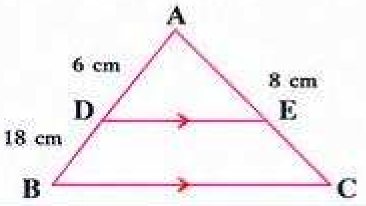
So every side of the large triangle is 3 times the corresponding side of the small triangle.
$AD = 3 \times 2 = 6$ m ✓

## Practice Yourself — Answer the following:

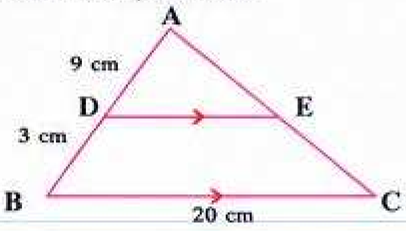
1. In the figure, $DE \parallel BC$.
If $AD = 5$ cm, $DB = 10$ cm, $DE = 6$ cm, $BC = 18$ cm, find $AE$.




2. In the figure, $DE \parallel BC$.
If $AD = 6$ cm, $DB = 18$ cm, $AE = 8$ cm, find $AC$.




4. In the figure, $DE \parallel BC$.
If $AB = 12$ cm, $BC = 20$ cm, $AD = 9$ cm, find $DE$.





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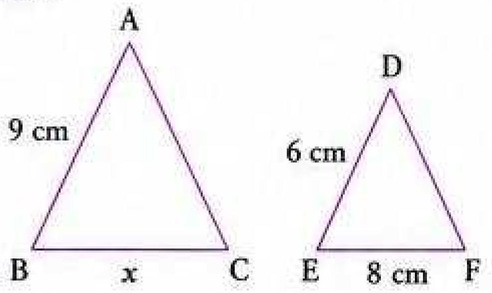
# Lesson Assessment
## Similarity of Triangles

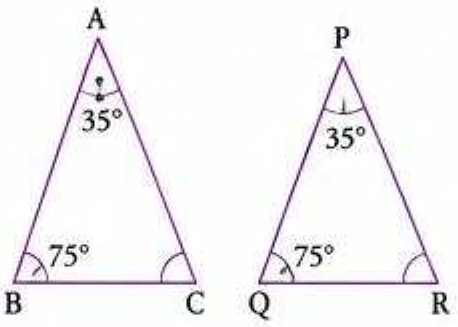
### A Choose the correct answer.

**1** In the figure, $\triangle ABC \sim \triangle DEF$.
If $AB = 9$ cm, $DE = 6$ cm and $EF = 8$ cm, find $BC$.



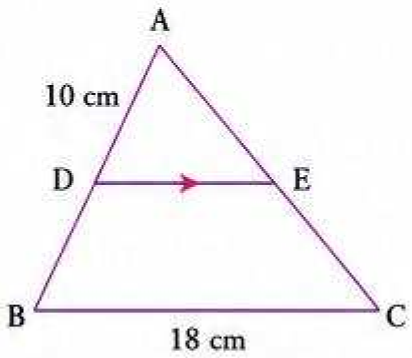

Ⓐ 10 cm Ⓑ 12 cm Ⓒ 14 cm Ⓓ 16 cm

**2** Two triangles are similar.
One angle of the first triangle measures $35°$.
Another angle measures $75°$.
Find the corresponding third angle in the second triangle.



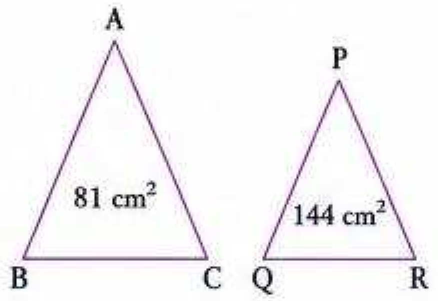

Ⓐ $35°$ Ⓑ $70°$ Ⓒ $75°$ Ⓓ $110°$

**3** In $\triangle ABC$, $DE \parallel BC$.
If $AB = 15$ cm, $AD = 10$ cm and $BC = 18$ cm, find $DE$.



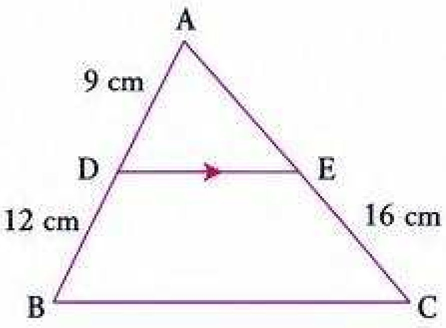

Ⓐ 10 cm Ⓑ 12 cm Ⓒ 15 cm Ⓓ 20 cm

**4** $\triangle ABC \sim \triangle PQR$.
The area of $\triangle ABC$ is $81\ cm^2$.
The area of $\triangle PQR$ is $144\ cm^2$.
The similarity ratio ($ABC : PQR$) equals:



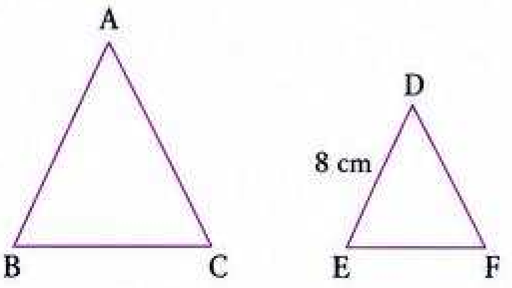

Ⓐ 3 : 4 Ⓑ 4 : 3 Ⓒ 9 : 16 Ⓓ 16 : 9

**5** In the figure, $AB = 12$ cm, $AC = 16$ cm and $BC \parallel BC$.
If $AD = 9$ cm, find $AE$.




Ⓐ 10 cm Ⓑ 11 cm Ⓒ 12 cm Ⓓ 13 cm

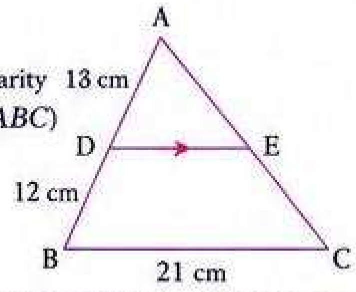
**6** If $AB : DE = 5 : 2$ and $DE = 8$ cm, find $AB$.



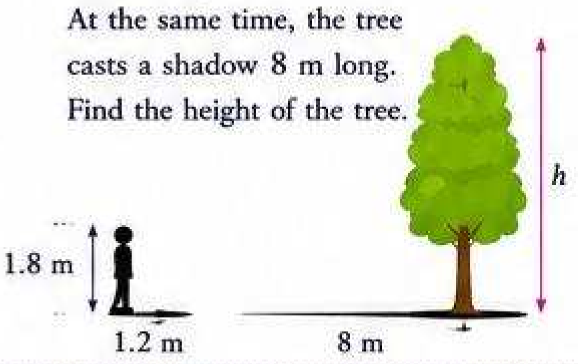

Ⓐ 16 cm Ⓑ 18 cm Ⓒ 20 cm Ⓓ 24 cm

### B Solve the following:

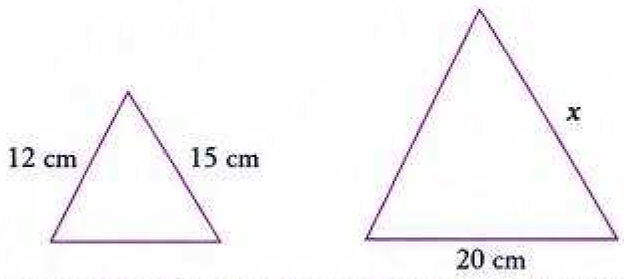
**7** In $\triangle ABC$, $D$ lies on $\overline{AB}$ and $E$ lies on $\overline{AC}$ such that $DE \parallel BC$.
If $AB = 18$ cm, $AD = 12$ cm and $BC = 21$ cm,
find:
(a) $DE$
(b) The similarity ($ADE : ABC$)



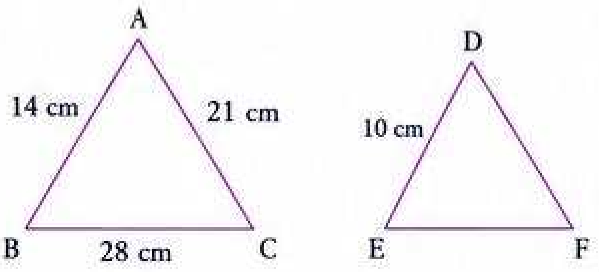

**8** The height of a tree is unknown.
A 1.8 m student casts a shadow 1.2 m long.
At the same time, the tree casts a shadow 8 m long.
Find the height of the tree.



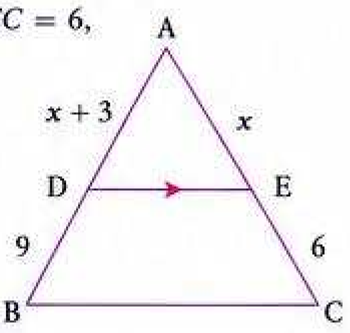

**9** Two similar triangles have corresponding sides 12 cm, 15 cm and 20 cm.
Find the corresponding side to 20 cm if the scale factor is 3 : 5.



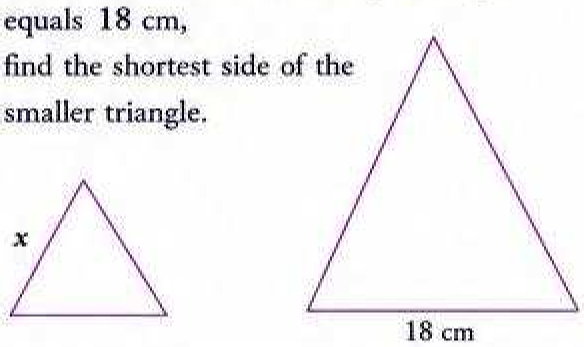

**10** In $ABC \sim \triangle DEF$.
If $AB = 14$ cm, $BC = 21$ cm, $CA = 28$ cm and $DE = 10$ cm, find $EF$ and $DF$.




**11** In $\triangle ABC$, $D$ lies on $\overline{AB}$ and $E$ lies on $\overline{AC}$ such that $DE \parallel BC$.
If $AD = x + 3$, $DB = 9$, $AE = x$, $EC = 6$, find $x$.




**12** The perimeters of two similar triangles are 42 cm and 63 cm.
If the shortest side of the larger triangle equals 18 cm, find the shortest side of the smaller triangle.





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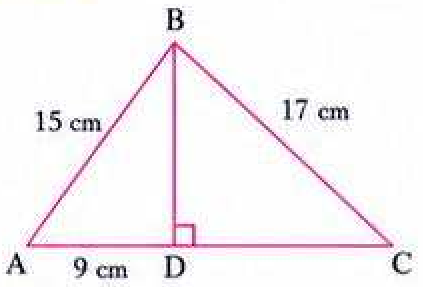
# Exercises

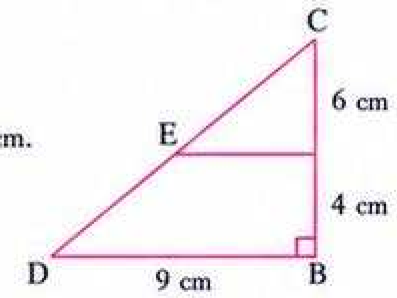
## A Choose the correct answer.

**1** In the figure:
$BD \perp AC$, $AB = 15$ cm,
$BC = 17$ cm, $AD = 9$ cm.
What is the length of $DC$ ?



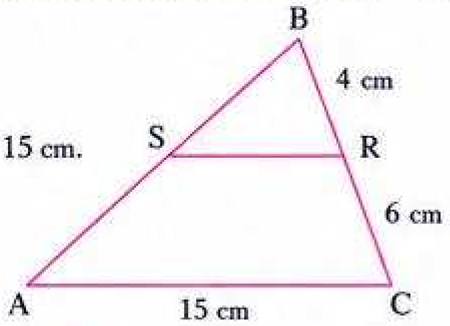

(A) 12 cm (B) 14 cm (C) 16 cm (D) 20 cm

**2** In the figure:
$DE \parallel CB$, $EB = 4$ cm,
$EC = 6$ cm, $DB = 6$ cm, $DC = 9$ cm.
Find the value of $\frac{ED}{CB}$ .



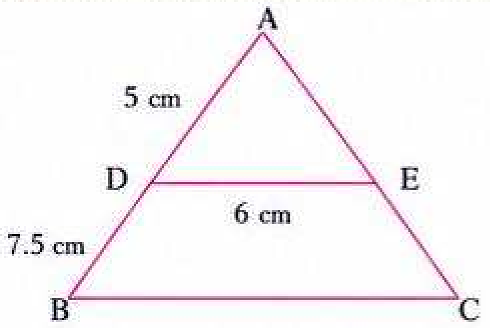

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

**3** In the figure:
$RS \parallel AC$, $BR = 4$ cm,
$RC = 6$ cm, $SR = 6$ cm, $AC = 15$ cm.
Find $AS$.



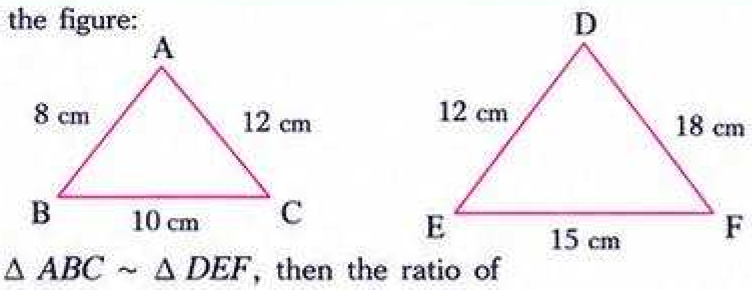

(A) 6 cm (B) 7.5 cm (C) 8 cm (D) 9 cm

**4** In the figure:
$DE \parallel BC$, $AD = 5$ cm,
$DB = 7.5$ cm, $DE = 6$ cm.
Find $BC$.




(A) 15 cm (B) 16.5 cm (C) 18 cm (D) 20 cm

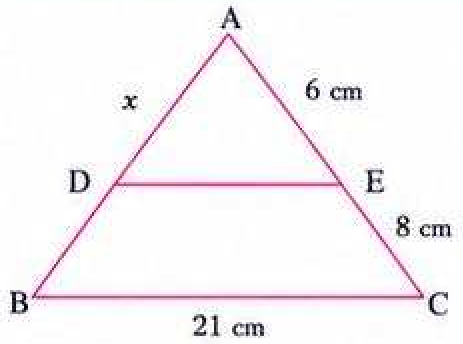
**5** In the figure:




If $\triangle ABC \sim \triangle DEF$, then the ratio of the perimeters $(ABC : DEF)$ equals:

(A) 2 : 3 (B) 3 . 2 (C) 4 . 5 (D) 5 : 4

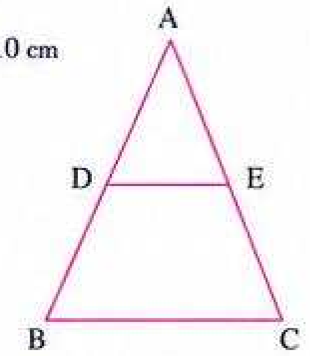
**6** In the figure:
$DE \parallel BC$, $AD = x$ cm,
$AE = 6$ cm, $DC = 8$ cm,
$BC = 21$ cm.
Find $x$ .



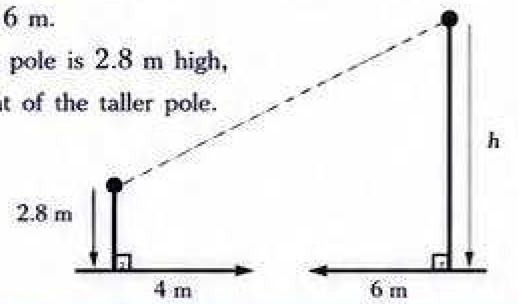

(A) 6 cm (B) 7 cm (C) 8 cm (D) 9 cm

## B Solve the following:

**7** In $\Delta ABC$, $DE \parallel BC$.
If $AB = 18$ cm, $AD = 10$ cm and $BC = 27$ cm, find:
(a) $DE$.
(b) The similarity ratio $(ADE : ABC)$.

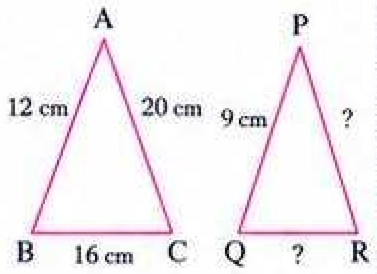



**8** Two poles stand on the same horizontal ground.
At the same time, their shadows are 4 m and 6 m.
If the shorter pole is 2.8 m high, find the height of the taller pole.

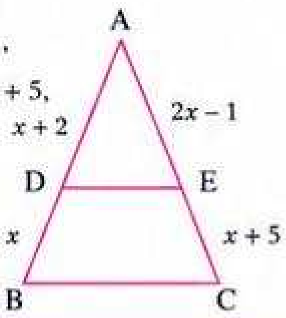



**9** Two similar triangles have corresponding sides 9 cm, 12 cm and 15 cm.
If the scale factor from the first triangle to the second is $\frac{3}{4}$, find the side in the second triangle that corresponds to 15 cm.

**10** In $\Delta ABC \sim \Delta PQR$,
$AB = 12$ cm, $BC = 16$ cm,
$CA = 20$ cm and $PQ = 9$ cm.
Find $QR$ and $PR$ .



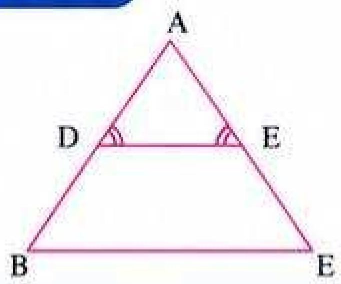

**11** In the figure, $DE \parallel BC$.
If $AD = x + 2$, $DB = x$,
$AE = 2x - 1$ and $EC = x + 5$,
find the value of $x$.




**12** The perimeters of two similar triangles are 36 cm and 54 cm.
If the shortest side of the larger triangle is 15 cm,
find the shortest side of the smaller triangle.

## C Prove that:

**13**




In the opposite figure:
$DE \parallel BC$, $BE \parallel AB$ and $m(\angle ADE) = m(\angle BDC)$.
Prove that $\Delta ADE \sim \Delta DBC$ and hence $\frac{AD}{DB} = \frac{AE}{DC}$ .

To prove similarity show:
- Two pairs of equal angles, or
- Two pairs of proportional sides and the included angle equal, or
- Three pairs of proportional sides.



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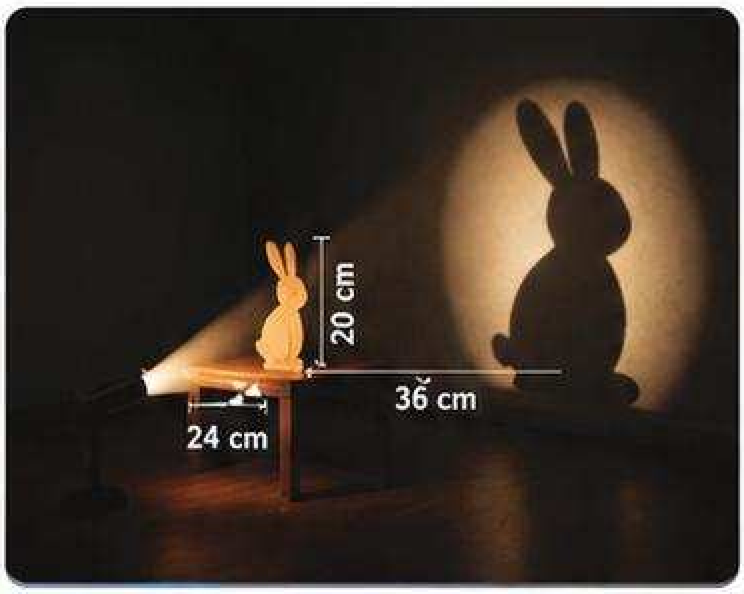



## Get ready!

When a transparent object is placed near a light source, a shadow is formed on the wall. The shadow is similar to the object, but different in size.

In the picture, the rabbit's height is 20 cm and its shadow is 36 cm. The table's length is 24 cm and its shadow is 36 cm. What is the dilation factor in this case?

In this lesson, you will learn the concept of dilation and how to solve related problems.

### Learning Outcomes

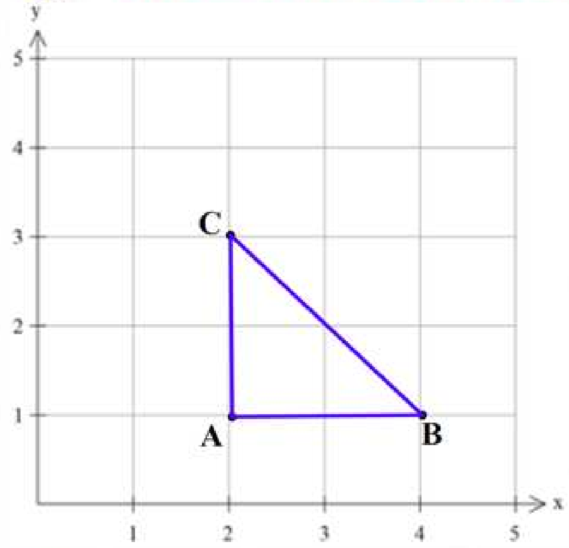
By the end of this lesson, you will be able to:

- Define the concept of dilation.
- Identify the center of dilation.
- Use the dilation factor.
- Enlarge or reduce a shape.
- Solve problems using dilation.

### Key Vocabulary

- Dilation
- Center of dilation
- Dilation factor
- Enlargement
- Reduction

## Think and Discuss!




Using a coordinate program, the triangle ABC is drawn. If it is dilated by a scale factor of $2:1$, what are the coordinates of the vertices of the image $A'B'C'$? What do you notice?

| Original | | Image |
|---|---|---|
| $A(2,1)$ | → | $A'(4,2)$ |
| $B(4,1)$ | | $B'(8,2)$ |
| $C(2,3)$ | | $C'(4,6)$ |

## Let's Learn!

### 1 The Concept of Dilation

- Dilation is a transformation that changes the size of a shape.
- It changes the lengths of the sides of the shape. The image may be larger or smaller than the original shape.
- The image is similar to the original shape. The corresponding angles are equal, and the corresponding sides are in the same ratio.

### 2 Dilation in the Coordinate Plane

The dilation that has center C and transforms any point $P$ to a point $P'$ is denoted by $D_C$. The properties of this dilation are:

1. C is the midpoint of $\overline{PP'}$ (i.e. P, C, P′ are collinear and $CP = \overline{CP'}$).
2. If P is not the center C, then $\frac{CP'}{CP} = k$, where $k$ is the scale factor.
   - If $0 < k < 1$, the dilation is a reduction.
   - If $k > 1$, the dilation is an enlargement.

### Take Note

- The scale factor $k$ is a positive number only.
- The lengths of the sides in the image are $k$ times the lengths of the corresponding sides in the original shape.
- The coordinates of the vertices of the image change according to the scale factor $k$.

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## 3 | Dilation in the Coordinate Plane (General Case)

If a dilation has center $C(a, b)$ and scale factor $k$, then for any point $P(x, y)$, its image $P'(x', y')$ is given by:

$$x' = a + k(x - a) \quad , \quad y' = b + k(y - b)$$

**Where:**

- $(x, y)$ are the coordinates of the original point $P$.
- $(x', y')$ are the coordinates of the image point $P'$.
- $(a, b)$ are the coordinates of the center of dilation $C$.
- $k$ is the scale factor.

### Key Idea

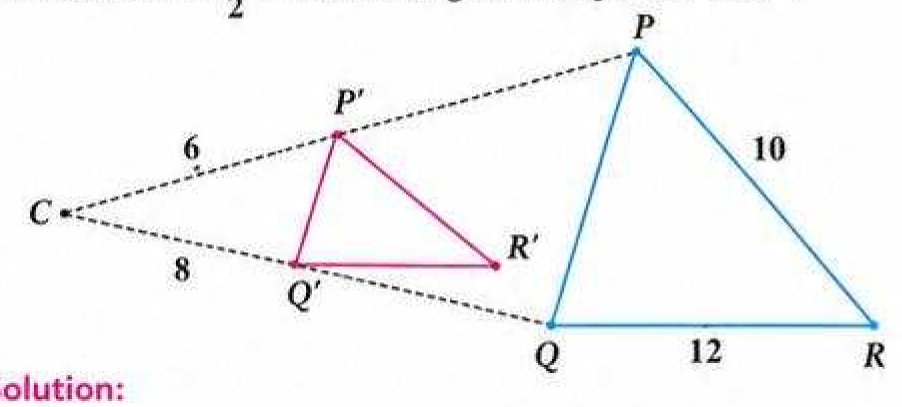
The dilation factor $k$ controls the size of the image.

- $k > 1$ → **Enlargement**
  The image is larger than the original shape.
- $k = 1$ → **Same size**
  The image is identical to the original shape.
- $0 < k < 1$ → **Reduction**
  The image is smaller than the original shape.

## Examples

### Example 1

In the figure, $\triangle PQR$ is dilated about point $C$ by a scale factor $k = \frac{1}{2}$. Find the lengths of $CQ'$, $CR'$, $CP'$.




**Solution:**

Since $k = \frac{1}{2}$, then the image lengths are half the original lengths.

$CQ' = \frac{1}{2} \times CQ = \frac{1}{2} \times 8 = 4$

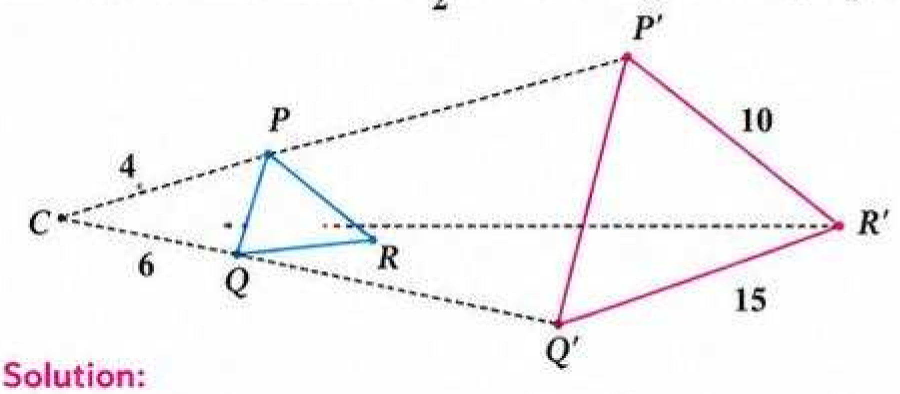
$CR' = \frac{1}{2} \times CR = \frac{1}{2} \times 10 = 5$

$CP' = \frac{1}{2} \times CP = \frac{1}{2} \times 6 = 3$

$CQ' = 4,\ CR' = 5,\ CP' = 3$

### Example 2

In the figure, $\triangle P'R'Q'$ is the image of $\triangle PRQ$ under a dilation about $C$ with scale factor $k = \frac{5}{2}$. Find the lengths of $CP$, $CQ$, $CR$.




**Solution:**

Since $k = \frac{5}{2}$, then the original lengths are $\frac{2}{5}$ of the image lengths.

$CP = \frac{2}{5} \times CP' = \frac{2}{5} \times 10 = 4$

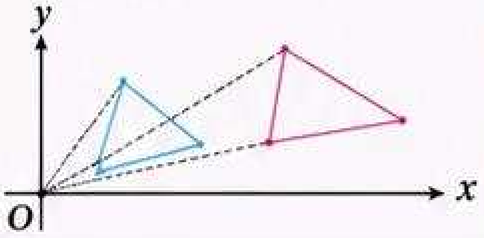
$CQ = \frac{2}{5} \times CQ' = \frac{2}{5} \times 15 = 6$

$CR = \frac{2}{5} \times CR' = \frac{2}{5} \times 3 = \frac{6}{5} = 1.2$

$CP = 4,\ CQ = 6,\ CR = 1.2$

## Common Mistakes

✗ **Mistake 1:** Forgetting the center of dilation.
Not all dilations are about the origin. Always check the center!




✓ **Correct:**
If the center is $O(0, 0)$, the coordinates are multiplied by $k$.

✗ **Mistake 2:** Using the scale factor incorrectly.
Do not add or subtract the scale factor from the lengths.
You should multiply all lengths by $k$.

Original length = 5

| ✗ Wrong | ✓ Correct |
|---|---|
| $5 + 2 = 7$ or $5 - 2 = 3$ | $5 \times 2 = 10$ |

### Quick Comparison Table

| Scale Factor $k$ | Type of Dilation | Size of Image |
|---|---|---|
| $k > 1$ | Enlargement | Larger |
| $k = 1$ | Same size | Same as original |
| $0 < k < 1$ | Reduction | Smaller |

### Think!

If a square has a side length of 6 cm and is dillated by a scale factor $k = \frac{3}{2}$ about any point, what will be the side length of its image?

______________________________



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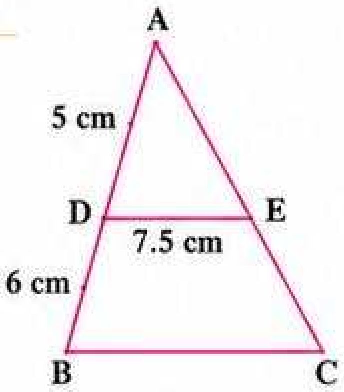
## Example 2

In the figure:
△ ADE is the image of △ ABC with center A such that DE // BC.
Find the scale factor and the lengths: DE and BC.




**Solution:**

Since △ ADE ~ △ ABC with center A,

then: $\frac{AD}{AB} = \frac{DE}{BC} = k$

AB = AD + DB = 5 + 6 = 11 cm

$\therefore \frac{AD}{AB} = \frac{5}{11}$ and $\frac{DE}{BC} = k$

$\therefore \frac{5}{11} = \frac{7.5}{BC}$

Cross multiply:

$5 \times BC = 7.5 \times 11$

$BC = \frac{82.5}{5} = 16.5$ cm

The scale factor:

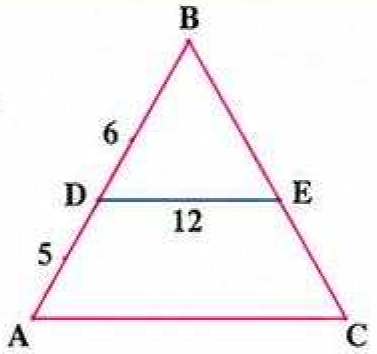
$k = \frac{AD}{AB} = \frac{5}{11}$

DE = 7.5 cm

BC = 16.5 cm

### Think

In △ ABC shown, DE // AC.
If AD = 6, DB = 5, and DE = 12, find AC.



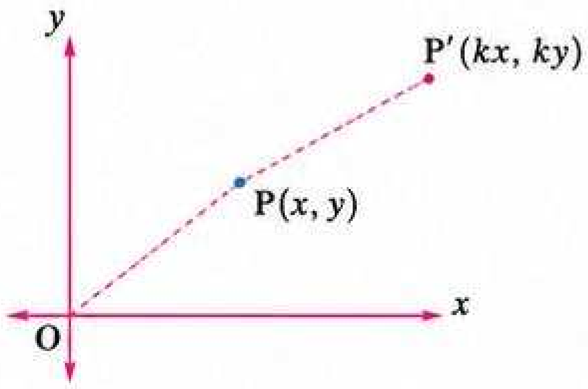

Since △ BDE ~ △ BAC with center B,

then

$$\frac{DE}{AC} = \frac{DB}{BA}$$

$$AC = \frac{DE \times BA}{DB} = \frac{12 \times (6+5)}{5} = 26.4$$

## The Dilation in the Coordinate Plane (In Detail)




If point $P(x, y)$ is on the coordinate plane, its image $P'$ after a dilation centered at the origin $O(0, 0)$ with scale factor $k$ is $P'(kx, ky)$.

**General rule:**

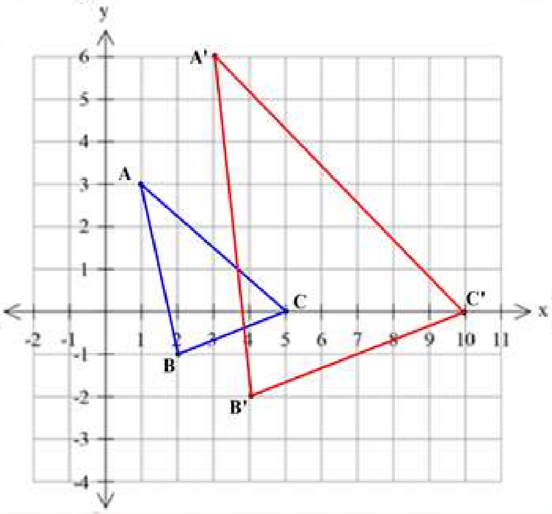
$(x, y) \longrightarrow (kx, ky)$

### Take Note

- If $k > 1$, the image is larger.
- If $k = 1$, the image is the same size.
- If $0 < k < 1$, the image is smaller.
- If $k < 0$, the image is on the opposite side of the center.

## Example 3

Draw triangle ABC with vertices: $A(1, 3)$, $B(2, -1)$, $C(5, 0)$ on the coordinate plane.
Then draw its image under a dilation centered at $O(0, 0)$ with scale factor 2.




**Solution:**

Using the rule $(kx, ky)$ with $k = 2$

A (1, 3) ⟶ A′ (2, 6)

B (2, −1) ⟶ B′ (4, −2)

C (5, 0) ⟶ C′ (10, 0)

### Key Idea

If the dilation center is $O(0, 0)$, multiply each coordinate by $k$.

- For any point $P(x, y)$: Image is $P'(kx, ky)$.
- For any polygon: Multiply all vertices by $k$.
- Negative $k$ reflects the image through the center.

### Summary from Previous Examples

1. If △ ABC is dilated by scale factor $k$, then △ A'B'C' ~ △ ABC.
2. Corresponding sides are in the ratio $k$.
3. All corresponding angles are equal.
4. The type of dilation depends on $k$:
   - $k > 1$ → Enlargement
   - $k = 1$ → Same size
   - $0 < k < 1$ → Reduction
   - $k < 0$ → Enlargement with reflection through the center.

### Check Your Understanding

A line segment has length 9 cm. What is the length of its image after a dilation with scale factor $k = \frac{2}{3}$ about any point?



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## Example 4

Draw quadrilateral $ABCD$ with vertices:
$A(-3, 6)$, $B(0, 3)$, $C(3, -3)$, $D(-3, -3)$
on the coordinate plane.
Then draw its image after a dilation centered at $O(0, 0)$ with scale factor $\frac{1}{3}$.

**Solution:**

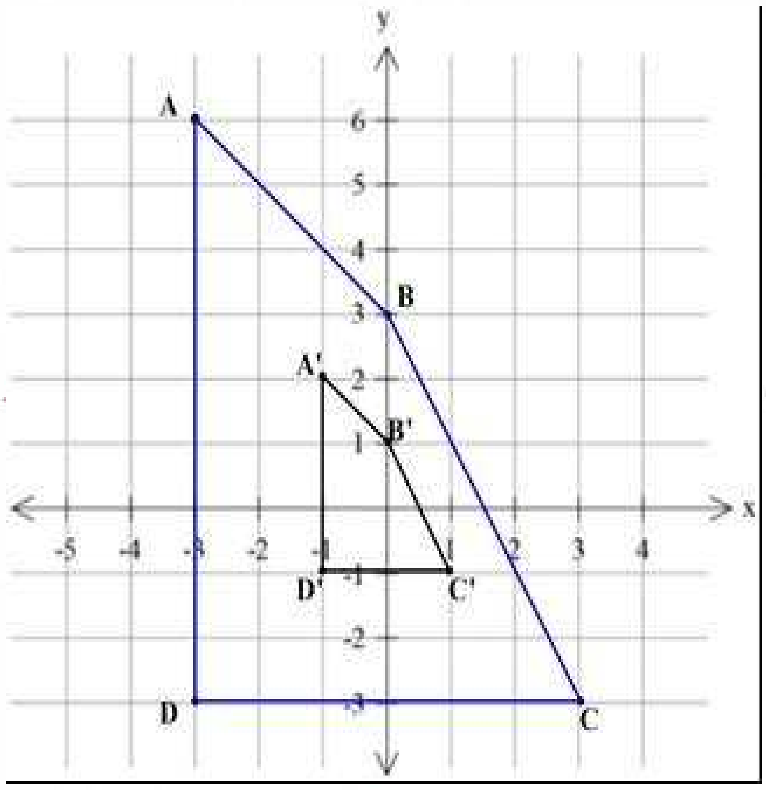
Using the rule $(kx, ky)$ with $k = \frac{1}{3}$

$A(-3, 6) \longrightarrow A'(-1, 2)$

$B(0, 3) \longrightarrow B'(0, 1)$

$C(3, -3) \longrightarrow C'(1, -1)$

$D(-3, -3) \longrightarrow D'(-1, -1)$




### Think!

If triangle ABC has vertices $A(2, 4)$, $B(-1, 2)$, $C(3, -2)$ and is dilated about $O(0, 0)$ by scale factor $k = -2$, what are the coordinates of $A'$, $B'$, $C'$ ?

______
______
______

### Take Note

- A negative scale factor produces an enlargement and a reflection through the center.
- The image appears on the opposite side of the center from the original shape.

## Steps to Solve Dilations in the Coordinate Plane

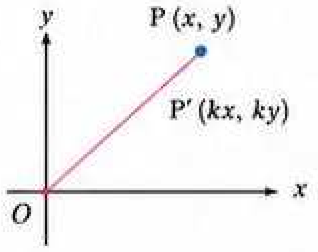
**Follow these steps:**

1. Identify the center of dilation $C(a, b)$.
2. Identify the scale factor $k$.
3. For each point $P(x, y)$, use the rule $(x', y') = (a + k(x - a),\ b + k(y - b))$.
4. Plot the image points and connect them in order.

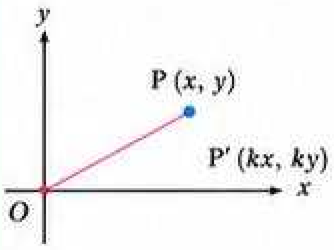
### Special Case: Center at Origin O (0, 0)

Use the rule $(kx, ky)$ for every point.

$k > 0$
Same direction




$k < 0$
Opposite direction




### Common Mistakes

✗ **Mistake 1:** Adding $k$ instead of multiplying.

$P(2, 3)$, $k = 2$

✗ Wrong
$(2 + 2, 3 + 2) = (4, 5)$

✓ Correct
$(2 \times 2, 3 \times 2) = (4, 6)$

✗ **Mistake 2:** Forgetting the sign of $k$.

$k = -2$ $P(1, -2)$

✗ Wrong $(2, -4)$ ✓ Correct $(-2, 4)$

## Practice Yourself

**Direct** 1. If triangle $ABC$ is dilated by a scale factor of 4, and $AB = 3$ cm, what is the length of $A'B'$ ? ______

**Understand** 2. Why does the image appear on the opposite side of the center when $k$ is negative? ______

**Apply** 3. Triangle $PQR$ has vertices $P(2, 1)$, $Q(4, 3)$, $R(5, -1)$. Draw its image after a dilation about $O(0, 0)$ with scale factor $k = 2$. List the coordinates of $P'$, $Q'$, $R'$. ______

**Real Life** 4. A map has a scale of 1 : 50,000. If the distance between two cities on the map is 4 cm, what is the actual distance between them? ______

**Higher Thinking** 5. If a shape remains unchanged after a dilation about $O(0, 0)$, what is the possible value of $k$? ______

## Test Yourself

**1 Choose the correct answer:**

If $k = \frac{3}{2}$, then the dilation is ( )

a. enlargement b. reduction
c. reflection d. same size

**2 Complete:**

The image of $P(-2, 5)$ under a dilation about $O(0, 0)$ with scale factor $\frac{1}{2}$ is ______.

**3 True or False:**

If $k < 0$, the image will be on the same side of the center. ( )

**4 Solve:**

Triangle ABC has $A(1, 2)$, $B(3, 1)$, $C(2, -2)$. Draw its image under a dilation about $O(0, 0)$ with $k = -2$. Write the coordinates of $A'$, $B'$, $C'$.

______
______
______

**5 Challenge:**

A point $P(x, y)$ is dilated about $O(0, 0)$ by $k = 3$, then about $O$ again by $k = \frac{1}{3}$.
What is the final image of P?

______

## Summary

- Dilation changes the size of a figure using a scale factor $k$.
- Center of dilation can be any point $(a, b)$.
- In the coordinate plane:

$(x, y) \longrightarrow (a + k(x - a),\ b + k(y - b))$

- If center is $O(0, 0)$, then $(x, y) \longrightarrow (kx, ky)$.
- $k > 1$ enlargement, $k = 1$ same size, $0 < k < 1$ reduction.
- $k < 0$ enlargement with reflection through the center.

## Homework

1. Draw triangle $XYZ$ with $X(-2, 4)$, $Y(1, 5)$, $Z(3, -1)$. Draw its image after a dilation about $O(0, 0)$ with $k = \frac{1}{2}$.
2. A rectangle has length 8 cm and width 5 cm. If it is dilated by $k = 3$, find the perimeter of its image.
3. Point $M(-4, 2)$ is dilated about $O(0, 0)$ by $k = -3$. Find the coordinates of $M'$.



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## Lesson Practice

### 1 Circle the correct answer:

1. If a figure is dilated by a scale factor $k = \frac{1}{4}$, then the image is ______.
   (a) enlarged (b) reduced (c) the same size (d) reflected
2. If $k = -2$ in a dilation, then the image will be ______.
   (a) enlarged (b) the same size
   (c) reduced (d) on the opposite side of the center
3. In a dilation, the center $O$ is the ______ of segment $PP'$.
   (a) midpoint (b) endpoint
   (c) perpendicular bisector (d) intersection
4. If $P(2, -3)$ is taken to $P'(6, -9)$, then the scale factor is ______.
   (a) $-2$ (b) $2$ (c) $3$ (d) $-3$

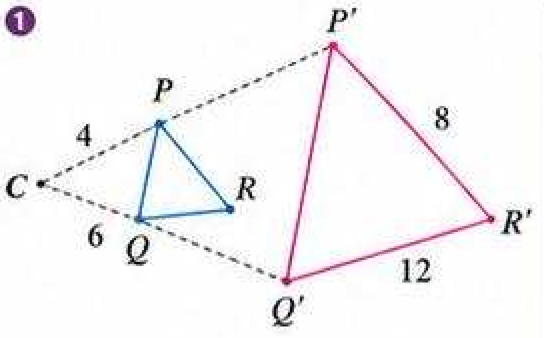
### 2 Complete the following:

1. In a dilation, the lengths of the sides in the image are ______ times the lengths of the corresponding sides in the original shape.
2. If the scale factor $k = 1$, the image is ______ the original shape.
3. If $k > 1$, the dilation is called ______.
4. If $0 < k < 1$, the dilation is called ______.
5. The dilation that has center $C(a, b)$ is denoted by ______.
6. If $P'(5, 10)$ is the image of $P(1, 2)$ under a dilation centered at $O(0, 0)$, then $k =$ ______.
7. If the image of $P(x, y)$ is $P'(-3x, -3y)$, then $k =$ ______.

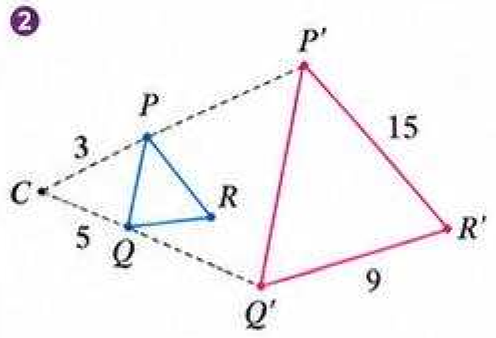
### 3 Find the coordinates of the image of each point under a dilation centered at $O(0, 0)$ with the given scale factor $k$.

| | Point $P(x, y)$ | Scale Factor $k$ | Image $P'(x', y')$ |
|---|---|---|---|
| 1 | $(2, 3)$ | $k = 3$ | (______ , ______) |
| 2 | $(-4, 1)$ | $k = \frac{1}{2}$ | (______ , ______) |
| 3 | $(5, -2)$ | $k = -2$ | (______ , ______) |
| 4 | $(-3, -4)$ | $k = -\frac{1}{3}$ | (______ , ______) |
| 5 | $(\frac{1}{2}, -6)$ | $k = 4$ | (______ , ______) |

### 4 Each figure is the image of the original under a dilation. Find the scale factor $k$.

1.



$k =$ ______

2.



$k =$ ______

### 5 Determine whether the following dilations are enlargements, reductions, or the same size.

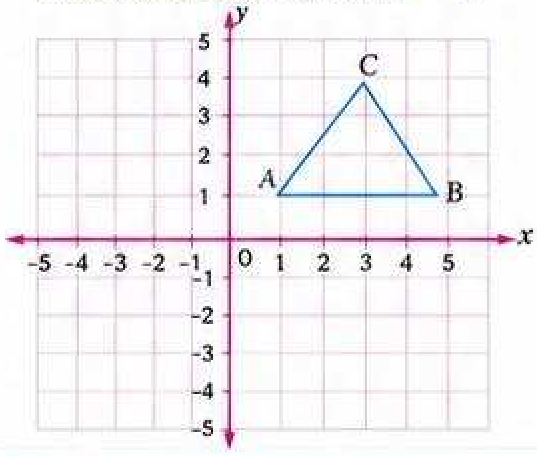
1. $k = 5$ ______
2. $k = \frac{2}{3}$ ______
3. $k = 1$ ______
4. $k = -\frac{1}{4}$ ______
5. $k = -3$ ______

### 6 Find the scale factor $k$ in each of the following.

1. $P(2, 4) \longrightarrow P'(6, 12)$
   $k =$ ______
2. $P(-3, 5) \longrightarrow P'(1.5, -2.5)$
   $k =$ ______
3. $P(-2, -1) \longrightarrow P'(4, 2)$
   $k =$ ______
4. $P(3, -6) \longrightarrow P'(-9, 18)$
   $k =$ ______
5. $P\left(\frac{1}{2}, -\frac{2}{3}\right) \longrightarrow P'\left(2, -\frac{8}{3}\right)$
   $k =$ ______

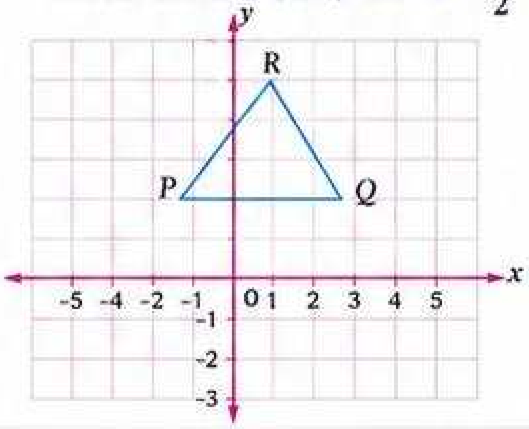
### 7 Find the coordinates of the image of each triangle under the given dilation.

1. Triangle $ABC$ with $A(1, 1)$, $B(4, 1)$, $C(2, 4)$ dilated about $O(0, 0)$ with $k = 2$.




$A'($ ; $)$
$B'($ , $)$
$C'($ , $)$

2. Triangle $PQR$ with $P(-2, 2)$, $Q(1, 2)$, $R\left(-\frac{1}{2}, 5\right)$ dilated about $O(0, 0)$ with $k = \frac{1}{2}$.




$P'($ , $)$
$Q'($ , $)$
$R'($ , $)$



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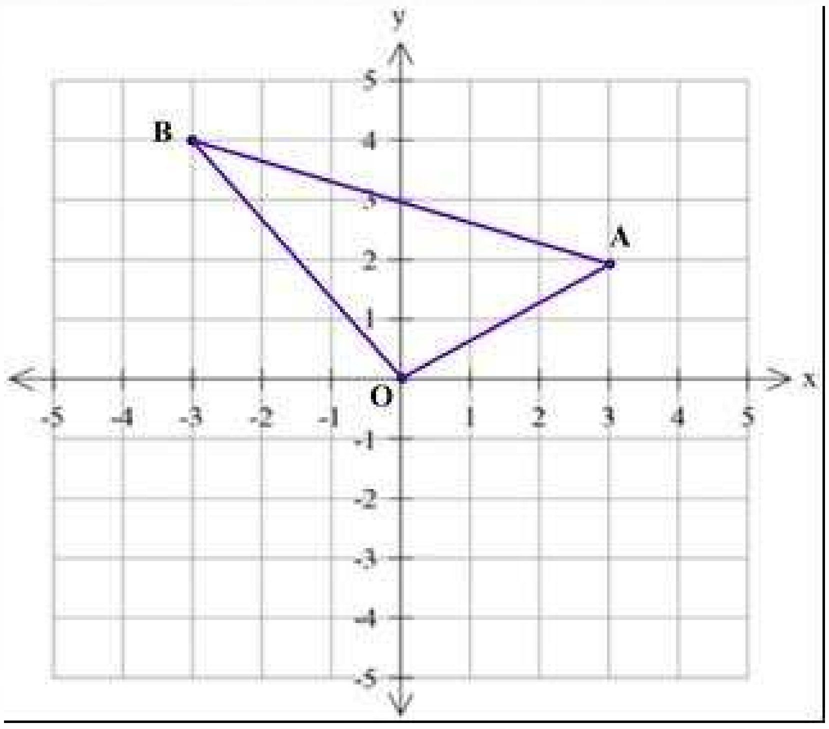
Solve each of the following problems.

**1** The figure shows $\Delta OAB$ with center of dilation $O$ and image $A'B'$. Use the mapping $(x, y) \longrightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$ to find the coordinates of $A'$ and $B'$.

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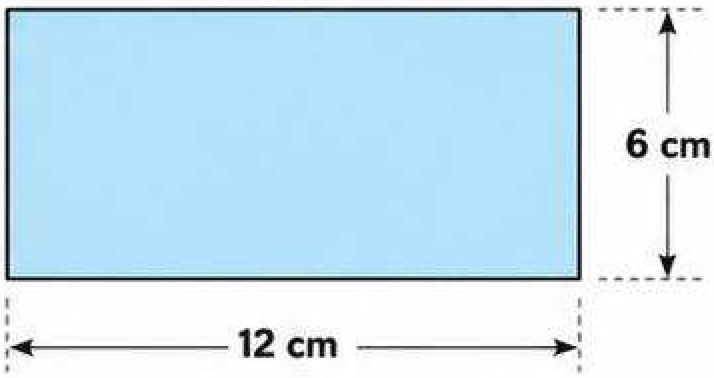
**2** A flea has a length of 2.5 mm under a microscope. If the image of the flea is enlarged by a scale factor of 7, what is the length of the flea in the enlarged image?

..................................................

..................................................

..................................................

**3** The area of a rectangle is 72 cm². If a dilation with scale factor $\frac{1}{3}$ is applied to the rectangle, what is the area of the image?




..................................................

..................................................

**4** If $\Delta PQR$ is dilated with center $C$ and scale factor 2 to get $\Delta PSD$ as shown in the figure, find the:

a) The coordinates of $S$.

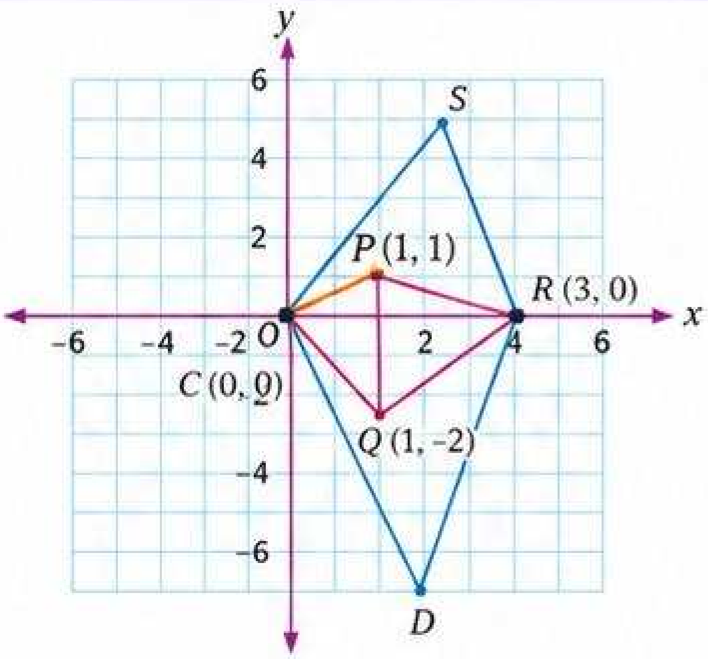
b) The coordinates of $D$.

a) ..................................................

..................................................

b) ..................................................

..................................................



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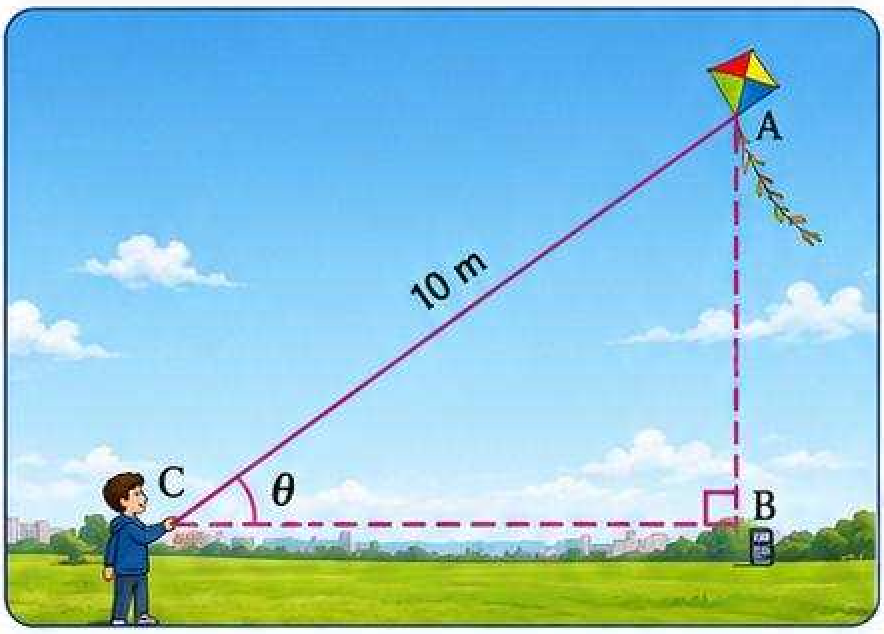
Lesson 4

# Trigonometric Ratios of Acute Angle




Omar flew a kite and measured the angle of elevation $\theta$ from his position to the kite. He found that:

$$\frac{AB}{AC} = \frac{3}{5}$$

If the length of the string AC is 10 m and AB is the height of the kite above the ground.

In this lesson, we will learn how to find the trigonometric ratios of the acute angle in a right triangle and how to use them to solve problems.

## Learning Objectives

- Define the trigonometric ratios of an acute angle in a right triangle.
- Find the trigonometric ratios of an acute angle.
- Use trigonometric ratios to find unknown sides and angles in right triangles.
- Solve real-life and mathematical problems involving trigonometric ratios.

## Key Vocabulary

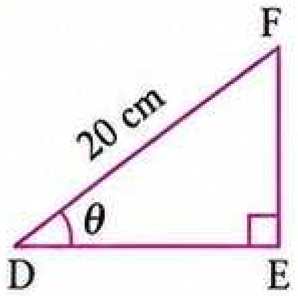
- Sine (sin)
- Cosine (cos)
- Tangent (tan)
- Trigonometric ratios
- Opposite side
- Adjacent side
- Hypotenuse
- Right triangle
- Acute angle
- Ratio

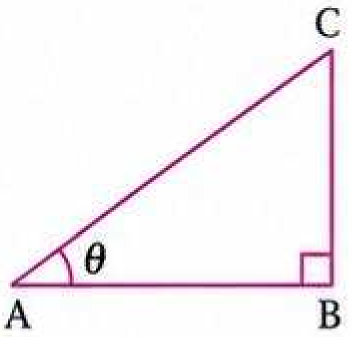
## Think and Discuss!

In the figures below:

AB : CB = 4 : 3

∠FE = ∠A = 90°




What is the relation between the two right triangles?

$\times 9$ $\times 12$ $\times 16$

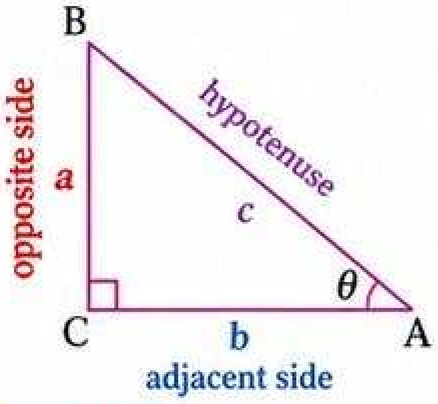
## Materials Needed

- Ruler
- Protractor
- Calculator

## Let's Learn!

### Trigonometric Ratios

The trigonometric ratios of an acute angle in a right triangle are the ratios between the lengths of its sides.

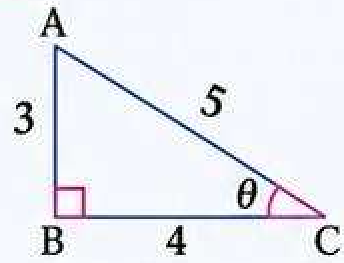



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

**Example:**




$$\frac{4}{5} = \frac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$$

$$\frac{3}{5} = \frac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$$

$$\frac{3}{4} = \frac{\text{opposite}}{\text{adjacent}} = \tan\theta$$

## Important Note

In a right triangle:

- The hypotenuse is the side opposite the right angle. It is always the longest side.
- The side opposite the angle $\theta$ is called the "opposite side".
- The side next to the angle $\theta$ (not the hypotenuse) is called the "adjacent side".



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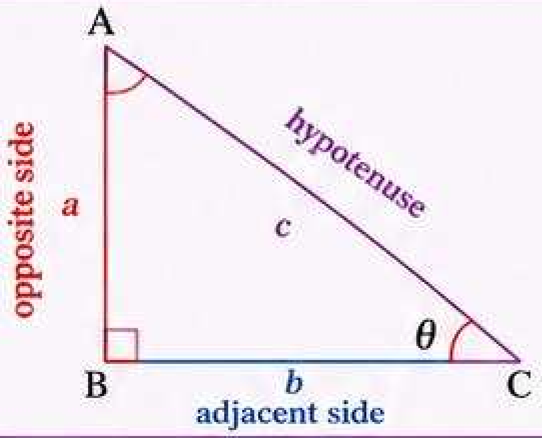
## Basic Rule

For any acute angle $\theta$ in a right triangle:

1. $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$
2. $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$
3. $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



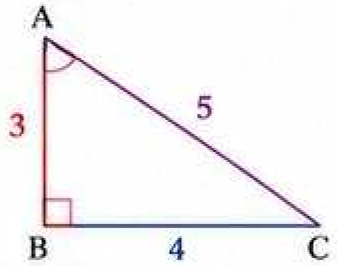

**Symbols Explanation**

- $\theta$ : an acute angle
- a : length of the opposite side
- b : length of the adjacent side
- c : length of the hypotenuse
- $\angle B = 90^\circ$
- $0^\circ < \theta < 90^\circ$

## Solved Examples

**Example 1**

In the right triangle ABC shown, find sin A, cos A and tan A.




**Solution:**

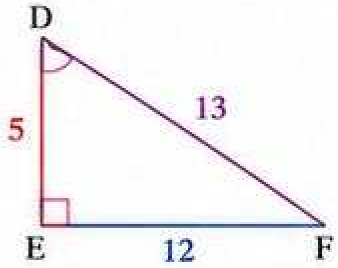
$\sin A = \dfrac{BC}{AC} = \dfrac{4}{5}$

$\cos A = \dfrac{AB}{AC} = \dfrac{3}{5}$

$\tan A = \dfrac{BC}{AB} = \dfrac{4}{3}$

**Example 2**

In the right triangle DEF shown, find sin D, cos D and tan D.




**Solution:**

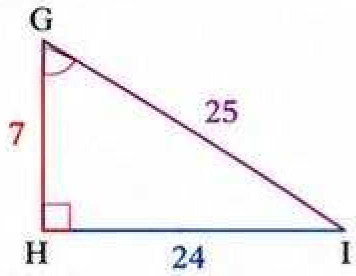
$\sin D = \dfrac{EF}{DF} = \dfrac{12}{13}$

$\cos D = \dfrac{DE}{DF} = \dfrac{5}{13}$

$\tan D = \dfrac{EF}{DE} = \dfrac{12}{5}$

**Example 3**

In the right triangle GHI shown, find sin G, cos G and tan G.




**Solution:**

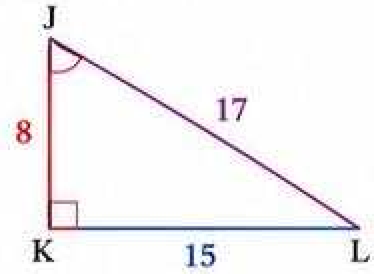
$\sin G = \dfrac{HI}{GI} = \dfrac{24}{25}$

$\cos G = \dfrac{GH}{GI} = \dfrac{7}{25}$

$\tan G = \dfrac{HI}{GH} = \dfrac{24}{7}$

**Example 4**

In the right triangle JKL shown, find sin J, cos J and tan J.




**Solution:**

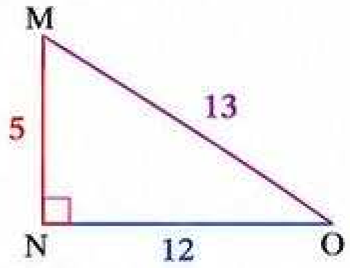
$\sin J = \dfrac{KL}{JL} = \dfrac{15}{17}$

$\cos J = \dfrac{JK}{JL} = \dfrac{8}{17}$

$\tan J = \dfrac{KL}{JK} = \dfrac{15}{8}$

**Example 5**

In the right triangle MNO shown, find sin N, cos N and tan N.




**Solution:**

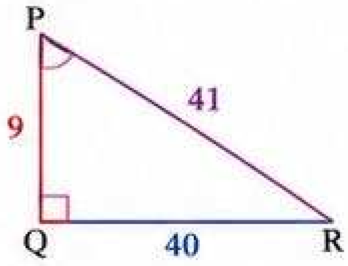
$\sin N = \dfrac{MO}{MO}$

$\cos N = \dfrac{NO}{12}$

$\tan N = \dfrac{MO}{NO} = \dfrac{13}{12}$

**Example 6**

In the right triangle PQR shown, find sin P, cos P and tan P.




**Solution:**

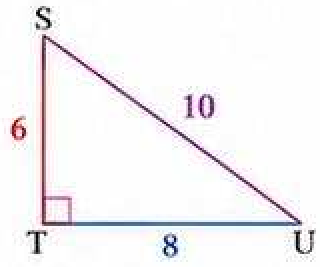
$\sin P = \dfrac{QR}{PR} = \dfrac{40}{41}$

$\cos P = \dfrac{PQ}{PR} = \dfrac{9}{41}$

$\tan P = \dfrac{QR}{PQ} = \dfrac{40}{9}$

**Example 7**

In the right triangle STU shown, find sin T, cos T and tan T.




**Solution:**

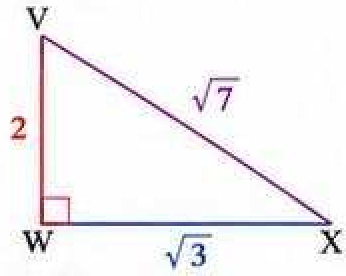
$\sin T = \dfrac{SU}{10}$

$\cos T = \dfrac{ST}{10}$

$\tan T = \dfrac{SU}{ST} = \dfrac{10}{6} = \dfrac{5}{3}$

**Example 8**

In the right triangle VWX shown, find sin V, cos V and tan V.




**Solution:**

$\sin V = \dfrac{WX}{VX} = \dfrac{\sqrt{3}}{\sqrt{7}}$

$\cos V = \dfrac{VW}{VX} = \dfrac{2}{\sqrt{7}}$

$\tan V = \dfrac{WX}{VW} = \dfrac{\sqrt{3}}{2}$

## Common Mistakes

- Confusing opposite and adjacent sides.
- Using the wrong side in the ratio.
- Forgetting that the hypotenuse is always the longest side.
- Mixing up sin, cos and tan.

## Comparison Table

| Ratio | Meaning | Formula | Uses |
|---|---|---|---|
| $\sin\theta$ | Ratio of opposite side to hypotenuse | $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ | Find heights, slopes, unknown sides |
| $\cos\theta$ | Ratio of adjacent side to hypotenuse | $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ | Find distances, unknown sides |
| $\tan\theta$ | Ratio of opposite side to adjacent side | $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ | Find slopes, angles of elevation |



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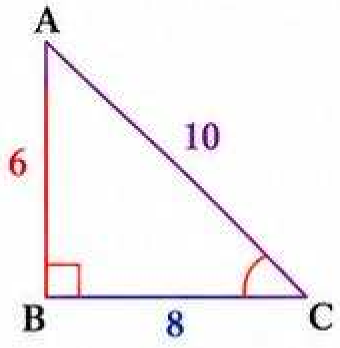
## More Solved Examples

### Example 9

In the right triangle ABC shown,
find sin B, cos B and tan B.




**Solution:**

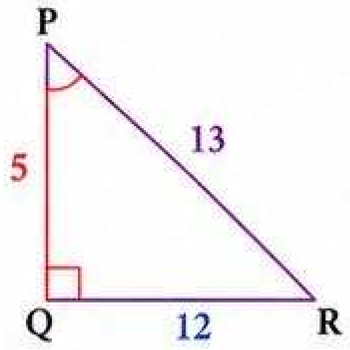
$\sin B = \dfrac{AC \text{ opposite to } B}{AC \text{ hypotenuse}} = \dfrac{6}{10} = \dfrac{3}{5}$

$\cos B = \dfrac{BC \text{ adjacent to } B}{AC \text{ hypotenuse}} = \dfrac{8}{10} = \dfrac{4}{5}$

$\tan B = \dfrac{AC \text{ opposite to } B}{BC \text{ adjacent to } B} = \dfrac{6}{8} = \dfrac{3}{4}$

### Example 10

In the right triangle PQR shown,
find sin P, cos P and tan P.




**Solution:**

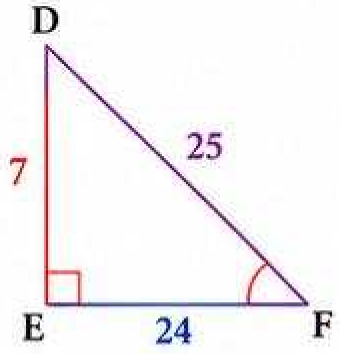
$\sin P = \dfrac{QR \text{ opposite to } P}{PR \text{ hypotenuse}} = \dfrac{12}{13}$

$\cos P = \dfrac{PQ \text{ adjacent to } P}{PR \text{ hypotenuse}} = \dfrac{5}{13}$

$\tan P = \dfrac{QR \text{ opposite to } P}{PQ \text{ adjacent to } P} = \dfrac{12}{5}$

### Example 11

In the right triangle DEF shown,
find sin E, cos E and tan E.




**Solution:**

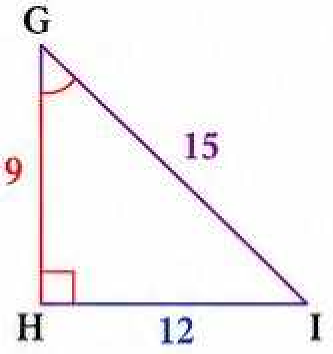
$\sin E = \dfrac{DF \text{ opposite to } E}{DF \text{ hypotenuse}} = \dfrac{24}{25}$

$\cos E = \dfrac{DE \text{ adjacent to } E}{DF \text{ hypotenuse}} = \dfrac{7}{25}$

$\tan E = \dfrac{DF \text{ opposite to } E}{DE \text{ adjacent to } E} = \dfrac{24}{7}$

### Example 12

In the right triangle GHI shown,
find sin G, cos G and tan G.




**Solution:**

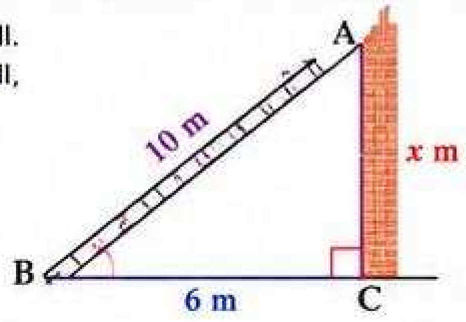
$\sin G = \dfrac{HI \text{ opposite to } G}{GI \text{ hypotenuse}} = \dfrac{12}{15} = \dfrac{4}{5}$

$\cos G = \dfrac{GH \text{ adjacent to } G}{GI \text{ hypotenuse}} = \dfrac{9}{15} = \dfrac{3}{5}$

$\tan G = \dfrac{HI \text{ opposite to } G}{GH \text{ adjacent to } G} = \dfrac{12}{9} = \dfrac{4}{3}$

## Real-Life Applications

**1** A ladder 10 m long is placed against a vertical wall. If the foot of the ladder is 6 m away from the wall, find the height it reaches on the wall.
(Use trigonometric ratios)




**Solution:**

In right triangle ABC at angle B:

$\cos B = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{BC}{AB} = \dfrac{6}{10} = \dfrac{3}{5}$

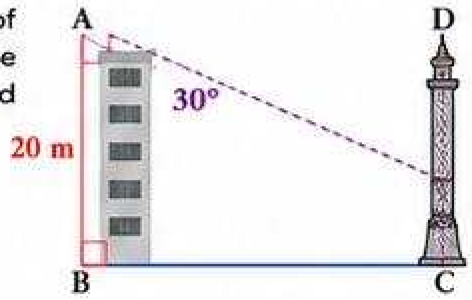
Now AB is the hypotenuse and AC is the height.
Using Pythagoras:

$AC^2 = AB^2 - BC^2 = 10^2 - 6^2 = 100 - 36 = 64$

$\therefore AC = \sqrt{64} = 8$ m

The ladder reaches 8 m high on the wall.

**2** From the top of a 20 m high building, the angle of elevation of the top of a tower is 30°. Find the horizontal distance between the building and the foot of the tower.
(Use trigonometric ratios)




**Solution:**

In right triangle ABC:

$\tan 30° = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{BC}{AB} = \dfrac{1}{\sqrt{3}}$

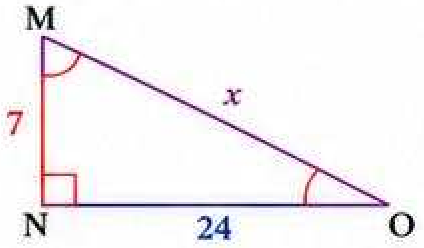
But AB = 20 m (opposite side)

$\therefore BC = 20 \times \sqrt{3} \approx 20 \times 1.732 = 34.64$ m

The horizontal distance is about 34.64 m.

## Think!

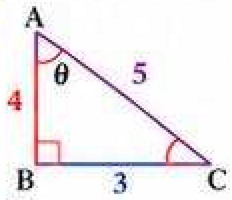
In right triangle MNO,
$\angle M = 90°$, $MO = 7$, $ON = 24$.
Find sin N, cos N and tan N.




## Quick Check

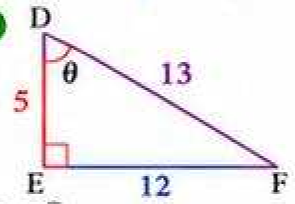
Find the trigonometric ratios for the given angle in each right triangle.

**1**



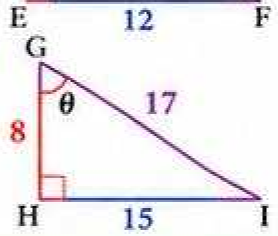

$\sin \theta =$ ______
$\cos \theta =$ ______
$\tan \theta =$ ______

**2**




$\sin \theta =$ ______
$\cos \theta =$ ______
$\tan \theta =$ ______

**3**




$\sin \theta =$ ______
$\cos \theta =$ ______
$\tan \theta =$ ______

**Key Point**

To find a trigonometric ratio of an angle, identify the opposite side, adjacent side, and hypotenuse relative to that angle.

**Remember**

- SOH – CAH – TOA
- sin θ uses: opposite & hypotenuse
- cos θ uses: adjacent & hypotenuse
- tan θ uses: opposite & adjacent

- $\sin \theta = \dfrac{opp}{hyp}$
- $\cos \theta = \dfrac{adj}{hyp}$
- $\tan \theta = \dfrac{opp}{adj}$



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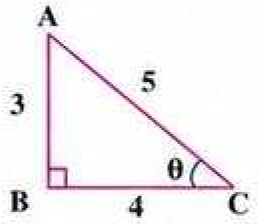
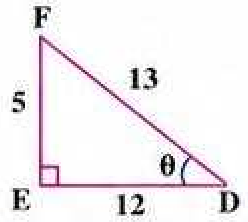
## A Choose the correct answer :

1. In the opposite figure : $\sin\theta = ?$



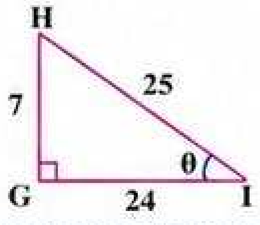

ⓐ $\frac{3}{4}$ ⓑ $\frac{4}{5}$ ⓒ $\frac{3}{5}$ ⓓ $\frac{4}{3}$

2. In the opposite figure : $\cos\theta = ?$



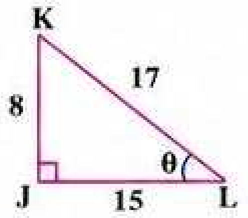

ⓐ $\frac{5}{12}$ ⓑ $\frac{12}{13}$ ⓒ $\frac{5}{13}$ ⓓ $\frac{12}{5}$

3. In the opposite figure : $\tan\theta = ?$



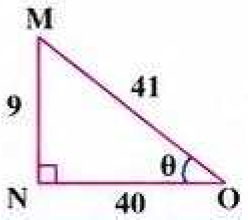

ⓐ $\frac{7}{24}$ ⓑ $\frac{24}{7}$ ⓒ $\frac{7}{25}$ ⓓ $\frac{24}{25}$

4. In the opposite figure : $\sin\theta = ?$



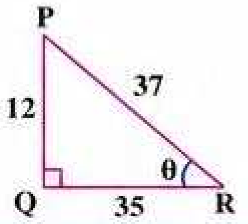

ⓐ $\frac{8}{15}$ ⓑ $\frac{15}{17}$ ⓒ $\frac{8}{17}$ ⓓ $\frac{17}{8}$

5. In the opposite figure : $\cos\theta = ?$



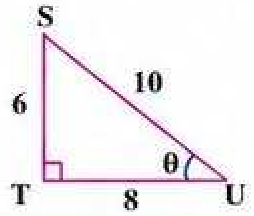

ⓐ $\frac{9}{40}$ ⓑ $\frac{40}{41}$ ⓒ $\frac{9}{41}$ ⓓ $\frac{41}{9}$

6. In the opposite figure : $\tan\theta = ?$



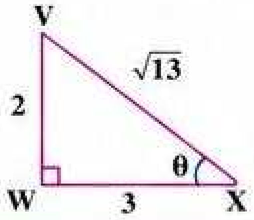

ⓐ $\frac{12}{35}$ ⓑ $\frac{35}{12}$ ⓒ $\frac{12}{37}$ ⓓ $\frac{37}{12}$

7. In the opposite figure : $\sin\theta = ?$




ⓐ $\frac{3}{5}$ ⓑ $\frac{4}{5}$ ⓒ $\frac{3}{4}$ ⓓ $\frac{4}{3}$

8. In the opposite figure : $\tan\theta = ?$




ⓐ $\frac{2}{\sqrt{13}}$ ⓑ $\frac{3}{\sqrt{13}}$ ⓒ $\frac{2}{3}$ ⓓ $\frac{3}{2}$

## B Complete the following :

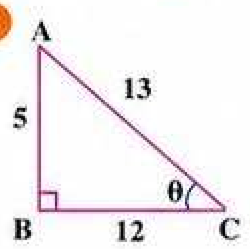
1. In a right triangle, if $\sin\theta = \frac{3}{5}$, then the ratio of the opposite side to the hypotenuse = ..............
2. In a right triangle, if $\cos\theta = \frac{4}{5}$, then the ratio of the adjacent side to the hypotenuse = ..............
3. In a right triangle, if $\tan\theta = \frac{7}{24}$, then the ratio of the opposite side to the adjacent side = ..............
4. In a right triangle, if the opposite side = 6 and the hypotenuse = 10, then $\sin\theta$ = ..............
5. In a right triangle, if the adjacent side = 12 and the hypotenuse = 13, then $\cos\theta$ = ..............
6. In a right triangle, if the opposite side = 9 and the adjacent side = 12, then $\tan\theta$ = ..............
7. In a right triangle, if the hypotenuse = 25 and $\cos\theta = \frac{7}{25}$, then the adjacent side = ..............
8. In a right triangle, if $\sin\theta = \frac{5}{13}$ and the opposite side = 5, then the hypotenuse = ..............
9. In a right triangle, if $\tan\theta = \frac{3}{4}$ and the adjacent side = 12, then the opposite side = ..............
10. In a right triangle, if the opposite side = 5 and the adjacent side = 12, then $\sin\theta$ = .............. and $\cos\theta$ = ..............

## C Write ( ✓ ) or ( X ) :

1. In a right triangle, $\sin\theta$ is the ratio of the opposite side to the hypotenuse. ( )
2. In a right triangle, $\cos\theta$ is the ratio of the opposite side to the hypotenuse. ( )
3. In a right triangle, $\tan\theta$ is the ratio of the opposite side to the adjacent side. ( )
4. The hypotenuse is the longest side in a right triangle. ( )
5. In a right triangle, $\sin\theta$ can be greater than 1. ( )
6. The ratio $\tan\theta$ does not use the hypotenuse. ( )
7. If $\cos\theta = \frac{1}{2}$, then the adjacent side is half the hypotenuse. ( )
8. If $\sin\theta = 1$, then the triangle is a right triangle. ( )
9. If $\tan\theta = 1$, then the opposite side equals the adjacent side. ( )
10. In a right triangle, $\cos^2\theta + \sin^2\theta = 1$. ( )

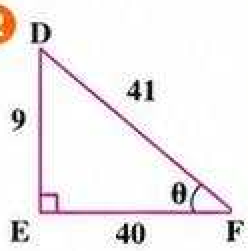
## D Find the trigonometric ratios for the marked angle θ in each triangle.

1.



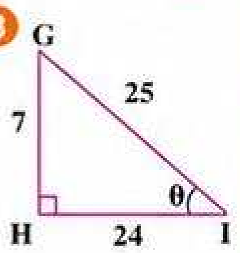

$\sin\theta$ = ________
$\cos\theta$ = ________
$\tan\theta$ = ________

2.




$\sin\theta$ = ________
$\cos\theta$ = ________
$\tan\theta$ = ________

3.




$\sin\theta$ = ________
$\cos\theta$ = ________
$\tan\theta$ = ________

## E Solve the following problems.

1. A ladder 15 m long is placed against a vertical wall. If the foot of the ladder is 9 m away from the wall, find the height it reaches on the wall. (Use trigonometric ratios)
..............................................................................................
2. From the top of a 30 m building, the angle of depression of a car on the ground is 30°. Find the horizontal distance between the building and the car. (Use trigonometric ratios)
..............................................................................................
3. A flagpole casts a shadow of 15 m when the angle of elevation of the sun is 60°. Find the height of the flagpole. (Use trigonometric ratios)
..............................................................................................



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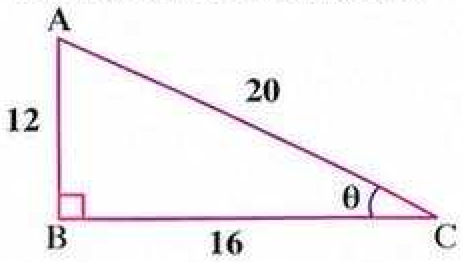
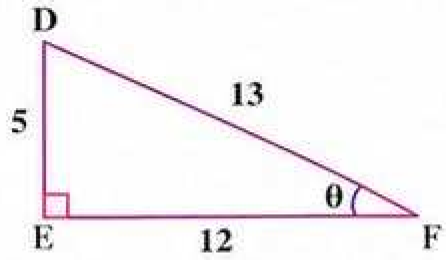
## F Find the value of each of the following:

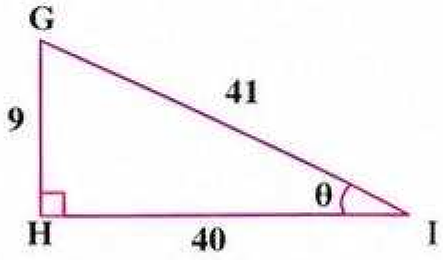
1. Find sin θ , cos θ and tan θ.



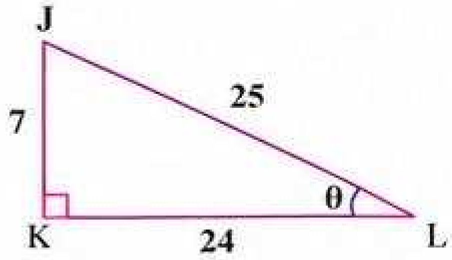

2. Find sin θ , cos θ and tan θ.



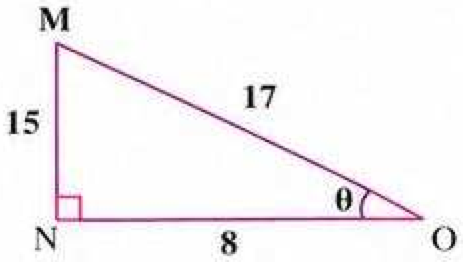

3. Find sin θ , cos θ and tan θ.



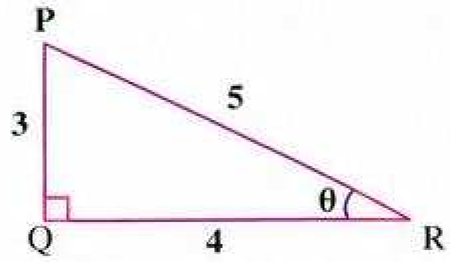

4. Find sin θ , cos θ and tan θ.



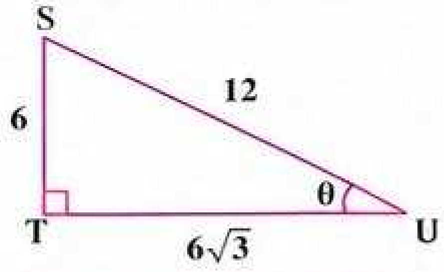

5. Find sin θ , cos θ and tan θ.



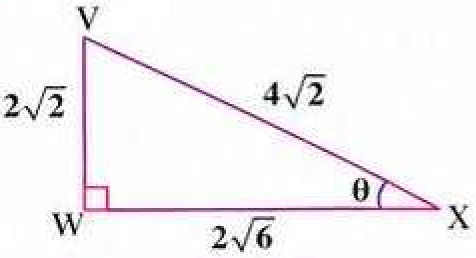

6. Find sin θ , cos θ and tan θ.



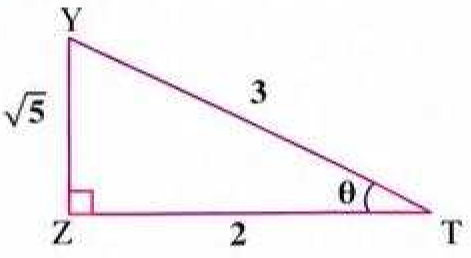

7. Find sin θ , cos θ and tan θ.




8. Find sin θ , cos θ and tan θ.

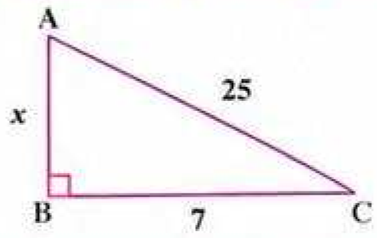



9. Find sin θ , cos θ and tan θ.

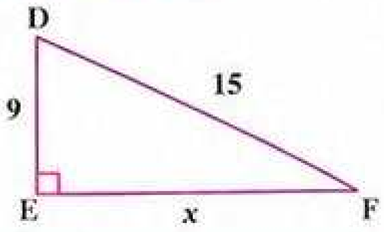



## G Solve for $x$ in each of the following right triangles:

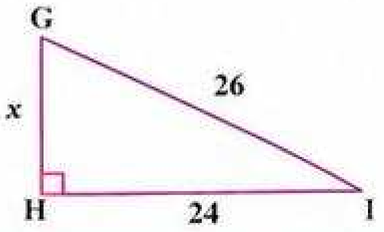
1.



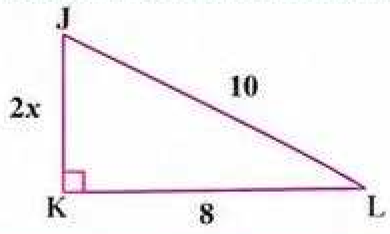

2.



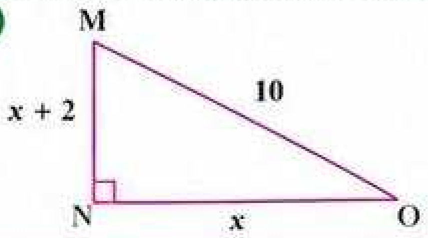

3.



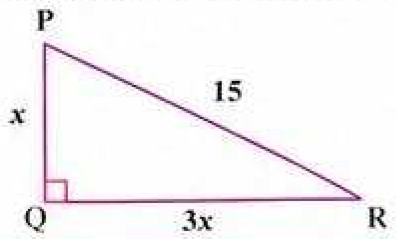

4.




5.




6.




## H Word Problems (Use trigonometric ratios)

1. A flagpole casts a shadow of 18 m when the angle of elevation of the sun is 30°. Find the height of the flagpole.

Solution :

..............................................................................................

..............................................................................................

2. From a point on the ground, the angle of elevation of the top of a tower is 45°. If the horizontal distance between the point and the base of the tower is 40 m, find the height of the tower.

Solution :

..............................................................................................

..............................................................................................

3. A man is standing 25 m away from a tree. The angle of elevation of the top of the tree from his eye is 60°. Find the height of the tree.

Solution :

..............................................................................................

..............................................................................................

4. A boy 1.6 m tall is standing 20 m away from a lamppost. The angle of elevation from his eyes to the top of the lamppost is 30°. Find the height of the lamppost.

Solution :

..............................................................................................

..............................................................................................

## I Problem Solving

1. In a right triangle, $\sin \theta = \frac{3}{5}$.
What can be the possible lengths of the sides?
(Write more than one possible set)

..............................................................................................

..............................................................................................

2. In a right triangle, $\cos \theta = \frac{5}{13}$.
What are the possible lengths of the sides?
(Write more than one possible set)

..............................................................................................

..............................................................................................



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## A Use Pythagoras Theorem to find the missing side, then find the trigonometric ratios for the marked angle.

**Example:**

In right triangle ABC at B, AB = 6 cm, BC = 8 cm.

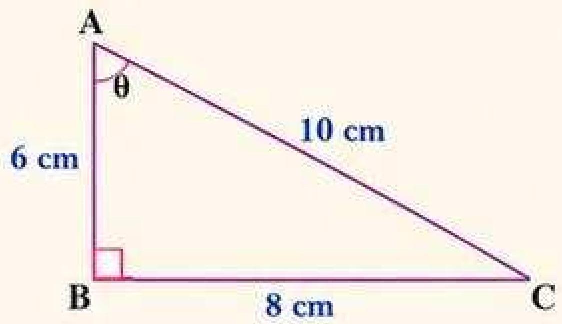
1) Find AC.

$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\therefore AC = \sqrt{100} = 10$ cm

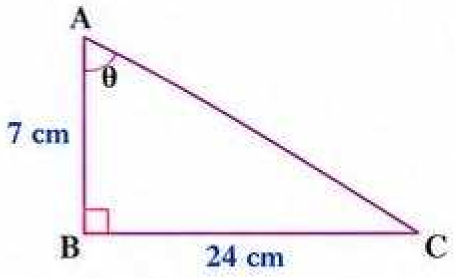
2) For angle $A$:

$\sin A = \frac{BC}{AC} = \frac{8}{10} = \frac{4}{5}$ , $\cos A = \frac{AB}{AC} = \frac{6}{10} = \frac{3}{5}$ , $\tan A = \frac{BC}{AB} = \frac{8}{6} = \frac{4}{3}$




**1** In right triangle ABC at B, AB = 7 cm, BC = 24 cm.

1) Find AC.

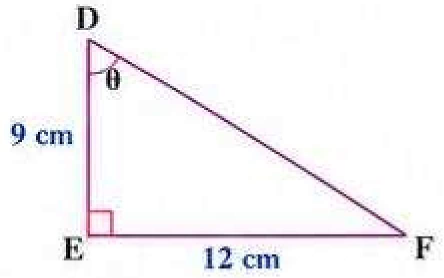



2) For angle A, find:

sin A = ____ cos A = ____ tan A = ____

**2** In right triangle DEF at E, DE = 9 cm, EF = 12 cm.

1) Find DF.

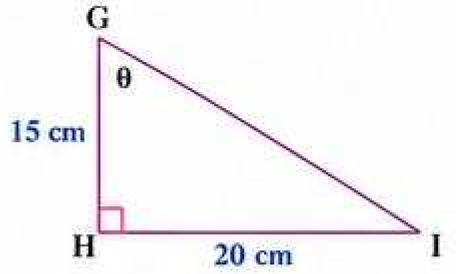



2) For angle D, find:

sin D = ____ cos D = ____ tan D = ____

**3** In right triangle GHI at H, GH = 15 cm, HI = 20 cm.

1) Find GI.

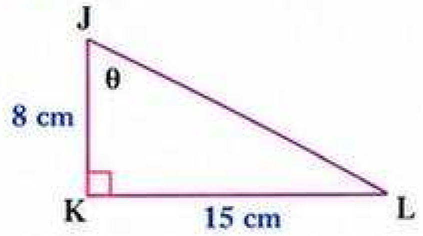



2) For angle G, find:

sin G = ____ cos G = ____ tan G = ____

**4** In right triangle JKL at K, JK = 8 cm, KL = 15 cm.

1) Find JL.

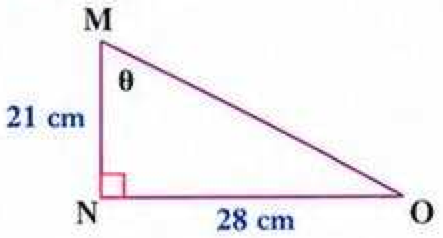



2) For angle J, find:

sin J = ____ cos J = ____ tan J = ____

**5** In right triangle MNO at N, MN = 21 cm, NO = 28 cm.

1) Find MO.

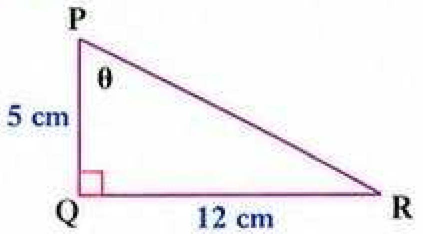



2) For angle $M$, find:

sin $M$ = ____ cos $M$ = ____ tan $M$ = ____

**6** In right triangle PQR at Q, PQ = 5 cm, QR = 12 cm.

1) Find PR.

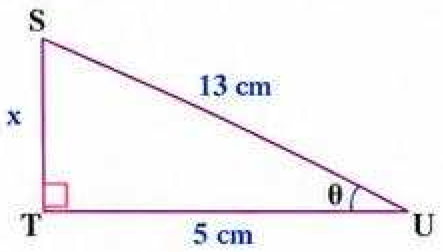



2) For angle P, find:

sin P = ____ cos P = ____ tan P = ____

## B Apply Pythagoras Theorem to find the missing side, then find the trigonometric ratios for the marked angle.

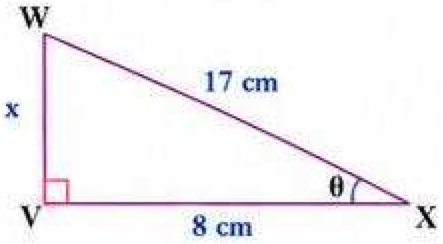
**1** In right triangle STU at T, SU = 13 cm, TU = 5 cm.
Find ST. Then find sin U , cos U and tan U.




ST = ______ cm

sin U = ____ cos U = ____ tan U = ____

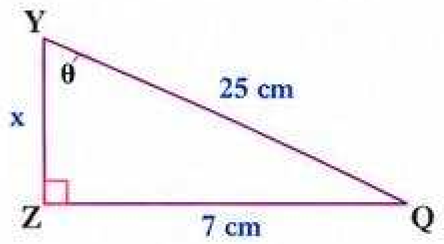
**2** In right triangle VWX at V, WX = 17 cm, VX = 8 cm.
Find VW. Then find sin X , cos X and tan X.




VW = ______ cm

sin X = ____ cos X = ____ tan X = ____

**3** In right triangle YZQ at Z, YQ = 25 cm, ZQ = 7 cm.
Find YZ. Then find sin Y , cos Y and tan Y.



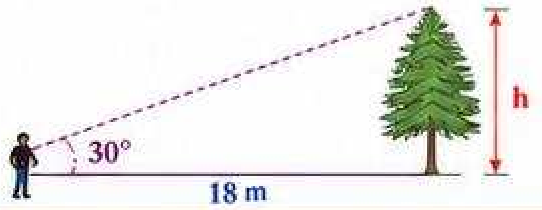

YZ = ______ cm

sin Y = ____ cos Y = ____ tan Y = ____

## C Real-Life Application (Use Pythagoras Theorem first, then trigonometric ratios)

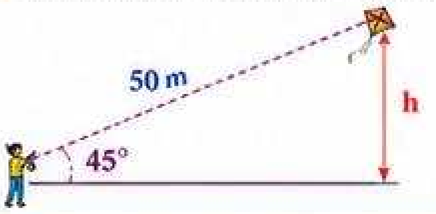
**1** A man stands 18 m away from the base of a tree. The angle of elevation from his eyes to the top of the tree is 30°. Find the height of the tree.

Answer : ..............................................................



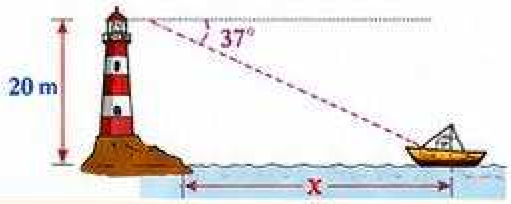

**2** A boy flying a kite lets out 50 m of string. The angle of elevation of the kite from the boy is 45°. How high is the kite above the ground?

Answer : ..............................................................




**3** From the top of a 20 m lighthouse, the angle of depression of a boat in the sea is 37°. Find the horizontal distance between the lighthouse and the boat. (Use trigonometric ratios)

Answer : ..............................................................



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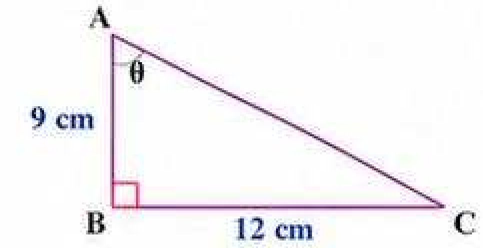
## A Use Pythagoras Theorem to find the missing side, then find the trigonometric ratios for the marked angle.

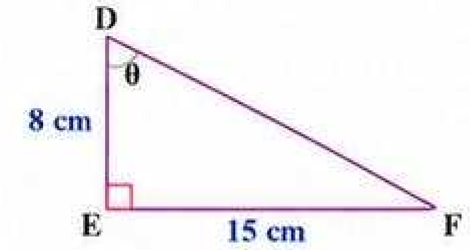
**1** In right triangle ABC at B,
AB = 9 cm, BC = 12 cm.
1) Find AC.




2) For angle A, find:

sin A = _____ cos A = _____ tan A = _____

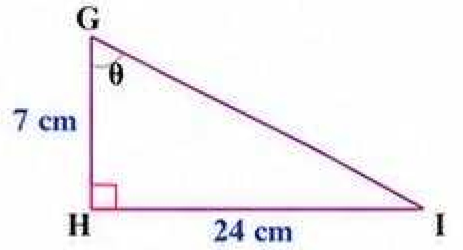
**2** In right triangle DEF at E,
DE = 8 cm, EF = 15 cm.
1) Find DF.




2) For angle D, find:

sin D = _____ cos D = _____ tan D = _____

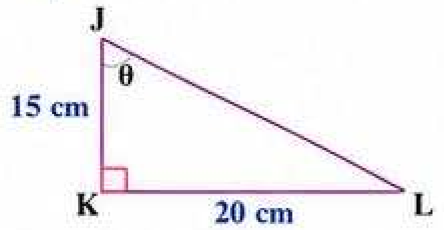
**3** In right triangle GHI at H,
GH = 7 cm, HI = 24 cm.
1) Find GI.




2) For angle G, find:

sin G = _____ cos G = _____ tan G = _____

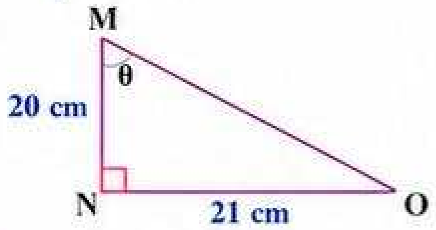
**4** In right triangle JKL at K,
JK = 15 cm, KL = 20 cm.
1) Find JL.




2) For angle J, find:

sin J = _____ cos J = _____ tan J = _____

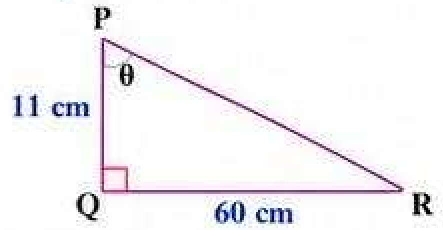
**5** In right triangle MNO at N,
MN = 20 cm, NO = 21 cm.
1) Find MO.




2) For angle M, find:

sin M = _____ cos M = _____ tan M = _____

**6** In right triangle PQR at Q,
PQ = 11 cm, QR = 60 cm.
1) Find PR.

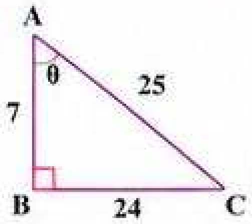



2) For angle P, find:

sin P = _____ cos P = _____ tan P = _____

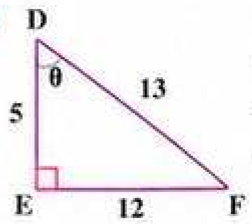
## B Find the trigonometric ratios for the marked angle θ in each right triangle.

**1**



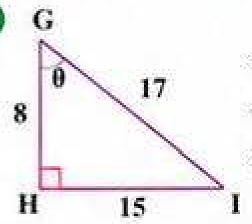

sin θ = _____
cos θ = _____
tan θ = _____

**2**



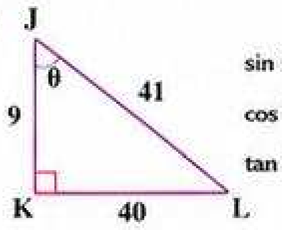

sin θ = _____
cos θ = _____
tan θ = _____

**3**



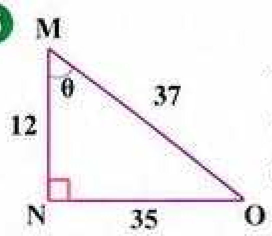

sin θ = _____
cos θ = _____
tan θ = _____

**4**



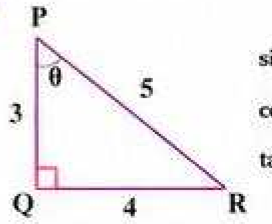

sin θ = _____
cos θ = _____
tan θ = _____

**5**



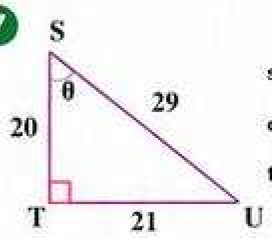

sin θ = _____
cos θ = _____
tan θ = _____

**6**



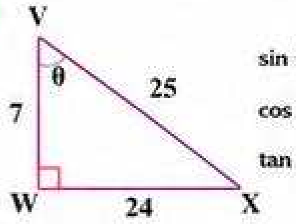

sin θ = _____
cos θ = _____
tan θ = _____

**7**




sin θ = _____
cos θ = _____
tan θ = _____

**8**

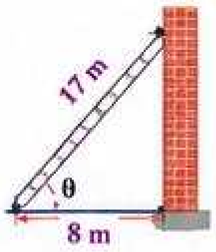



sin θ = _____
cos θ = _____
tan θ = _____

## C Word Problems (Use Pythagoras Theorem then trigonometric ratios)

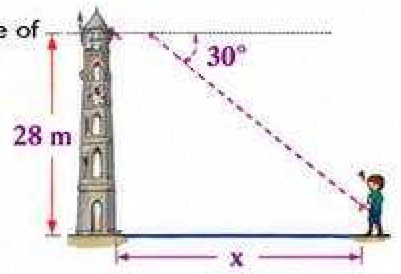
**1** A 17 m long ladder is leaning against a wall.
The foot of the ladder is 8 m away from the wall.
Find the height it reaches on the wall and then find sin θ, cos θ and tan θ.

Answer : ..........................................
..........................................



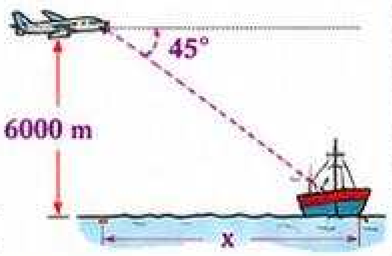

**2** From the top of a 28 m high tower, the angle of depression of a boy on the ground is 30°.
Find the horizontal distance between the boy and the base of the tower and then find sin 30°, cos 30° and tan 30°.

Answer : ..........................................
..........................................



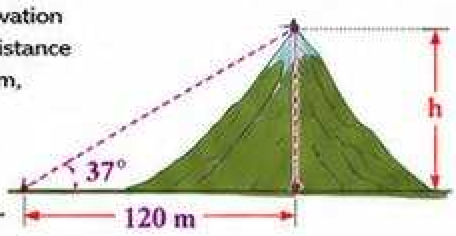

**3** An airplane is flying at a height of 6000 m.
The angle of depression of a ship is 45°.
Find the horizontal distance between the airplane and the ship and then find sin 45°, cos 45° and tan 45°.

Answer : ..........................................
..........................................




**4** From a point on the ground, the angle of elevation of the top of a hill is 37°. If the horizontal distance from the point to the foot of the hill is 120 m, find the height of the hill and then find sin 37°, cos 37° and tan 37°.

Answer : ..........................................
..........................................




## D Solve the following.

**1** In a right triangle, $\tan\theta = \frac{4}{3}$.
If the adjacent side is 9 cm,
find the lengths of the other two sides.
..........................................
..........................................

**2** In a right triangle, $\sin\theta = \frac{3}{5}$.
If the hypotenuse is 20 cm,
find the lengths of the other two sides.
..........................................
..........................................

**3** In a right triangle, $\cos\theta = \frac{5}{13}$.
If the adjacent side is 10 cm,
find the lengths of the other two sides.
..........................................
..........................................



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Egyptian Obelisks

## Get Started!

Egyptian obelisks are tall stone monuments that have held historical and cultural significance for thousands of years. Many of them were designed with specific inclinations using special angles such as 30°, 45°, and 60° to achieve both stability and beauty.

In this lesson, we will learn how to find trigonometric ratios of some special angles and use them to solve problems.

### In this lesson, you will learn:

- Find the trigonometric ratios of the special angles (30°, 45°, 60°).
- Use the trigonometric ratios to solve problems.
- Find unknown sides or angles in right triangles.
- Solve problems in real-life situations using the **ratios** of special angles.

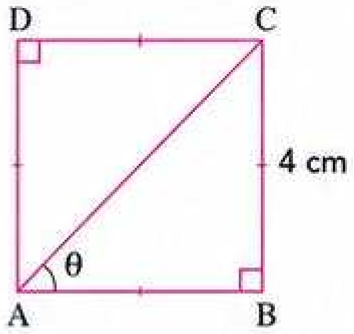
### Key Words

- Special angles

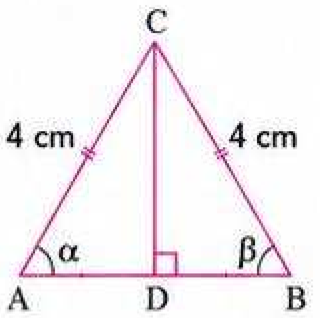
## Think & Discuss!

**In the figures:**

- What is the value of each of $\alpha$ , $\beta$ and $\theta$ ?
- How can we find the trigonometric ratios of the special angles: 30°, 45°, 60° ?




Square ABCD



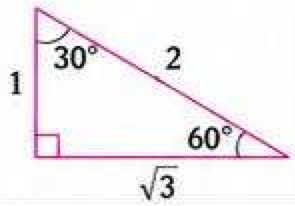
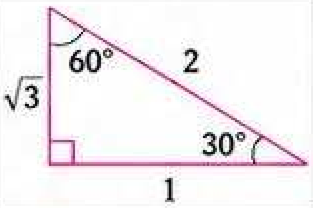
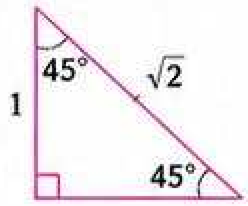

Isosceles triangle ABC

### Why do we need this?

- To find unknown sides or angles in right triangles.
- In engineering and architecture to find heights and distances.
- In navigation and surveying to measure distances and angles.
- In daily life situations such as calculating the height of a tree or the slope of a roof.

## Trigonometric Ratios of Some Special Angles

We can use the right triangles that contain the angles (30°, 60°, 90°) and (45°, 45°, 90°) to find the trigonometric ratios of the special angles 30°, 45°, 60° as shown in the table below.

| Ratio | sin (θ) | cos (θ) | tan (θ) | Right Triangle |
|---|---|---|---|---|
| 30° | $\sin 30° = \frac{1}{2}$ | $\cos 30° = \frac{\sqrt{3}}{2}$ | $\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ |   |
| 60° | $\sin 60° = \frac{\sqrt{3}}{2}$ | $\cos 60° = \frac{1}{2}$ | $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$ |   |
| 45° | $\sin 45° = \frac{\sqrt{2}}{2}$ | $\cos 45° = \frac{\sqrt{2}}{2}$ | $\tan 45° = \frac{1}{1} = 1$ |   |

### Remember

**Facts about right triangles:**

- The sum of the acute angles in a right triangle = **90°**. ($A + B = 90°$)
- The measure of each acute angle in an isosceles right triangle = **45°**. ($A = B = 45°$)
- The measure of the two acute angles in a right triangle that complement each other: ($A$, $B$) ($A + B = 90°$, $A \neq B$)

Example:
If $A = 30° \Rightarrow B = 60°$
If $A = 60° \Rightarrow B = 30°$

### Math Tip

- You can find the trigonometric ratios of **30°**, **45°** and **60°** using:
- The known values of sin, cos, tan in special right triangles.
- The calculator in **degree mode**.



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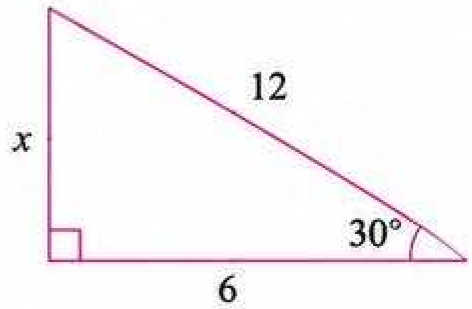
## Examples

Use the trigonometric ratios of 30°, 45° and 60° to find the value of $x$ in each of the following:

**Example 1**

In the right triangle below, find the value of $x$.




**Solution**

Since the triangle is a 30°-60°-90° right triangle.

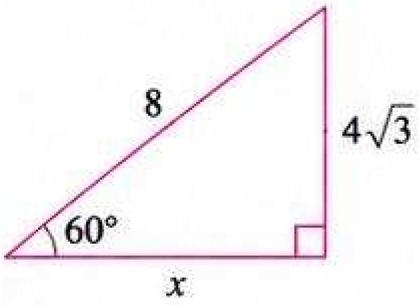
$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$

$\therefore \frac{1}{2} = \frac{x}{12}$

$\therefore x = 12 \times \frac{1}{2} = 6$

**Example 2**

In the right triangle below, find the value of $x$.




**Solution**

Since the triangle is a 30°-60°-90° right triangle.

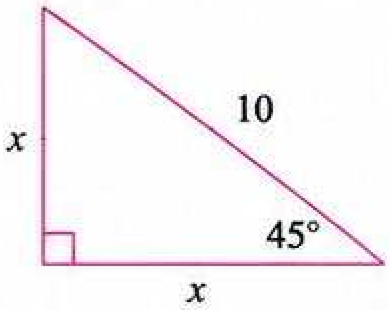
$\cos 60° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{2}$

$\therefore \frac{1}{2} = \frac{x}{8}$

$\therefore x = 8 \times \frac{1}{2} = 4$

**Example 3**

In the right isosceles triangle below, find the value of $x$.




**Solution**

Since the triangle is a 45°-45°-90° right triangle.

$\cos 45° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{2}}{2}$

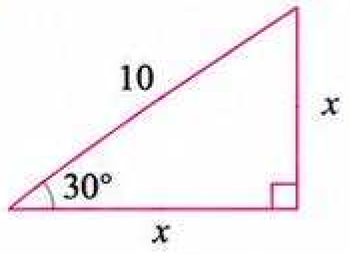
$\therefore \frac{\sqrt{2}}{2} = \frac{x}{10}$

$\therefore x = 10 \times \frac{\sqrt{2}}{2} = 5\sqrt{2}$

## Your Turn

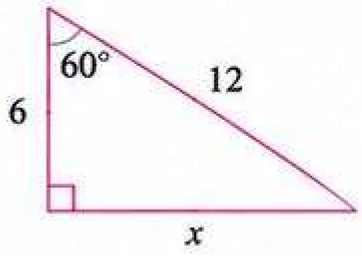
Find the value of $x$ in each of the following:

1.



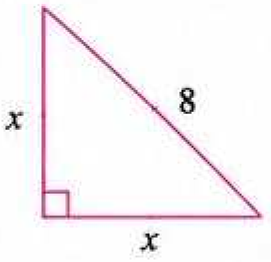

$x =$ ..............................

2.




$x =$ ..............................

3.




$x =$ ..............................

### Study Tip

Memorize the exact values of sin, cos, tan for 30°, 45°, 60°.
This will help you solve many problems quickly and accurately.

## Try This!

**Find the value of each of the following without using a calculator:**

1. $\sin 30°$ ..............................
2. $\cos 60°$ ..............................
3. $\tan 45°$ ..............................
4. $\cos 45°$ ..............................
5. $\tan 60°$ ..............................
6. $\sin 45°$ ..............................

### Common Mistake

Don't confuse the ratios!

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

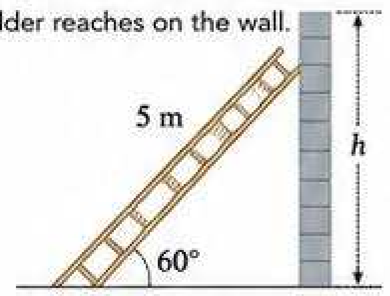
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Always identify the side related to the given angle.

## Real Life

A ladder 5 meters long is leaning against a wall. The angle between the ladder and the ground is 60°. Find the height ($h$) that the ladder reaches on the wall.




**Solution**

$\sin 60° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$

$\therefore \frac{\sqrt{3}}{2} = \frac{h}{5}$

$\therefore h = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$ meters

### Challenge

A flagpole casts a shadow of $4\sqrt{3}$ meters on the ground when the angle of elevation of the sun is 30°. Find the height of the flagpole.

Answer: ..................



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## Example 3

Find the value of $x$ in each of the following:

**1**

**Solution**

$$\sin 26° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{8}$$

$$\therefore\ x = 8 \sin 26° \approx 3.51 \text{ m}$$

**2**

**Solution**

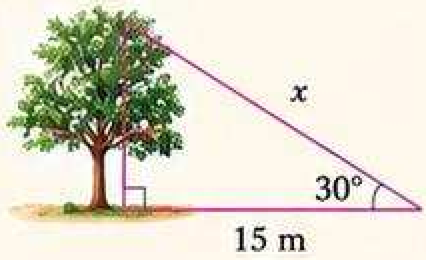
$$\tan 54° = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{x}$$

$$\therefore\ x = \frac{6}{\tan 54°} \approx 4.36 \text{ m}$$

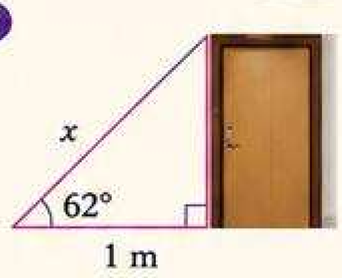
## Example 3 (Continued)

Find the value of $x$ in each of the following:

**1**




**2**




## Measuring an Angle When One Trigonometric Ratio Is Known

If one trigonometric ratio of an acute angle θ is known, we can find θ as follows:

If $\theta = \sin^{-1}(x)$ then $\sin\theta = x$

If $\theta = \cos^{-1}(x)$ then $\cos\theta = x$

If $\theta = \tan^{-1}(x)$ then $\tan\theta = x$

### Study Tip

When you know one trigonometric ratio of an angle, you can find the angle using the inverse functions on your calculator.

**Example:**

If $\sin\theta = 0.834$, find the measure of θ.

To find the measure of this angle, use the calculator as follows:

SHIFT | $\sin^{-1}$ | 0 | · | 8 | 3 | 4 | = | 56.30°

Hence, the measure of the angle is approximately $56°\ 30'\ 43''$.

### Common Mistake

Do not confuse opposite, adjacent and hypotenuse. Always identify the side related to the given angle.

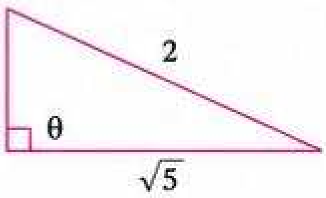
### Remember

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
- All the ratios are used only in right triangles.

## Example 4

Find the value of θ in each of the following:

**1**



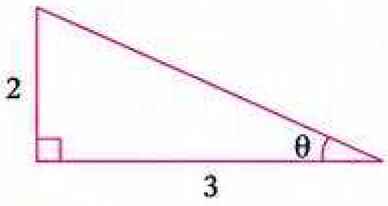

**Solution**

$$\sin\theta = \frac{\sqrt{5}}{2}$$

$$\therefore\ \theta = \sin^{-1}\left(\frac{\sqrt{5}}{2}\right)$$

$$\therefore\ \theta \approx 63°\ 26'\ 6''$$

**2**




**Solution**

$$\tan\theta = \frac{2}{3}$$

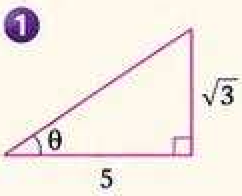
$$\therefore\ \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

$$\therefore\ \theta \approx 33°\ 41'\ 24''$$

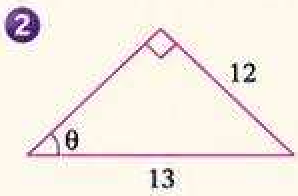
## Example 4 (Continued)

Find the value of θ in each of the following:

**1**




**2**




## Example 5

Find the value of θ if $2\cos(\theta + 20°) = \tan 45°$

$$2\cos(\theta + 20°) = \tan 45°$$

$$2\cos(\theta + 20°) = 1$$

$$\cos(\theta + 20°) = \frac{1}{2}$$

$$\theta + 20° = 60°$$

$$\therefore\ \theta = 40°$$

- $2\cos(\theta + 20°) = \tan 45°$
- $\tan 45° = 1$
- $\cos(\theta + 20°) = \frac{1}{2}$
- $\cos 60° = \frac{1}{2}$
- $\theta + 20° = 60°$
- $\theta = 40°$

## Example 5 (Continued)

Find the value of θ if:

$$\sqrt{3}\sin(\theta - 16°) = \sin 60°$$

## Example 6

In $\triangle ABC$, $\sin A = \cos 2A$ and $m(\angle B) = 72°$.

Find $m(\angle C)$.



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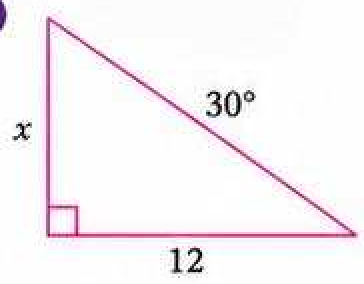
## Your Turn

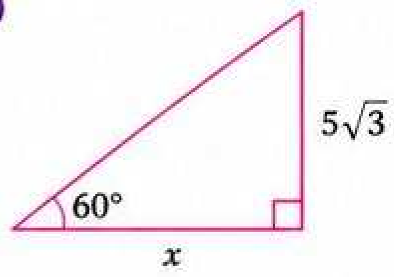
**1** Find the value of $x$ in each of the following:

**1**



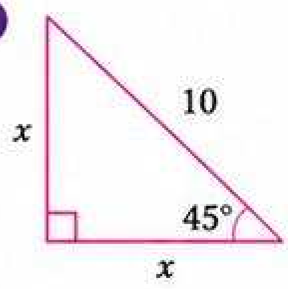

$x =$ ..........................

**2**



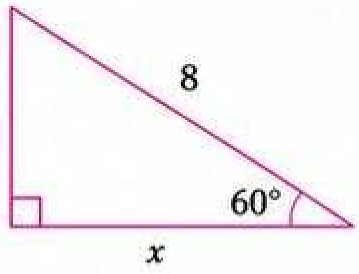

$x =$ ..........................

**3**




$x =$ ..........................

**4**




$x =$ ..........................

## Try This!

Find the value of each of the following without using a calculator.

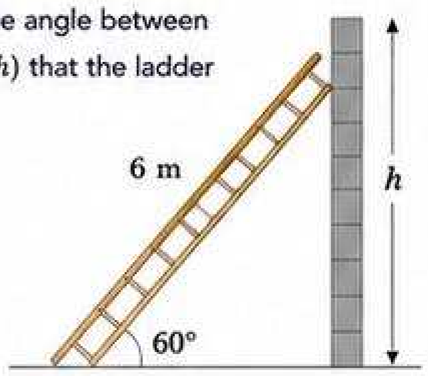
1. $\sin 30°$ ..........................
2. $\cos 60°$ ..........................
3. $\tan 45°$ ..........................
4. $\sin 60°$ ..........................
5. $\cos 45°$ ..........................
6. $\tan 30°$ ..........................
7. $\sin 45°$ ..........................
8. $\cos 30°$ ..........................

### Study Tip

Memorize the exact values of sin, cos, tan for 30°, 45°, 60°.
This will help you solve problems quickly and accurately.

## Real Life

A ladder 6 meters long is leaning against a wall. The angle between the ladder and the ground is 60°. Find the height ($h$) that the ladder reaches on the wall.




**Solution**

$\sin 60° = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{h}{6}$

$\dfrac{\sqrt{3}}{2} = \dfrac{h}{6}$

$h = 6 \times \dfrac{\sqrt{3}}{2} = 3\sqrt{3}$ m

Hence, the height is $3\sqrt{3}$ meters.

### Common Mistake

Don't confuse opposite, adjacent and hypotenuse. Always identify the side related to the given angle.

## Evaluate Yourself

**1 Choose the correct answer:**

(1) $\sin 60°$ equals:
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1

(2) In a 45°–45°–90° triangle, each acute angle equals:
(a) 30° (b) 45° (c) 60° (d) 90°

(3) $\cos 30°$ equals:
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{\sqrt{2}}{2}$

(4) $\tan 45°$ equals:
(a) $\frac{1}{2}$ (b) $\sqrt{3}$ (c) 1 (d) 0

**2 Complete:**

(1) $\sin 45° =$ ...............
(2) $\cos 60° =$ ...............
(3) $\tan 30° =$ ...............
(4) $\sin 30° =$ ...............

**3 True or False:**

(1) $\sin 30° = \cos 60°$. ( )
(2) $\cos 45° = \tan 45°$. ( )
(3) $\sin 60° = \frac{\sqrt{3}}{2}$. ( )
(4) $\tan 30° = 1$. ( )

**4 Solve the following:**

(1) In a right triangle, $\sin\theta = \frac{1}{2}$. If $\theta$ is an acute angle, find $\theta$.
..........................................................

(2) In a right triangle, $\cos\theta = \frac{\sqrt{2}}{2}$. Find $\theta$.
..........................................................

(3) In a right triangle, $\tan\theta = \sqrt{3}$. Find $\theta$.
..........................................................

## Challenge

**1** In a right triangle, $\sin A = \cos B$. If $A$ is an acute angle and $A \neq B$, find the measures of $A$ and $B$.
..........................................................

**2** A flagpole casts a shadow of $8\sqrt{3}$ meters on the ground when the angle of elevation of the sun is 30°. Find the height of the flagpole.
..........................................................

**3** The angle of elevation of the top of a tower from a point on the ground is 45°. When the point moves 20 meters farther from the tower, the angle becomes 30°. Find the height of the tower.
..........................................................



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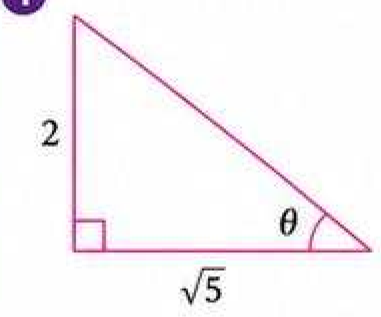
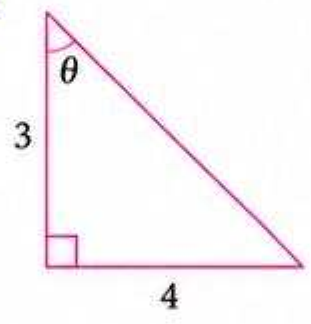
## Your Turn (Continued)

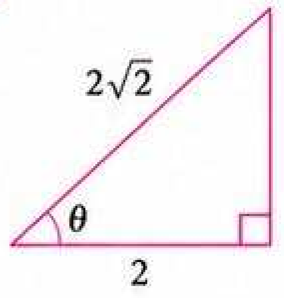
**2** Find the value of $\theta$ in each of the following:

1. 

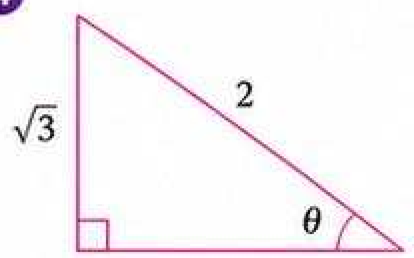
$\theta =$ ..........................

2. 

$\theta =$ ..........................

3. 

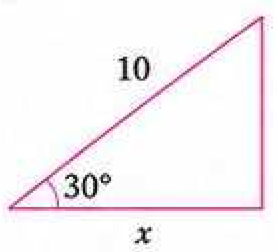
$\theta =$ ..........................

4. 

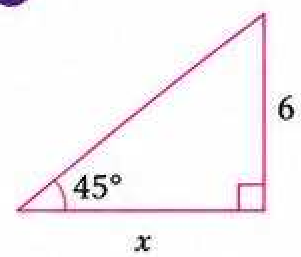
$\theta =$ ..........................

## More Practice

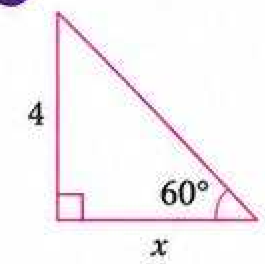
Find the value of $x$ in each of the following:

1. 

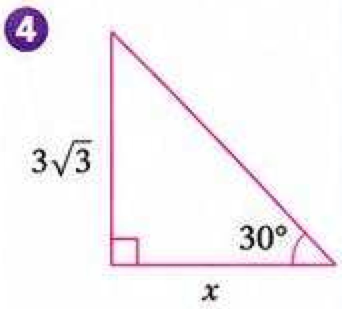
$x =$ ..........................

2. 

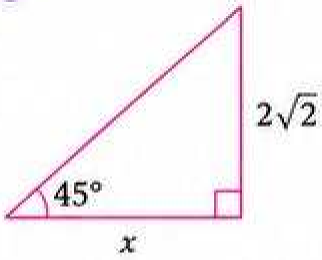
$x =$ ..........................

3. 

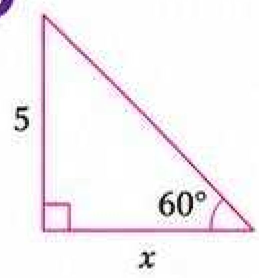
$x =$ ..........................

4. 

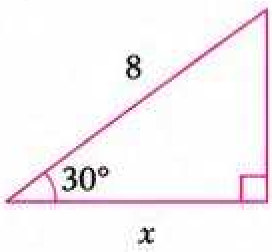
$x =$ ..........................

5. 

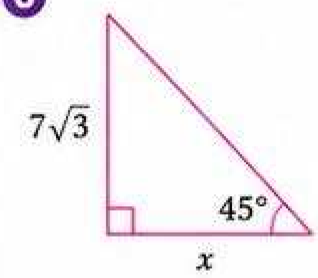
$x =$ ..........................

6. 

$x =$ ..........................

7. 

$x =$ ..........................

8. 

$x =$ ..........................

### Study Tip

For special right triangles:

- **30° - 60° - 90°**
  Sides are in the ratio $1 : \sqrt{3} : 2$
- **45° - 45° - 90°**
  Sides are in the ratio $1 : 1 : \sqrt{2}$

### Common Mistake

Do not mix up the ratios! Check the angle you are using and choose the correct ratio.

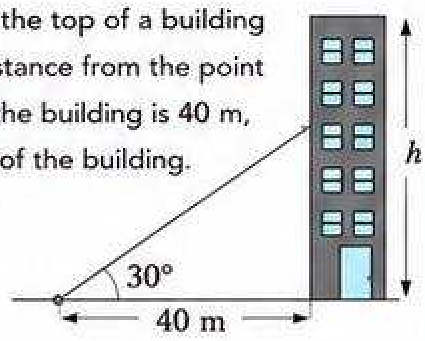
### Remember

- $\sin\theta$ = opposite / hypotenuse
- $\cos\theta$ = adjacent / hypotenuse
- $\tan\theta$ = opposite / adjacent

Only use these ratios in right triangles.

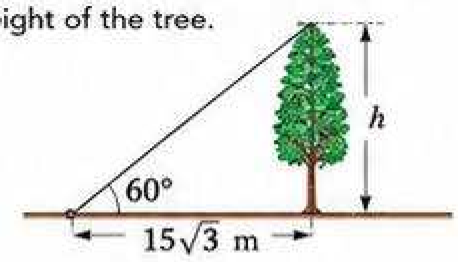
## Real-Life Problems

**1** From a point on the ground, the angle of elevation of the top of a building is 30°. If the distance from the point to the base of the building is 40 m, find the height of the building.



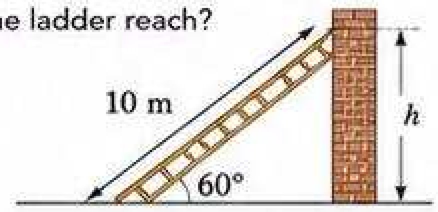

Answer: $h =$ ...................... m

**2** A tree casts a shadow of $15\sqrt{3}$ meters on the ground when the angle of elevation of the sun is 60°. Find the height of the tree.




Answer: $h =$ ...................... m

**3** A ladder 10 meters long is leaning against a wall. If the angle between the ladder and the ground is 60°, how high up the wall does the top of the ladder reach?




Answer: $h =$ ...................... m

## Think & Solve

**1** In a right triangle, $\sin\theta = \frac{1}{2}$ and $\theta$ is an acute angle. What are the two possible values of $\theta$?

Answer: $\theta =$ ........ , ........

**2** In a right triangle, $\cos\theta = \frac{\sqrt{3}}{2}$ and $\theta$ is an acute angle. What is the value of $\tan\theta$?

Answer: $\tan\theta =$ ..............

**3** If sin A = cos B and A is an acute angle, what is the relationship between A and B?

Answer: ..........................

**4** If $\tan\theta = 1$, where $\theta$ is an acute angle, what is the measure of $\theta$?

Answer: $\theta =$ ..................



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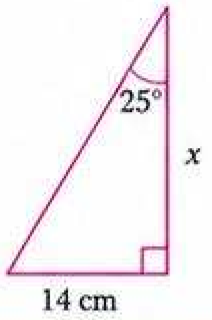
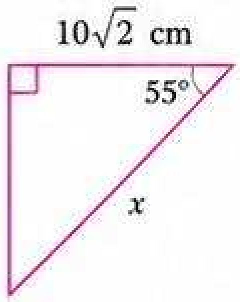
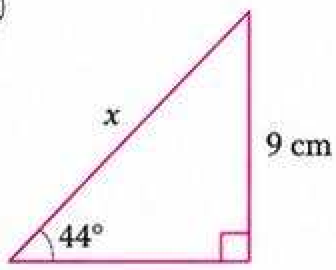
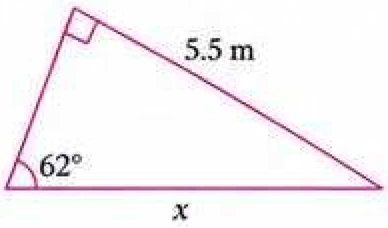
**1** In each of the following figures, find the value of $x$. (Round each answer to two decimal places)

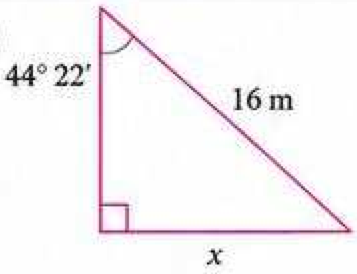
① 

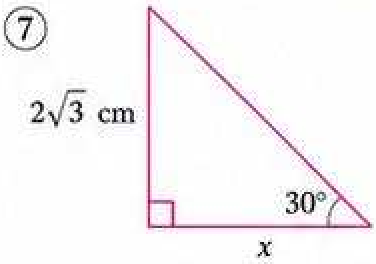

② 

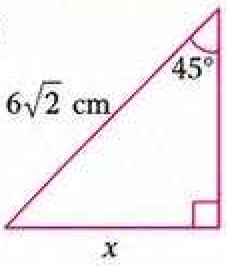

③


④ 

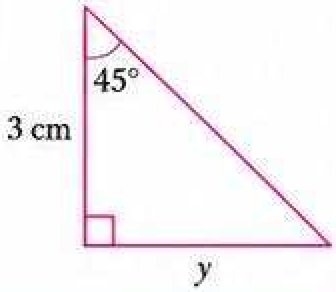

⑤ 

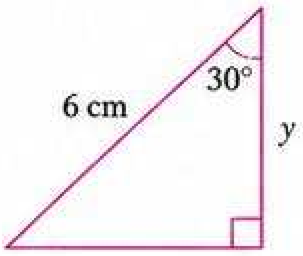

⑥ 

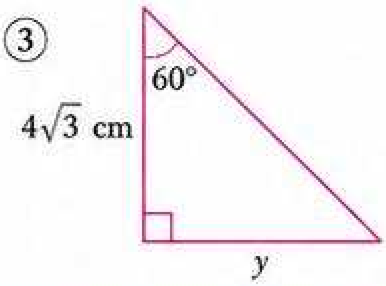

⑦ 

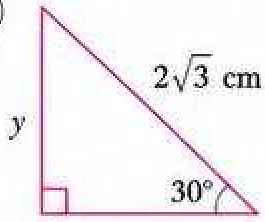

⑧ 


**2** In each of the following figures, find the value of $y$.

① 


② 


③ 


④ 


**3** Find the value of $x$ in each of the following:

① $x \tan 45° \cos 30° = \tan 60°$ ② $x \sin 45° \cos 45° \tan 60° = (\tan 45°)^2 - (\cos 60°)^2$

**4** Find the value of $x$ in each of the following:

① $x \tan 45° \cos 30° = \tan 60°$ ② $x \sin 45° \cos 45° \tan 60° = (\tan 45°)^2 - (\cos 60°)^2$

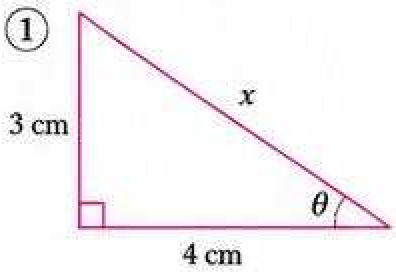
**5** Find the value of $\theta$ in each of the following:

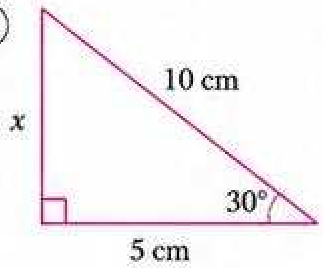
① $\sin 30° = \frac{1}{2}$ ② $\tan 39° = \sqrt{3}$ ③ $\cos 48° = \frac{1}{2}$ ④ $\tan 48° = \frac{\sqrt{3}}{2}$

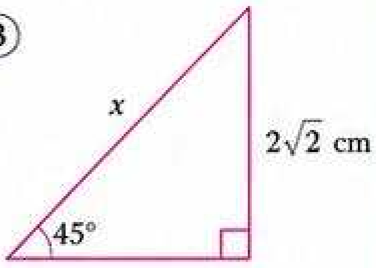
**6** Find the value of $\theta$ in each of the following:

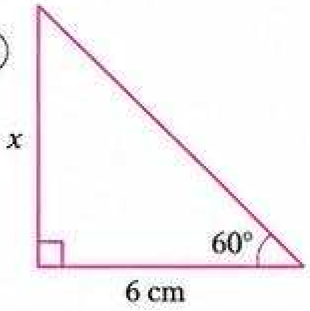
① $\sin 60° = \frac{\sqrt{3}}{2}$ ② $\cos 45° = \frac{\sqrt{2}}{2}$ ③ $\tan \theta = 1$ ④ $\sin 45° = \frac{\sqrt{2}}{2}$

**7** Find the value of $x$ in each of the following:

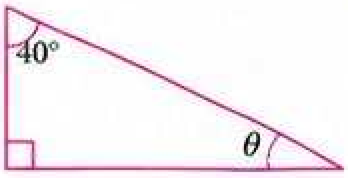
① 

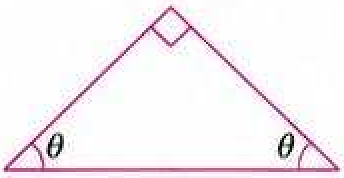

② 

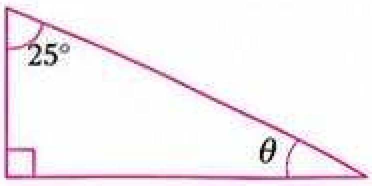

③ 


④ 


**8** In each of the following figures, find the measure of the unknown angle.

① 


② 


③ 


**9** In $\triangle ABC$, $\sin A = \cos 2A$ and $m(\angle B) = 72°$. Find $m(\angle C)$.

**10** In $\triangle ABC$, $\cos B = \sin 2B$ and $A$ is an acute angle. Find the measure of $\angle A$.



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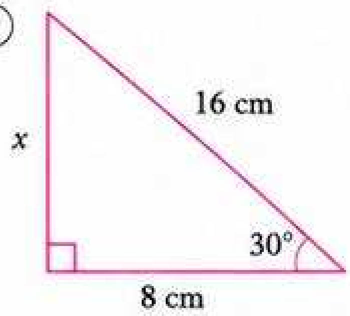
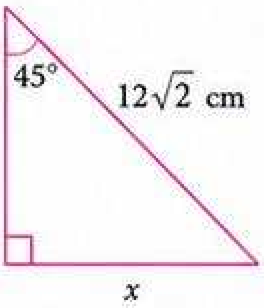
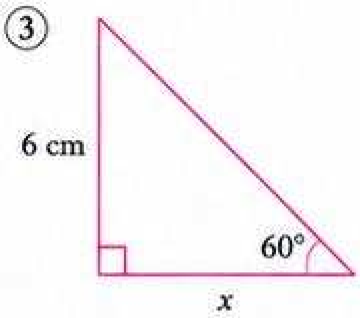
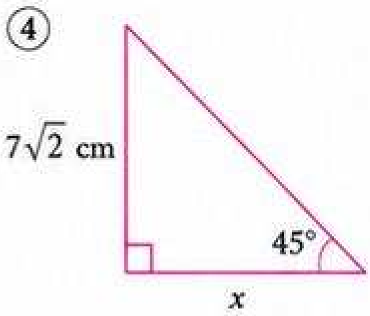
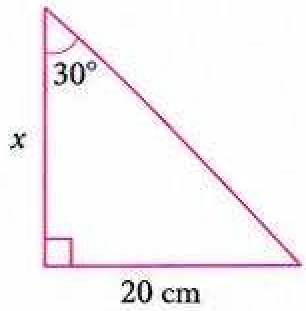
## 1 In each of the following right triangles, find the value of $x$.

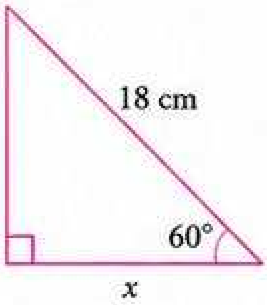
① 

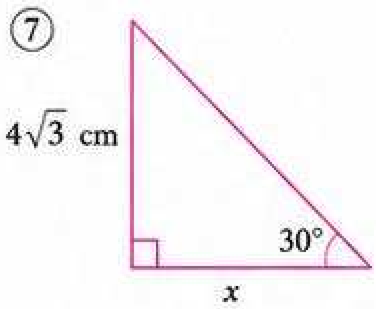

② 

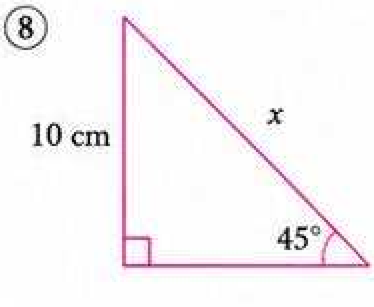

③ 


④ 

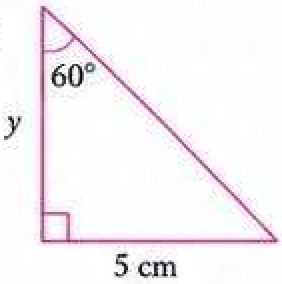

⑤ 

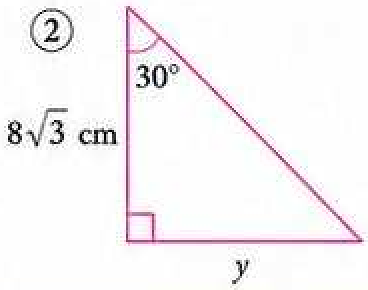

⑥ 

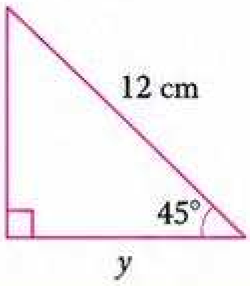

⑦ 

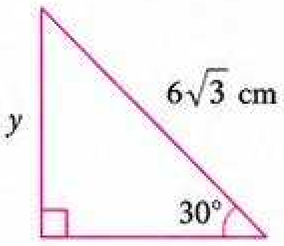

⑧ 


## 2 In each of the following triangles, find the value of $y$.

① 


② 


③ 


④ 


## 3 Evaluate the following expressions.

① $\sin 60° + \cos 45°$ | ② $\cos 30° + \sin 30° - \tan 45°$ | ③ $\tan 45° \times \cos 60°$ | ④ $\sin 30° + \cos 60° + \tan 45°$

## 4 Simplify the following expressions.

① $\tan 45° \cos 60°$ | ② $\sin 45° \cos 45°$ | ③ $\tan 60° - \sin 30°$ | ④ $\cos 45° + \sin 30°$

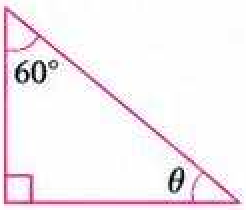
## 5 Find the value of $\theta$ in each of the following.

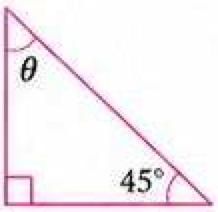
① $\sin \theta = \frac{\sqrt{3}}{2}$ | ② $\cos \theta = \frac{1}{2}$ | ③ $\tan \theta = \sqrt{3}$ | ④ $\tan \theta = 1$

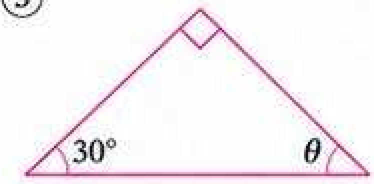
## 6 Find the value of $\theta$ in each of the following.

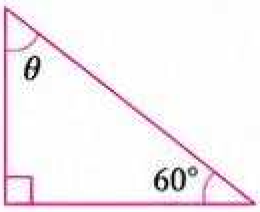
① $\sin \theta = \frac{1}{2}$ | ② $\cos \theta = \frac{\sqrt{2}}{2}$ | ③ $\tan \theta = \frac{\sqrt{3}}{3}$ | ④ $\sin \theta = \frac{\sqrt{3}}{2}$

## 7 Find the measure of the unknown angle in each of the following figures.

① 


② 


③ 


④ 


**8** A ladder 8 meters long is leaning against a wall. The angle between the ladder and the ground is 60°. Find the distance of the foot of the ladder from the wall.

**2** The angle of elevation of the top of a tower from a point on the ground is 30°. If the distance from the point to the base of the tower is 30 meters, find the height of the tower.

**3** A tree casts a shadow of 10 meters when the angle of elevation of the sun is 45°. Find the height of the tree.

**9** In $\triangle ABC$, $\sin A = \cos 2A$ and $m(\angle B) = 72°$. Find $m(\angle C)$.

**10** In $\triangle ABC$, $\cos B = \sin 2B$ and $A$ is an acute angle. Find the measure of $\angle A$.



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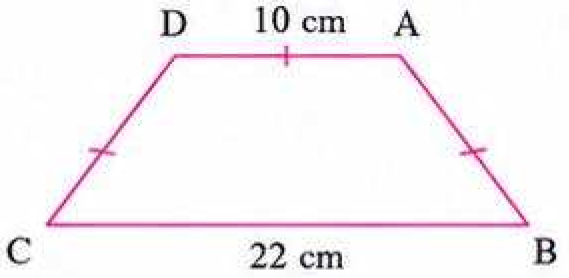
**13** In the figure opposite:

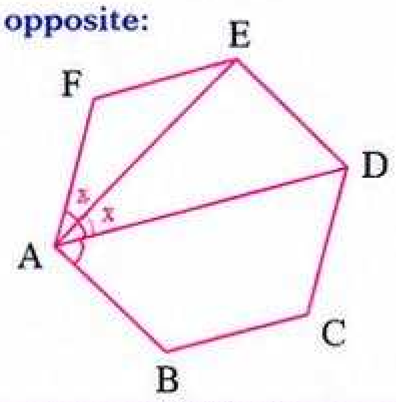



ABCD is an isosceles trapezoid where: BC = 22 cm , 10 = DC = AD = AB

Find $m(\angle C)$.

Then, find the area of trapezoid ABCD.

**14** In the figure opposite:



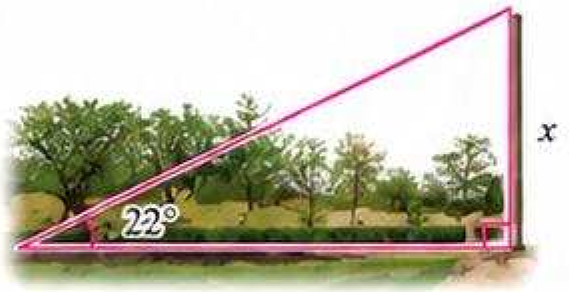

If ABCDEF is a regular hexagon,

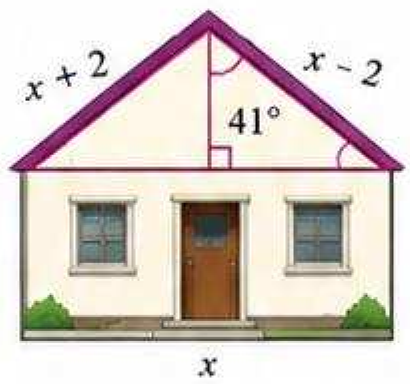
find in terms of $x$ , $y$ , $z$ :

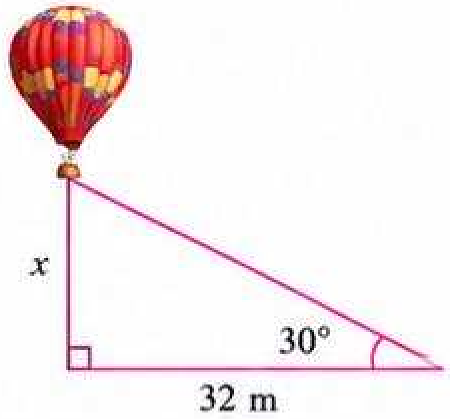
$2 \sin x + 4 \sin y - 6 \cos z$

where the angles are as shown in the figure.

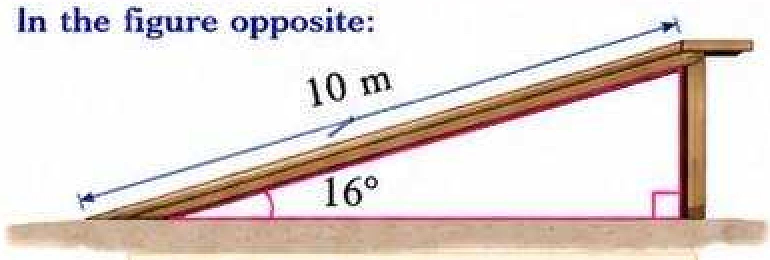
**15** Find the value of $x$ in each of the following, where $x$ is a positive number.

① 


② 


③ 


**16** In the figure opposite:




The figure shows a ramp used to load equipment onto a truck.

The ramp makes an angle of 16° with the ground.

If the length of the ramp is 10 meters,

find the height of the truck.

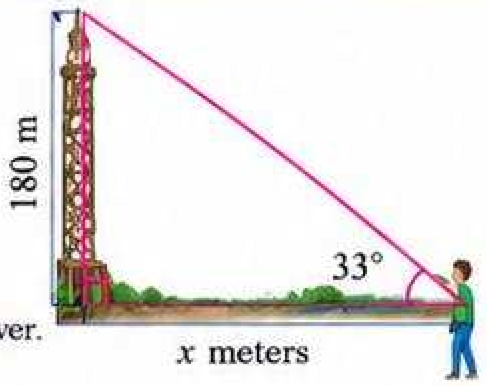
**17** In the figure opposite:

If the height of the tower is 180 meters,

and a person on the ground observes the

top of the tower at an angle of elevation of 33°,

find the distance between the person and the base of the tower.

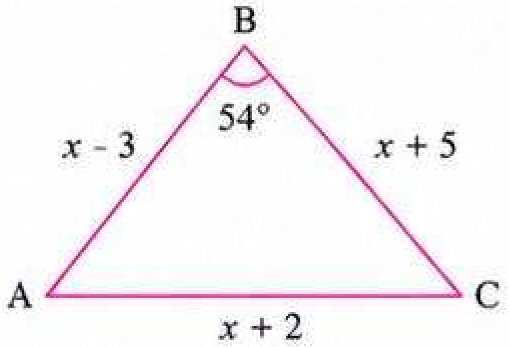



The figure shows the Cairo Tower in Egypt, one of the tallest towers in Africa and the Arab world.

**18** In the figure opposite:

If $\sin 54° \approx 0.8$ ,

find the perimeter of triangle ABC.




**How did I do?**

How well can you use trigonometric ratios?

☐ ☐ ☐ ☐



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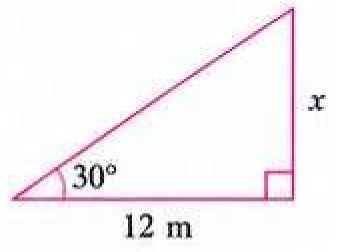
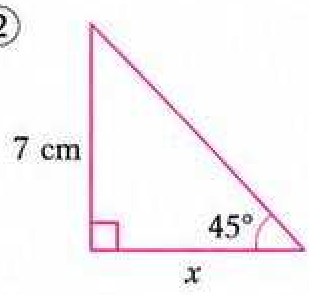
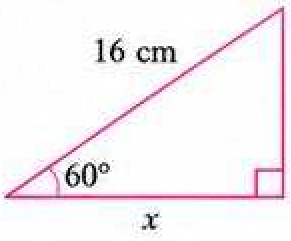
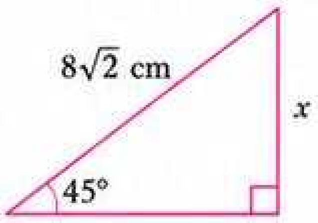
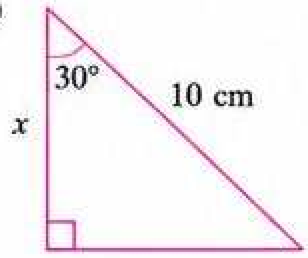
**1** Find the value of $x$ in each of the following figures. (Round each answer to two decimal places)

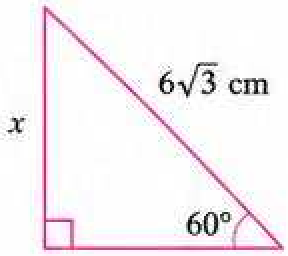
① 

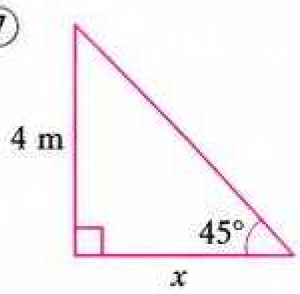

② 

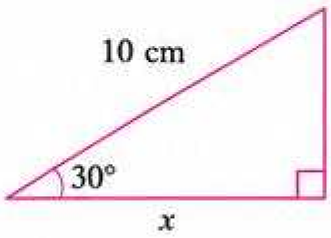

③ 


④ 

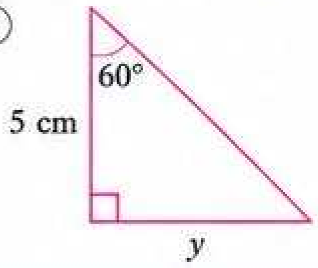

⑤ 

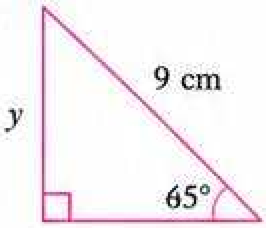

⑥ 

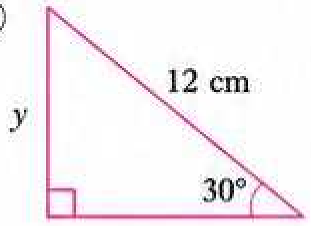

⑦ 

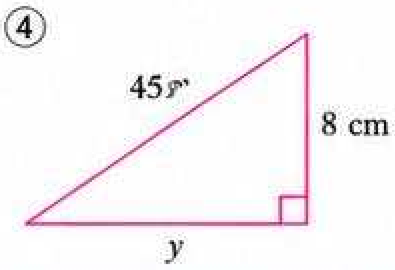

⑧ 


**2** Find the value of $y$ in each of the following figures.

① 


② 


③ 


④ 


**3** Evaluate the following expressions.

① $\sin 60° - \cos 30°$ ② $\cos 45° + \sin 30°$ ③ $\tan 45° \times \cos 60°$ ④ $\sin 30° + \tan 60° - \cos 45°$

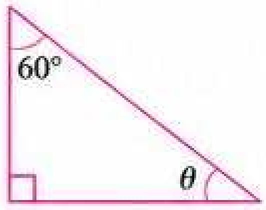
**4** Simplify the following expressions.

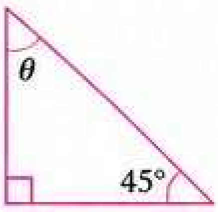
① $\tan 45° \times \cos 60°$ ② $\sin 60° \div \cos 30°$ ③ $(\sin 30° + \cos 60°)^2$ ④ $\tan 60° - \sin 45°$

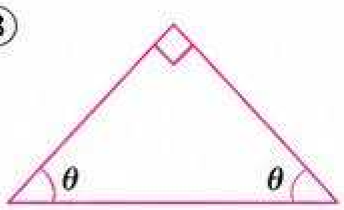
**5** Find the value of $\theta$ in each of the following.

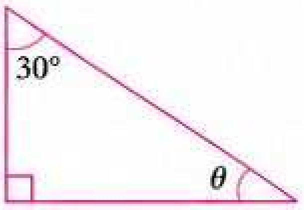
① $\sin \theta = \frac{\sqrt{2}}{2}$ ② $\cos \theta = \frac{1}{2}$ ③ $\tan \theta = \sqrt{3}$ ④ $\cos \theta = \frac{\sqrt{3}}{2}$

**6** Find the angle measure $\theta$ in each of the following figures.

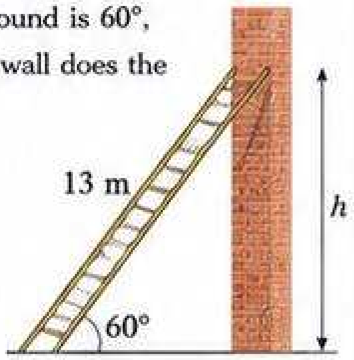
① 


② 

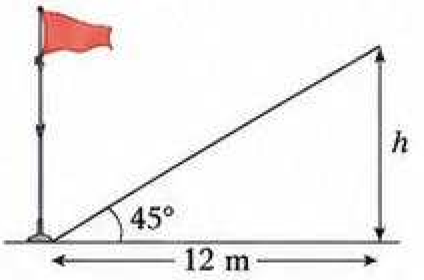

③ 


④ 

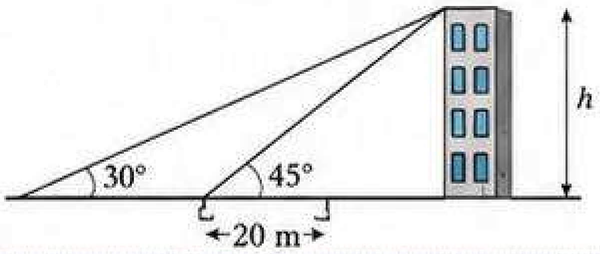

**7** A ladder 13 meters long is leaning against a wall. If the angle between the ladder and the ground is 60°, find how high up the wall does the ladder reach.




**2** A flagpole casts a shadow of 12 meters on the ground when the angle of elevation of the sun is 45°. Find the height of the flagpole.




**3** From a point on the ground, the angle of elevation of the top of a building is 30°. If you move 20 meters closer to the building, the angle of elevation becomes 45°. Find the height of the building.




**8** In △ABC, $\sin A = \cos 2A$ and $m(\angle B) = 72°$. Find $m(\angle C)$.

**9** In △ABC, $\cos B = \sin 2B$ and $A$ is an acute angle. Find the measure of $\angle A$.



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## 1 Evaluate the following expressions.

① $2\cos(\theta + 20°) = \tan 45°$. Find $\theta$ if $\theta$ is acute.

② $2\cos(\theta + 20°) = 1$. Find $\theta$ if $\theta$ is acute.

③ $2\cos(\theta + 20°) = \sqrt{3}$. Find $\theta$ if $\theta$ is acute.

④ $\tan(\theta + 15°) = 1$. Find $\theta$ if $\theta$ is acute.

⑤ $\tan(\theta + 30°) = \sqrt{3}$. Find $\theta$ if $\theta$ is acute.

⑥ $\sin(30° + \theta) = \frac{1}{2}$. Find $\theta$ if $\theta$ is acute.

⑦ $\cos(15° + \theta) = \frac{\sqrt{2}}{2}$. Find $\theta$ if $\theta$ is acute.

## 2 Evaluate the following without using a calculator.

① $\sin 30° + \cos 30°$

② $\cos 30° - \sin 60°$

③ $\tan 45° + \tan 60°$

④ $\cot 60° - \tan 30°$

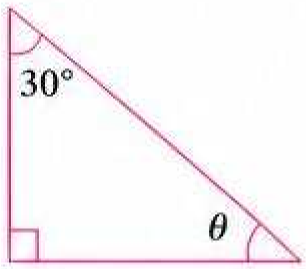
⑤ $\sin 60° \cos 30°$

⑥ $\sin^2 30° + \cos^2 60°$

⑦ $\frac{\sin 60°}{\cos 30°}$

⑧ $\frac{\cos 45°}{\sin 45°}$

## 3 Find the measure of $\theta$ in each of the following figures.

①
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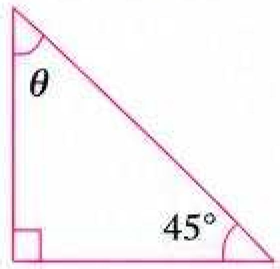


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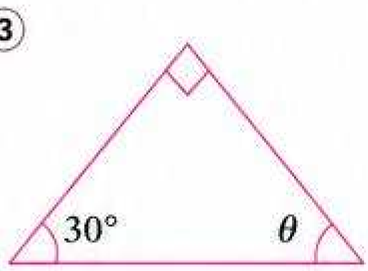


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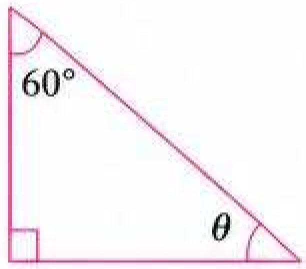
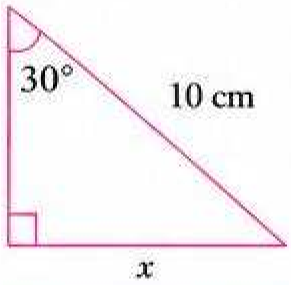



## 4 Solve for $x$ in each of the following right triangles.

①
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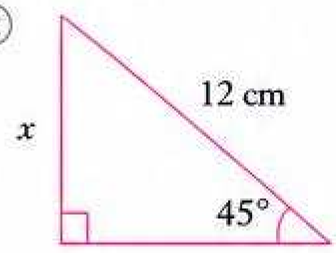


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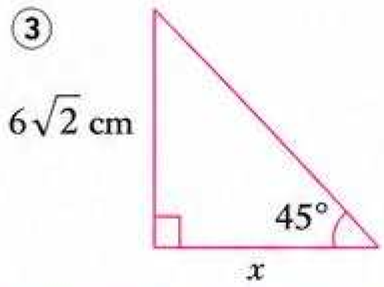


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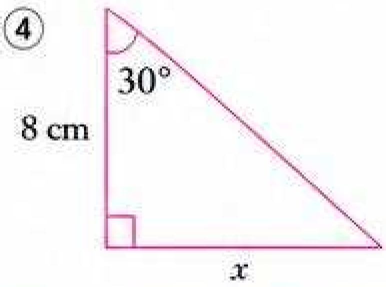
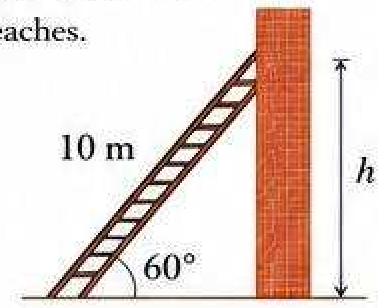
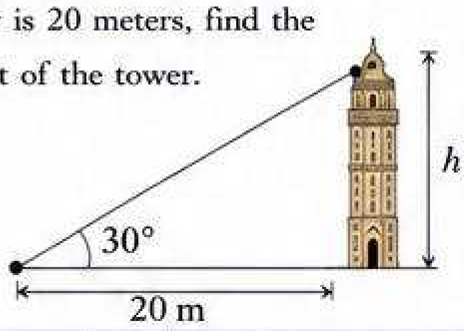



## 5 Find the length of the unknown side in each of the following.

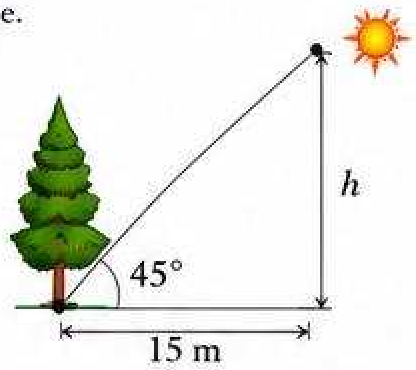
① A ladder 10 meters long is leaning against a wall. The angle between the ladder and the ground is 60°. Find how high up the wall the ladder reaches.




② From a point on the ground, the angle of elevation of the top of a tower is 30°. If the distance from the point to the base of the tower is 20 meters, find the height of the tower.




③ A tree casts a shadow of 15 meters when the angle of elevation of the sun is 45°. Find the height of the tree.




## 6 In $\triangle ABC$, find $m(\angle C)$ in each of the following.

① $\sin A = \cos 2A$, $m(\angle B) = 72°$

② $\cos B = \sin 2B$, $A$ is an acute angle.

③ $\tan A = \tan 2A$, $m(\angle B) = 45°$

## 7 Find $\theta$ if $\theta$ is an acute angle in each of the following.

① $2\cos(\theta + 20°) = \tan 45°$

② $2\cos(\theta + 20°) = 1$

③ $2\cos(\theta + 20°) = \sqrt{3}$

④ $\tan(\theta + 15°) = 1$

⑤ $\tan(\theta + 30°) = \sqrt{3}$

⑥ $\sin(30° + \theta) = \frac{1}{2}$

⑦ $\cos(15° + \theta) = \frac{\sqrt{2}}{2}$

⑧ $2\cos(\theta + 20°) = 0$



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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## Get Started!

On the 6th of October every year, Egypt celebrates the October victories that restored our pride and dignity.
On that day, our heroes wrote a great epic with determination and will.
Let's learn today a new mathematical concept that has many useful applications in our real life, such as in the design of roads, buildings, and even in the flight paths of planes.

## Think & Discuss!

If we draw two straight lines $l_1$ and $l_2$ in the coordinate plane passing through the points (5, −1), (7, 5) and (2, 3), (−4, 4) respectively, what is the relationship between their slopes?

$m_1 = m_2$ $m_1 = -m_2$ $m_1 \times m_2 = 1$ $m_1 \div m_2 = -1$

## Learning Outcomes

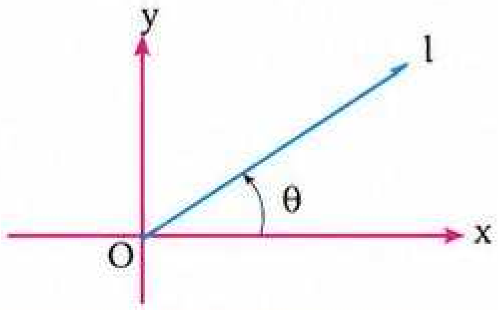
**By the end of this lesson, you will be able to:**

- Find the slope of a straight line.
- Find the angle that a straight line makes with the positive direction of the x−axis.
- State the relation between the slopes of two parallel straight lines.
- State the relation between the slopes of two perpendicular straight lines.
- Solve problems using the slopes of straight lines.

## Learn!

### Angle of Inclination of a Straight Line

The angle of inclination of a straight line is the angle that the line makes with the positive direction of the $x$ = axis measured counterclockwise.




Angle of inclination = θ

### Slope of a Straight Line

The slope ($m$) of a straight line is the tangent of its angle of inclination.

$$m = \tan\theta$$

**Example:** A line makes an angle of 45° with the positive direction of the $x$-axis. Find its slope.

**Solution:**

$m = \tan 45° = 1$

## Key Vocabulary

- Slope of straight line
- Angle of inclination
- Two parallel straight lines
- Two perpendicular straight lines

## Remember!

Slope of a line ($m$) = tan (θ)
where θ is the angle of inclination.

## Note!

- $90° < \theta < 180°$
  $\Rightarrow \tan\theta < 0$ (slope is negative)
- $0° < \theta < 90°$
  $\Rightarrow \tan\theta > 0$ (slope is positive)
- $\theta = 0°$ or $180° \Rightarrow \tan\theta = 0$
- $\theta = 90° \Rightarrow \tan\theta$ is not defined.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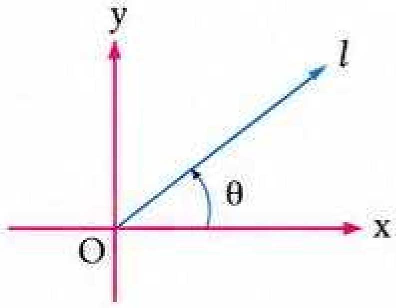
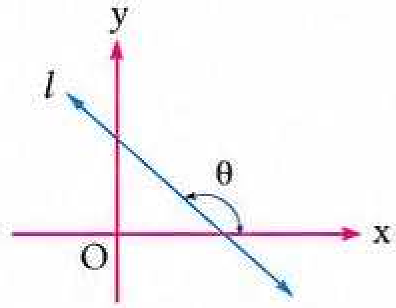
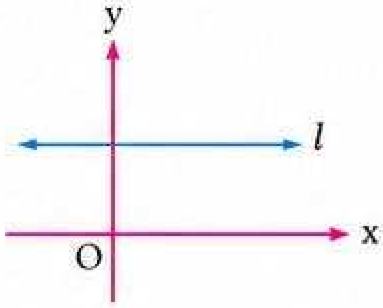
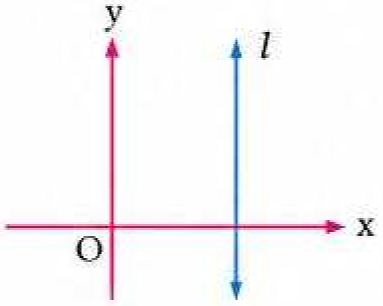
# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## Note:

The angle of inclination and the slope of a straight line:

| Positive Slope | Negative Slope | Zero Slope | Undefined Slope |
|---|---|---|---|
|  |  |  |  |
| $0° < \theta < 90°$ | $90° < \theta < 180°$ | $\theta = 0°$ or $180°$ | $\theta = 90°$ |
| $\tan\theta > 0$ | $\tan\theta < 0$ | $\tan\theta = 0$ | $\tan\theta$ is not defined |



**Examples:**

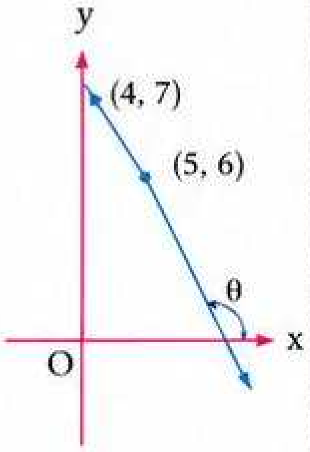
- $(\tan 60° = \sqrt{3})$
  Slope of the line that makes an angle of 60° is $\sqrt{3}$
- $\left(\tan 150° = -\frac{\sqrt{3}}{3}\right)$
  Slope of the line that makes an angle of 150° is $-\frac{\sqrt{3}}{3}$

## Example 1

Find the slope and the angle of inclination of each line if:

1. The line passes through the points $(4, 7)$ and $(5, 6)$

**Solution:**



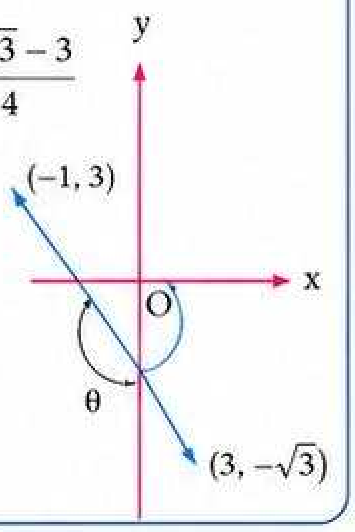

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-7}{5-4} = -1$$

$$m = -1 \Rightarrow \theta = \tan^{-1}(-1) = \boxed{135°}$$

The angle of inclination is 135°.

2. The line passes through the points $(-1, 3)$ and $(3, -\sqrt{3})$

**Solution:**




$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\sqrt{3}-3}{3-(-1)} = \frac{-\sqrt{3}-3}{4}$$

$$m = -\frac{3+\sqrt{3}}{4}$$

$$m < 0 \Rightarrow \theta = 180° + \tan^{-1}(m)$$

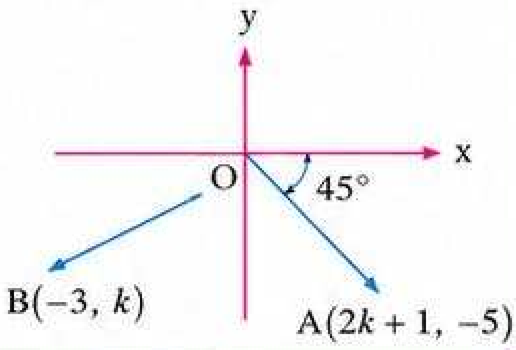
$$\tan^{-1}\left(\frac{3+\sqrt{3}}{4}\right) \approx 57.9°$$

$$\theta \approx 180° + 57.9° = 237.9°$$

## Example 2

If the line passes through the points $B(-3, k)$ and $A(2k+1, -5)$ and its angle of inclination is 45°, find the value of $k$.

**Solution:**




Since $\theta = 45° \Rightarrow \tan 45° = 1 \Rightarrow m = 1$

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$1 = \frac{-5-k}{(2k+1)-(-3)} = \frac{-5-k}{2k+4}$$

$$2k + 4 = -5 - k$$

$$2k + k = -5 - 4$$

$$3k = -9$$

$$\boxed{k = -3}$$

## Check Your Understanding

1. If the line passes through the points $(1, 5)$ and $(4, -1)$, find its slope and the measure of its angle of inclination.
2. If the line passes through the points $(k, 2)$ and $(1, k)$ and its slope equals $-2$, find the value of $k$.



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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## The Relation between the Slopes of Two Parallel Lines

If $l_1$ and $l_2$ are two distinct parallel lines with slopes $m_1$ and $m_2$ respectively,

$$m_1 = m_2 \qquad (l_1 \parallel l_2)$$

That is, the slopes of two parallel lines are equal.

**The Converse:**

If $l_1$ and $l_2$ are two distinct lines and $m_1 = m_2$,
then $l_1 \parallel l_2$.

That is, if the slopes of two distinct lines are equal, then the two lines are parallel.

### Example 3

Find the measure of the angle that the line passing through $A(2, 1)$ and $B(4, -1)$ makes with the positive direction of the $x$-axis.

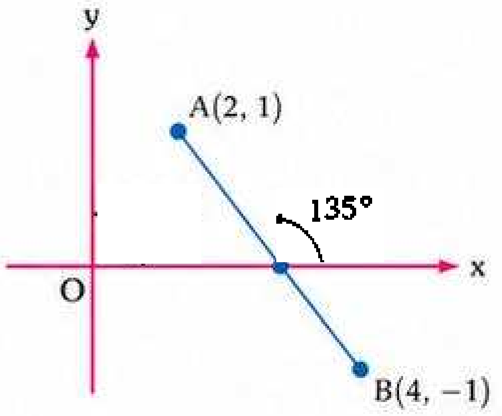
**Solution:**

Slope of $\overleftrightarrow{AB}$, $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-1}{4-2}$

$= \dfrac{-2}{2} = -1$

$m_1 = -1 = \tan 135°$

$\therefore$ The required angle $= 135°$.




### Example 4

Show that the points $A(4, 3)$, $B(2, 7)$, $C(8, -5)$ are collinear.

**Solution:**

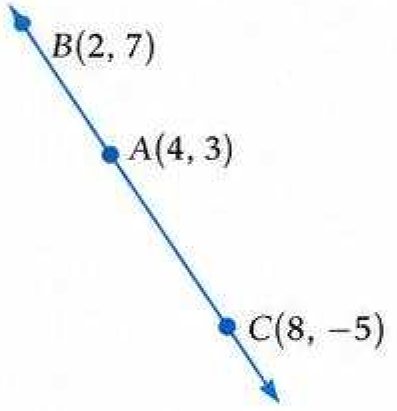
Slope of $\overleftrightarrow{AB}$, $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-3}{2-4} = \dfrac{4}{-2} = -2$

Slope of $\overleftrightarrow{AC}$, $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-5-3}{8-4} = \dfrac{-8}{4} = -2$

$m_1 = m_2 = -2$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{AC}$

And the points $A$, $B$, $C$ lie on the same straight line.




## The Relation between the Slopes of Two Perpendicular Lines

If $l_1$ and $l_2$ are two distinct perpendicular lines with slopes $m_1$ and $m_2$ respectively,

$$m_1 \times m_2 = -1 \qquad (l_1 \perp l_2)$$

That is, the product of the slopes of two perpendicular lines is $-1$.

**The Converse:**

If $l_1$ and $l_2$ are two distinct lines and $m_1 \times m_2 = -1$,
then $l_1 \perp l_2$.

That is, if the product of the slopes of two distinct lines is $-1$, then the two lines are perpendicular.

### Important Note

- Two distinct lines may be:
  - Parallel
  - Perpendicular

### Example 3

If the line passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$, then:

Slope of $\overleftrightarrow{AB}$, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

and the angle $\theta$ it makes with the positive $x$-axis is:

$\theta = \tan^{-1}(m)$
$(0° \le \theta < 180°)$

### Example 4

If the points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are collinear, then:

Slope of $\overleftrightarrow{AB}$ = Slope of $\overleftrightarrow{AC}$

That is, $m_1 = m_2$.

### Example 5

If the line passes through the points $(x_1, y_1)$ and $(x_2, y_2)$, and the line passing through the points $(x_3, y_3)$ and $(x_4, y_4)$ is perpendicular to it, then the angle between them is 90°.

### Think about it!

1. Can two distinct lines have the same slope and be perpendicular at the same time?
2. What is the slope of a line perpendicular to the line whose slope is 2?



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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## Example 5

Show that the line passing through the points $(5, 2)$ and $(-1, 6)$ is perpendicular to the line passing through the points $(-2, -4)$ and $(6, 8)$.

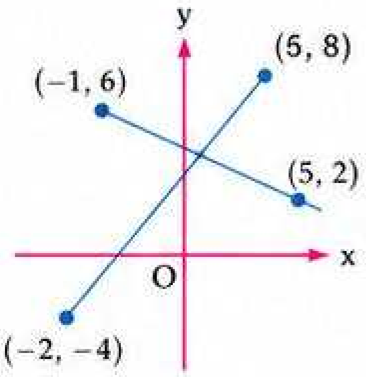
**Solution:**

Slope of the first line, $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{-1-5} = \frac{4}{-6} = -\frac{2}{3}$

Slope of the second line, $m_2 = \frac{8-(-4)}{6-(-2)} = \frac{12}{8} = \frac{3}{2}$

Then $m_1 \times m_2 = \left(-\frac{2}{3}\right) \times \left(\frac{3}{2}\right) = -1$

$\therefore$ The two lines are perpendicular.




## Common Mistakes

- Forgetting the order in substituting the points in the slope formula.
- Thinking that lines with equal slopes are perpendicular.
- Not multiplying the slopes when checking perpendicular lines.
- Confusing parallel lines with perpendicular lines.

## Example 6

Find the value of $k$ so that the line passing through $A(k, 2)$ and $B(4, 8)$ is perpendicular to the line passing through $C(-12, k)$ and $D(4, 8)$.

**Solution:**

Slope of $\overline{AB}$, $m_1 = \frac{8-2}{4-k} = \frac{6}{4-k}$

Slope of $\overline{CD}$, $m_2 = \frac{8-k}{4-(-12)} = \frac{8-k}{16}$

Since the two lines are perpendicular,

$m_1 \times m_2 = -1$

$\frac{6}{4-k} \times \frac{8-k}{16} = -1 \Rightarrow 6(8-k) = -16(4-k) \Rightarrow 48 - 6k = -64 + 16k$

$112 = 22k \Rightarrow k = \frac{56}{11}$

## Key Summary

- Parallel lines have equal slopes.

$$m_1 = m_2$$

- Perpendicular lines have slopes whose product is $-1$.

$$m_1 \times m_2 = -1$$

- The angle between perpendicular lines is $90°$.

## Practice Yourself

**1 Direct Question**

Find the slope of the line that makes an angle of $150°$ with the positive direction of the $x$ – axis.

..............................

**2 Understanding**

If the slope of a line is 0, what is the angle it makes with the positive direction of the $x$ – axis?

..............................

**3 Application**

Find the slope of the line passing through the points $(1, -2)$ and $(4, 7)$.

..............................

**4 Real-life Problem**

A ramp rises 3 meters for every 4 meters it runs horizontally. Find the slope of the ramp.

..............................

**5 Higher Thinking**

Two lines have slopes $m$ and $-\frac{1}{m}$. What is the angle between them?

..............................

## Test Yourself

**1 Choose the correct answer:**

If $m_1 = \frac{3}{2}$ and $m_2 = -\frac{2}{3}$, then the two lines are .....

ⓐ Parallel
ⓑ Perpendicular
ⓒ Intersecting but not perpendicular
ⓓ Cannot be determined

**2 Complete:**

The slope of a line that makes an angle of 45° with the positive direction of the $x$ – axis is ......

**3 True or False:**

If two lines are parallel, their slopes are perpendicular.

..........................

**4 Solve:**

Find the slope of the line passing through the points $(3, -1)$ and $(7, 5)$.

..........................

**5 Challenge:**

Find the slope of a line perpendicular to the line whose slope is $-\frac{4}{5}$.

..........................



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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## Key Concepts Review

- The slope of a line that makes an angle $\theta$ with the positive direction of the $x$−axis is $m = \tan\theta$.
- Two lines $l_1$ and $l_2$ are parallel if and only if $m_1 = m_2$.
- Two lines $l_1$ and $l_2$ are perpendicular if and only if $m_1 \times m_2 = -1$.
- The angle between two perpendicular lines is $90°$.

## Remember!

- If $m = 0$ $\theta = 0°$ or $180°$
- If $\theta = 45°$ $m = 1$
- If $\theta = 135°$ $m = -1$
- If $\theta = 60°$ $m = \sqrt{3}$
- If $\theta = 150°$ $m = -\frac{\sqrt{3}}{3}$

## Practice More

**A) Find the slope of each line:**

1. The line passing through $(2, 5)$ and $(6, 9)$.
   ..........................
2. The line passing through $(-4, 3)$ and $(0, -1)$.
   ..........................
3. The line passing through $(1, -2)$ and $(5, -2)$.
   ..........................
4. The line passing through $(3, 7)$ and $(3, -5)$.
   ..........................
5. The line passing through $(0, 4)$ and $(-2, -8)$.
   ..........................

**B) Find the value of $k$:**

1. If the line passing through $(k, 2)$ and $(4, 6)$ has slope 2.
   ..........................
2. If the line passing through $(-1, k)$ and $(3, 2)$ has slope $-\frac{1}{2}$.
   ..........................
3. If the line passing through $(2, 3)$ and $(k, 7)$ has slope 1.
   ..........................
4. If the line passing through $(-2, k)$ and $(2, -6)$ has slope $-3$.
   ..........................
5. If the line passing through $(k, 1)$ and $(5, 1)$ is horizontal.
   ..........................

## Attention!

- When two lines are parallel, they never meet.
- When two lines are perpendicular, they meet at right angles.
- Lines can be neither parallel nor perpendicular; they may intersect at any other angle.

## Math in Life

City planners use the idea of parallel and perpendicular lines in designing roads and buildings to make sure streets are straight and meet at right angles.

## Challenge Yourself

1. The line $l_1$ passes through $(1, 2)$ and $(3, 6)$, and the line $l_2$ passes through $(2, k)$ and $(6, 10)$. Find the value of $k$ so that $l_1 \parallel l_2$.
   ..........................
2. The line $l_1$ passes through $(m, 1)$ and $(2, 3)$, and the line $l_2$ passes through $(1, -2)$ and $(4, n)$. If $l_1 \perp l_2$, find the values of $m$ and $n$.
   ..........................
3. Find the slope of the line that is perpendicular to the line passing through $(-1, 4)$ and $(5, -2)$.
   ..........................
4. The line $l_1$ passes through $(2, -3)$ and $(x, 5)$, and the line $l_2$ passes through $(-4, y)$ and $(2, 1)$. If $l_1 \perp l_2$, $x \neq 2$ and $y \neq 2$, find $x$ and $y$.
   ..........................
5. The line passing through $(a, 1)$ and $(5, 7)$ is perpendicular to the line passing through $(2, b)$ and $(6, 2)$. Find $a$ and $b$.
   ..........................
6. Two lines have slopes $m$ and $-3m$. If the angle between them is $90°$, find the value of $m$.
   ..........................

## Self Check

- I can find the slope of a straight line. ☆☆☆
- I can state the relation between the slopes of parallel lines. ☆☆☆
- I can state the relation between the slopes of perpendicular lines. ☆☆☆
- I can solve problems using these relations. ☆☆☆

## Homework

1. Find the slope of each line:
   ⓐ Through $(2, 1)$ and $(5, 7)$
   ⓑ Through $(-3, 4)$ and $(1, -2)$
   ⓒ Through $(0, 6)$ and $(4, 2)$
   ⓓ Through $(-2, -1)$ and $(-2, 5)$
2. Find the value of $k$:
   ⓐ Through $(k, 1)$ and $(3, 5)$ has slope 2
   ⓑ Through $(-4, k)$ and $(2, -2)$ has slope $-1$
   ⓒ Through $(1, k)$ and $(5, -3)$ is horizontal
   ⓓ Through $(2, 3)$ and $(k, 9)$ has slope $\frac{3}{2}$
3. Determine whether the lines are parallel, perpendicular, or neither:
   ⓐ $(1, 2)$ and $(4, 5)$ with $(2, -1)$ and $(5, 2)$
   ⓑ $(-2, 1)$ and $(2, 5)$ with $(0, -3)$ and $(4, 1)$
   ⓒ $(3, -2)$ and $(6, 1)$ with $(-1, 4)$ and $(2, -1)$
   ⓓ $(0, 0)$ and $(2, 2)$ with $(1, -1)$ and $(3, -3)$


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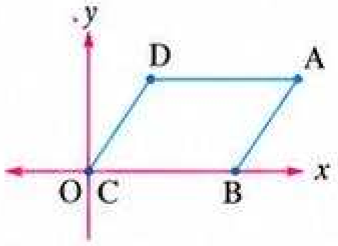
# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

① In the figure, $\overrightarrow{ABCD}$ is a parallelogram.
$A = (2, 4)$ and the coordinates of $B$ are $(4, 0)$.
Find the slope of $\overrightarrow{BC}$.




ⓐ $-\frac{1}{2}$ ⓑ $\frac{1}{2}$
ⓒ 2 ⓓ −2

② If $\overrightarrow{ABCD}$ is a parallelogram, then $\overrightarrow{BD}$ equals _______ of $\overrightarrow{AC}$.

ⓐ $\frac{3}{4}$ ⓑ $-\frac{3}{4}$ ⓒ $\frac{3}{4}$ ⓓ $-\frac{4}{3}$

③ If $\overrightarrow{ABCD}$ is a parallelogram, then $\overrightarrow{BD}$ equals $\frac{3}{4}$ of $\overrightarrow{AC}$.
Choose the correct relation:

ⓐ $\frac{4}{3}$ ⓑ $-\frac{3}{4}$ ⓒ $\frac{3}{4}$ ⓓ $-\frac{4}{3}$

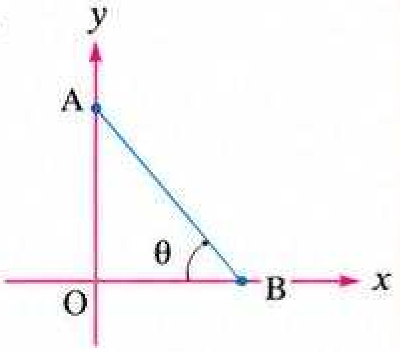
④ If $\overrightarrow{ABCD}$ is a parallelogram, the measure of $\angle \overrightarrow{BCD}$ equals:

ⓐ 60° ⓑ 75° ⓒ 30° ⓓ 45°

⑤ If the slope of the line is $\frac{k}{6} - \frac{5}{3}$, and its perpendicular line has slope 4, find $k$.

ⓐ −5 ⓑ 5 ⓒ −10 ⓓ 10

⑥ In the figure, the angle of inclination $\theta$ equals:
given that $A = (2, 4)$ and $B = (4, 0)$.




ⓐ $\tan^{-1}\left(\frac{1}{2}\right)$
ⓑ $\tan^{-1}(2)$
ⓒ $\tan^{-1}\left(\frac{3}{2}\right)$
ⓓ $\tan^{-1}(3)$

⑦ If $\overrightarrow{ABCD}$ is a parallelogram, then $\overrightarrow{AB}$ equals:
_______ of $\overrightarrow{BC}$.

ⓐ $\frac{-\sqrt{3}}{3}$ ⓑ $\sqrt{3}$ ⓒ $-\sqrt{3}$ ⓓ $\frac{\sqrt{3}}{3}$

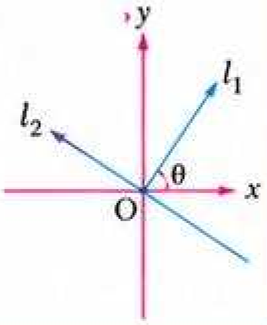
⑧ In $\overrightarrow{ABCD}$ parallelogram, if $\overrightarrow{BD}$ equals $\frac{4}{3}$ of $\overrightarrow{AC}$,
then $\overrightarrow{AB}$ equals _______ of $\overrightarrow{AC}$.

ⓐ $\frac{-\sqrt{3}}{3}$ ⓑ $\sqrt{3}$ ⓒ $-\sqrt{3}$ ⓓ $\frac{\sqrt{3}}{3}$

⑨ If the slope of line $l_1$ equals $m$, and the slope of line $l_2$ equals $\sqrt{3}$, and $l_1$ is perpendicular to $l_2$, then $m$ equals:

ⓐ tan 15° ⓑ tan 30° ⓒ tan 60° ⓓ tan 45°

⑩ In the figure, if lines $l_1$ and $l_2$ are perpendicular, and the angle of inclination of $l_1$ equals $\sqrt{3}$, then the slope of $l_2$ equals:




ⓐ $\frac{1}{\sqrt{3}}$ ⓑ $\sqrt{3}$
ⓒ $-\sqrt{3}$ ⓓ $-\frac{1}{\sqrt{3}}$

## It's Your Turn!

1. Find the slope of the line that makes an angle of 60° with the positive direction of the $x$−axis.
..............................
2. If the slopes of two lines are 2 and $-\frac{1}{2}$, are they perpendicular? Why?
..............................
3. If $\overrightarrow{AB}$ passes through (1, 2) and (4, 8), and $\overrightarrow{CD}$ passes through (2, −1) and (5, 5), are $\overrightarrow{AB}$ and $\overrightarrow{CD}$ parallel, perpendicular, or neither? Justify.
..............................
4. If two lines are perpendicular, and one of them has slope −4, find the slope of the other line.
..............................
5. Find the value of $k$ so that the line passing through $(k, 1)$ and (3, 5) is parallel to the line passing through (0, −2) and (4, 0).
..............................
6. Find the slope of the line perpendicular to the line passing through (2, 3) and (5, −1).
..............................

## Test Yourself

**1 Choose the correct answer:**
If $m_1 = 3$ and $m_2 = -\frac{1}{3}$, then the two lines are .....
ⓐ Parallel
ⓑ Perpendicular
ⓒ Intersecting but not perpendicular
ⓓ Cannot be determined

**2 Complete:**
The slope of a line that makes an angle of 135° with the positive direction of the $x$−axis is ......

**3 True or False:**
If two lines are perpendicular, the product of their slopes equals 1.
..............................

**4 Solve:**
Find the slope of the line passing through the points (2, −1) and (6, 5).
..............................

**5 Challenge:**
If two lines have slopes $m$ and $\frac{2}{m}$, what is the angle between them?
..............................



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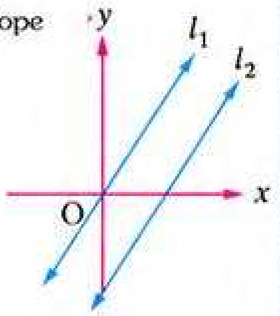
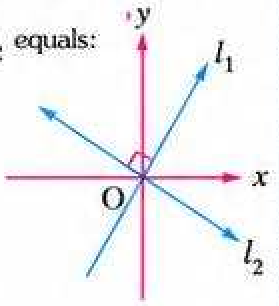
# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

**11** In the figure, $l_1$ and $l_2$ are parallel.
If the slope of $l_1$ equals $-2$, then the slope of $l_2$ equals:




ⓐ $2$ ⓑ $-2$
ⓒ $\frac{1}{2}$ ⓓ $-\frac{1}{2}$

**12** In the figure, $l_1$ is perpendicular to $l_2$. If the slope of $l_1$ equals $\frac{3}{2}$, then the slope of $l_2$ equals:




ⓐ $\frac{3}{2}$ ⓑ $-\frac{3}{2}$
ⓒ $\frac{2}{3}$ ⓓ $-\frac{2}{3}$

**13** If the line has slope $0$, then it is:
ⓐ parallel to the $x$−axis
ⓑ parallel to the $y$−axis
ⓒ perpendicular to the $x$−axis
ⓓ perpendicular to the $y$ −axis

**14** If the slope of a line is undefined, then it is:
ⓐ parallel to the $x$−axis
ⓑ parallel to the $y$−axis
ⓒ perpendicular to the $x$−axis
ⓓ perpendicular to the $y$−axis

**15** Which of the following pairs of lines are perpendicular?
ⓐ $m_1 = 3$ , $m_2 = -\frac{1}{3}$
ⓑ $m_1 = 2$ , $m_2 = 2$
ⓒ $m_1 = -4$ , $m_2 = -\frac{1}{4}$
ⓓ $m_1 = -5$ , $m_2 = -5$

**16** If two lines are perpendicular, and one of them makes an angle of $60°$ with the positive direction of the $x$−axis, then the angle made by the other line with the positive direction is:
ⓐ $30°$
ⓑ $60°$
ⓒ $90°$
ⓓ $120°$

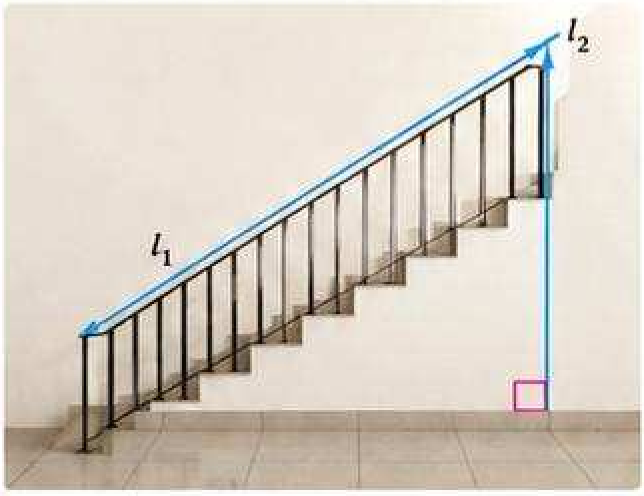
### Think More!

1. Can two distinct lines have the same slope and be perpendicular at the same time?
..............................
..............................
2. If one line is parallel to the $x$−axis and another line is perpendicular to it, what is the slope of the second line?
..............................
..............................

### Common Errors

- Thinking that slopes of perpendicular lines are negative reciprocals without using the negative sign.
- Forgetting that parallel lines have exactly equal slopes.
- Finding slopes correctly but not checking if the lines are parallel or perpendicular.

## Real-life Application




In the opposite figure, the handrail of a staircase is represented by the line $l_1$, and the wall is represented by the line $l_2$.

- If the slope of $l_1$ is $-\frac{3}{4}$, find the slope of $l_2$.
- Are the two lines parallel or perpendicular? Why?

**Solution:**
Slope of $l_1 = -\frac{3}{4}$
Since $l_1$ is perpendicular to $l_2$,
$m_1 \times m_2 = -1$
$-\frac{3}{4} \times m_2 = -1 \Rightarrow m_2 = \frac{4}{3}$
$\therefore$ The slope of $l_2$ is $\frac{4}{3}$ and the two lines are perpendicular.

### Math in Life

Engineers use the relationship between parallel and perpendicular lines when designing bridges, buildings, and roads to ensure stability and safe right angles.

## Challenge Yourself

1. The line $l_1$ passes through $(2, 3)$ and $(5, k)$, and the line $l_2$ passes through $(k, -1)$ and $(4, 7)$. If $l_1 \perp l_2$, find the value of $k$.
..............................
2. If two lines have slopes $m$ and $2m + 1$ and are perpendicular, find the value of $m$.
..............................
3. The line $l_1$ passes through $(1, 0)$ and $(4, 6)$. Find the slope of a line perpendicular to $l_1$. Then draw both lines on the same coordinate plane.
..............................
..............................
..............................

## Homework

1. Find the slope of each line:
ⓐ Through $(2, 1)$ and $(5, 9)$
ⓑ Through $(-3, -2)$ and $(1, -2)$
ⓒ Through $(0, 4)$ and $(0, -1)$
2. If the line passing through $(1, 2)$ and $(k, 5)$ is parallel to the line passing through $(0, -1)$ and $(3, 5)$, find $k$.
3. If the line passing through $(2, 1)$ and $(5, y)$ is perpendicular to the line passing through $(1, -2)$ and $(4, 1)$, find $y$.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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## Summary

- The slope of a line that makes an angle $\theta$ with the positive direction of the $x-$axis is $m = \tan\theta$.
- Two lines $l_1$ and $l_2$ are parallel if and only if $m_1 = m_2$.
- Two lines $l_1$ and $l_2$ are perpendicular if and only if $m_1 \times m_2 = -1$.
- The angle between two perpendicular lines is $90°$.
- Horizontal line: slope $= 0$.
- Vertical line: slope is undefined.
- If $m$ is the slope of a line, then:

| Angle $\theta$ | $0°$ | $45°$ | $60°$ | $90°$ | $135°$ | $150°$ |
|---|---|---|---|---|---|---|
| $m = \tan\theta$ | 0 | 1 | $\sqrt{3}$ | Undefined | $-1$ | $-\frac{\sqrt{3}}{3}$ |

## Concept Map

Relation between Slopes
→ Two Lines
- Parallel Lines → $m_1 = m_2$ → They never intersect.
- Perpendicular Lines → $m_1 \times m_2 = -1$ → They intersect at right angle ($90°$).

## Mixed Practice

**A) Choose the correct answer:**

1. If two lines are perpendicular and one of them has slope $-4$, then the slope of the other line is .......
   (a) $-\frac{1}{4}$ (b) $-4$ (c) $4$ (d) $\frac{1}{4}$
2. If two lines are parallel and one of them has slope $\frac{2}{3}$, then the slope of the other line is .......
   (a) $-\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{2}{3}$
3. The angle between two lines of slopes $2$ and $-\frac{1}{2}$ is .........
   (a) $0°$ (b) $45°$ (c) $90°$ (d) $135°$
4. The slope of a line perpendicular to the line of slope $\sqrt{3}$ is ........
   (a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $-\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{3}}{3}$

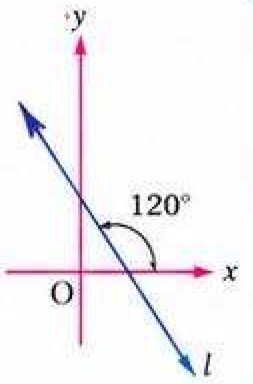
**B) Complete:**

1. If $m_1 = 5$, then the slope of a line perpendicular to it = ..........
2. If the slope of a line is $0$, then the line is ...............................
3. The angle of inclination of a line of slope $-1$ is ..............
4. If two lines are parallel, then the angle between them is ...........
5. If $m_1 = -3$, then to be perpendicular to it, the slope $m_2$ must be ............

## Higher Thinking

1. Two lines $l_1$ and $l_2$ have slopes $m$ and $2m-1$. If the angle between them is $45°$, find the possible values of $m$.

.................................................................



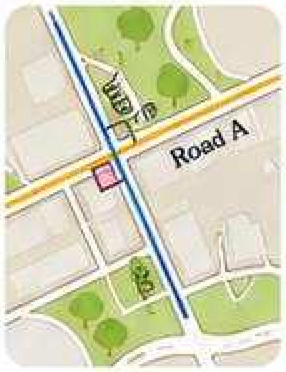

2. A line $l_1$ passes through the point $(2, 3)$ and makes an angle of $120°$ with the positive direction of the $x-$axis. Find the equation of the line.

.................................................................

## Real-world Problem

Two roads are designed as straight lines on a map.
Road A has a slope of $\frac{3}{5}$.
Road B is planned to be perpendicular to Road A.

1. What is the slope of Road B?
2. If Road B passes through the point $(10, 2)$, find the equation of Road B.

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.................................................................
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## Final Challenge

1. Prove that if $l_1$ and $l_2$ are perpendicular, then the angle of inclination of $l_1$ plus the angle of inclination of $l_2$ equals $90°$.

.................................................................
.................................................................

2. The line $l_1$ passes through $(1, 2)$ and $(4, 6)$, and the line $l_2$ passes through $(2, -1)$ and $(5, k)$. Find the value of $k$ such that $l_1 \perp l_2$.

.................................................................
.................................................................

## Self Check

- I can find the slope of a straight line. ★★★
- I can identify parallel lines. ★★★
- I can identify perpendicular lines. ★★★
- I can use these relations to solve problems. ★★★



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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## A Choose the correct answer:

1. If two lines are parallel and one of them has slope $-\frac{3}{4}$, then the slope of the other line is ........
 (a) $-\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $-\frac{3}{4}$ (d) $\frac{4}{3}$

2. If two lines are perpendicular and one of them has slope 2, then the slope of the other line is ........
 (a) 2 (b) −2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

3. The angle between two lines of slopes −2 and $\frac{1}{2}$ is ........
 (a) 0° (b) 45° (c) 90° (d) 135°

4. The slope of a line that makes an angle of 150° with the positive direction of the $x$−axis is ........
 (a) $\frac{\sqrt{3}}{3}$ (b) −1 (c) $-\frac{\sqrt{3}}{3}$ (d) 0

5. If the slope of a line is $\frac{5}{3}$, then the slope of a line perpendicular to it is ........
 (a) $-\frac{3}{5}$ (b) $\frac{3}{5}$ (c) $-\frac{5}{3}$ (d) $\frac{5}{3}$

6. The slope of a horizontal line is ........
 (a) 0 (b) 1 (c) Undefined (d) −1

7. The slope of a vertical line is ........
 (a) 0 (b) 1 (c) Undefined (d) −1

8. If the angle of inclination of a line is 60°, then its slope is ........
 (a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{3}$

## B Fill in the blanks:

1. If two lines are parallel, then their slopes are ........
2. If two lines are perpendicular, then the product of their slopes is ........
3. The angle between two perpendicular lines is ........
4. The slope of a line that makes an angle of 45° with the positive direction of the $x$−axis is ........
5. The slope of a line that makes an angle of 135° with the positive direction of the $x$−axis is ........
6. If the slope of a line is $m$, then the slope of a line perpendicular to it is ........
7. The slope of a line passing through two points with the same $x$−coordinate is ........
8. The slope of a line passing through two points with the same $y$−coordinate is ........
9. If two lines have slopes $m$ and $-3m$, then the angle between them is ........
10. If the angle of inclination of a line is $\theta$, then the slope of the line is ........

## C Find the slope of the line passing through each pair of points:

1. (2, 3) and (5, 7)
2. (−4, 1) and (2, −5)
3. (0, 2) and (6, −4)
4. (−3, −1) and (1, 5)
5. (4, 0) and (4, 7)
6. (2, −5) and (−1, −5)
7. (3, 6) and (3, −2)
8. (−6, 4) and (−2, −3)

## D Find the value of $k$ in each of the following:

1. The line passing through $(k, 2)$ and (3, 6) has slope 4.
2. The line passing through $(-1, k)$ and (5, 2) has slope $-\frac{1}{2}$.
3. The line passing through (2, 3) and $(k, 7)$ has slope 1.
4. The line passing through $(-2, k)$ and (2, −6) has slope −3.
5. The line passing through $(k, 1)$ and (5, 1) is horizontal.

## E Determine whether the lines are parallel, perpendicular, or neither:

1. $l_1$ passes through (1, 2) and (3, 6),
 $l_2$ passes through (2, −1) and (4, 5). ________
2. $l_1$ passes through (−3, 1) and (1, −2),
 $l_2$ passes through (0, −3) and (2, −5). ________
3. $l_1$ passes through (2, 4) and (6, 8),
 $l_2$ passes through (−1, −2) and (3, 2). ________
4. $l_1$ passes through (0, 0) and (2, 3),
 $l_2$ passes through (1, −2) and (3, 2). ________
5. $l_1$ passes through (−2, 1) and (2, −1),
 $l_2$ passes through (0, 3) and (3, 0). ________
6. $l_1$ passes through (5, 1) and (7, 3),
 $l_2$ passes through (1, −4) and (3, −2). ________
7. $l_1$ passes through (−4, −2) and (0, 2),
 $l_2$ passes through (−1, 1) and (2, −2). ________
8. $l_1$ passes through (3, −1) and (6, 2),
 $l_2$ passes through (2, 3) and (4, −1). ________
9. $l_1$ is parallel to the line passing through (1, 2) and (3, 6), and passes through (2, −1). Find the equation of $l_1$. ________
10. $l_1$ is perpendicular to the line passing through (−1, 2) and (3, −4), and passes through (2, 1). Find the equation of $l_1$. ________



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# The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

## A Solve the following:

1. Find the slope of the line which passes through $(2, 3)$ and $(6, 7)$. ..................
2. Find the slope of the line which passes through $(-1, 4)$ and $(3, -2)$. ..................
3. Find the slope of the line which makes an angle of $30°$ with the positive direction of the $x$-axis. ..................
4. Find the slope of the line perpendicular to the line of slope $\frac{5}{3}$. ..................
5. Find the slope of the line parallel to the line of slope $-4$. ..................
6. Find the slope of the line perpendicular to the line passing through $(0, 2)$ and $(4, 10)$. ..................

## B Determine whether the following lines are: parallel, perpendicular, or neither.

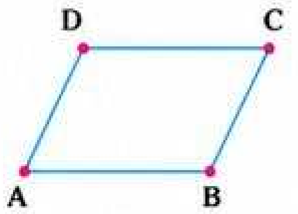
1. $l_1$ : through $(1, 2)$, $(4, 8)$
   $l_2$ : through $(-2, 1)$, $(1, 7)$ ..................
2. $l_1$ : through $(-1, 3)$, $(2, 0)$
   $l_2$ : through $(4, 5)$, $(1, -1)$ ..................
3. $l_1$ : through $(2, -1)$, $(5, 5)$
   $l_2$ : through $(-4, -7)$, $(-1, -1)$ ..................
4. $l_1$ : through $(0, 4)$, $(3, 4)$
   $l_2$ : through $(2, 1)$, $(2, -5)$ ..................
5. $l_1$ : through $(3, 2)$, $(6, 5)$
   $l_2$ : through $(-3, -4)$, $(0, -1)$ ..................

## C Find the value of k:

1. The line passing through $(k, 3)$ and $(5, 7)$ has slope 2. ..................
2. The line passing through $(1, k)$ and $(4, 2)$ has slope $-1$. ..................
3. The line passing through $(2, 3)$ and $(k, 7)$ has slope $\frac{4}{3}$. ..................
4. The line passing through $(-2, k)$ and $(2, -6)$ has slope $-2$. ..................
5. The line passing through $(k, 1)$ and $(3, 1)$ is horizontal. ..................
6. The line passing through $(-3, k)$ and $(-3, 5)$ is vertical. ..................

## D Geometric Applications

1. In the figure, ABCD is a parallelogram, A $(2, 1)$ and B $(6, 1)$. Find the slope of $\overline{BC}$.

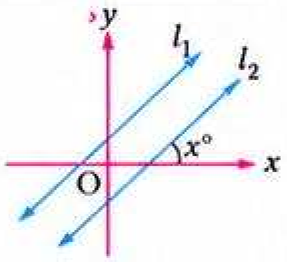



..................

2. If the slope of $\overline{AB}$ equals $\frac{3}{4}$ in a parallelogram, find the slope of $\overline{AD}$.

..................

3. In the figure, $l_1 \parallel l_2$ and the slope of $l_1$ is 2. Find the value of $x$.




..................

## E Choose the correct answer:

1. The slope of a line perpendicular to the line of slope $-2$ is ...... .
   (a) 2 (b) $-2$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
2. If two lines have slopes 4 and $m$ and are parallel, then $m =$ ...... .
   (a) $-4$ (b) 4 (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$
3. If the angle between two lines is $90°$, then the product of their slopes equals ...... .
   (a) 1 (b) $-1$ (c) 0 (d) 2
4. The slope of the line which passes through $(-2, 4)$ and $(2, 0)$ is ...... .
   (a) 1 (b) $-1$ (c) 2 (d) $-2$

## F Word Problems

1. A ramp makes an angle of $30°$ with the horizontal ground. Find its slope and the slope of a line perpendicular to it.

..................

2. The slope of a straight road on a map is $-\frac{3}{4}$. If another road is perpendicular to it at the same point, what is the slope of the second road ?

..................

3. A staircase is designed as a straight line that passes through $(0, 0)$ and $(4, 3)$. Another line is drawn through $(4, 3)$ and is parallel to the staircase. If it intersects the $y$-axis at point $(0, k)$, find $k$.

..................

## G Challenge

1. The line $l_1$ passes through $(1, 2)$ and $(4, 8)$, and the line $l_2$ passes through $(-2, -1)$ and $(k, 5)$. If $l_1 \perp l_2$, find $k$.

..................

2. Find the equation of the line which passes through $(2, 3)$ and is parallel to the line $y = -\frac{1}{2}x + 4$.

..................

## Self Check

- I can find the slope of a straight line. ★★★
- I can identify parallel and perpendicular lines. ★★★
- I can use slopes to solve geometric problems. ★★★



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# Equation of the Straight Line

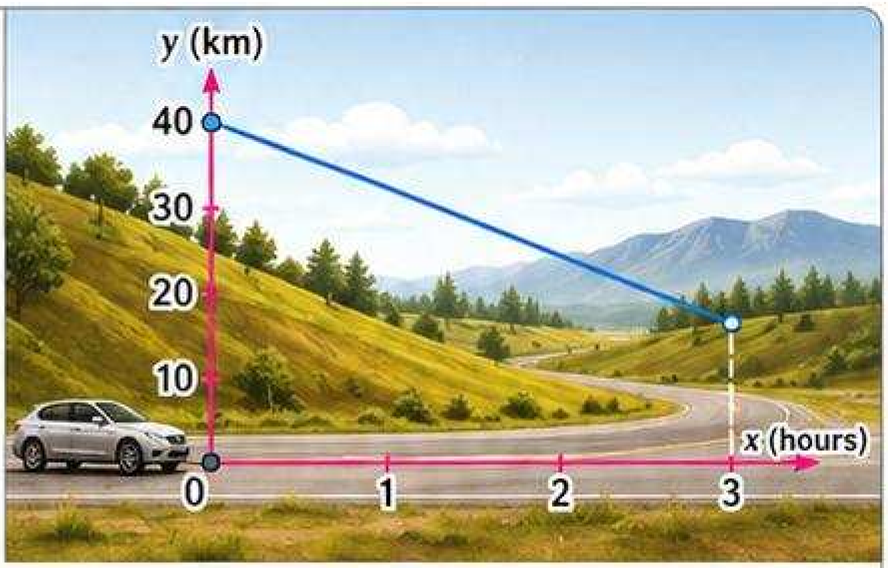
## Why learn this?

A shopkeeper wants to connect his store to a new supplier.
He needs to find the cheapest route on the map.
The equation of the straight line helps us describe that route accurately using mathematics.

A study showed that in **2020**, about **80** million tourists visited Luxor.
In **2021**, the number increased to **85** million.
Using the equation of a straight line, we can predict the number of tourists in the following years and study patterns.

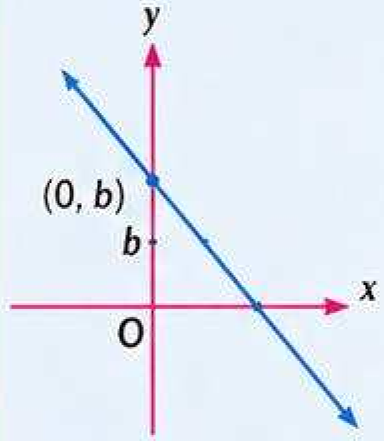
## Think and Discuss!

A car travels from point A for 3 hours. At the start, it was 40 km away from the city. After 3 hours, it is 15 km away.
Can you write an equation that represents the distance ($y$) in km from the city after ($x$) hours?




① $y = 40 - \frac{25}{3}x$ ② $y = 15x + 40$ ③ $y = x^2 - 40$

## Let's Learn!




### Equation of the Line in Slope–Intercept Form

If a line has slope $m$ and y–intercept $b$, then its equation is:

$$y = mx + b$$

slope ($m$), y–intercept ($b$)

Note: If the slope is $-\frac{4}{5}$ and the y–intercept is $-3$, then the equation is: $y = -\frac{4}{5}x - 3$

## Learning Outcomes

**By the end of this lesson, you will be able to:**

- Find the equation of a straight line in different forms.
- Use the slope and y-intercept to write the equation.
- Write the equation from two points.
- Find an equation from a table or a graph.
- Use the equation of a line to solve problems.
- Apply equations in real-life situations.

## Key Vocabulary

- Equation of a straight line
- Slope (m)
- y-intercept
- Point
- Graph

## You Need to Know

The equation of a line can be written in the form:

$$y = mx + b$$

where $m$ is the slope and $b$ is the y–intercept.

## Remember!

The y–intercept is the point where the line crosses the y–axis. Its coordinates are always (0, $b$).

## Accuracy in Graphing

- ☑ Plot the points carefully.
- ☑ Count the units correctly.
- ☑ Draw a straight line through the points using a ruler.
- ☑ Label the axes and points clearly.



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# Writing Equations of Straight Lines Using Intercepts and Standard Form

## Standard (Intercept) Form of a Line

$$Ax + By = C$$

where $A, B, C$ are real numbers and $A, B \neq 0$.
Usually $A, B > 0$ and the greatest common factor (G.C.F) of $A, B, C$ is $1$.

**Note:**
If $Ax + By = C$, then
- The $x-$intercept is $\frac{C}{A}$.
- The $y-$intercept is $\frac{C}{B}$.

## Slope from Intercepts

If a line cuts the $x-$axis at $\left(\frac{C}{A}, 0\right)$ and the $y-$axis at $\left(0, \frac{C}{B}\right)$, then the slope of the line is:

$$m = -\frac{A}{B}$$

## Example 1 ($Ax + By = C$) Intercepts Method

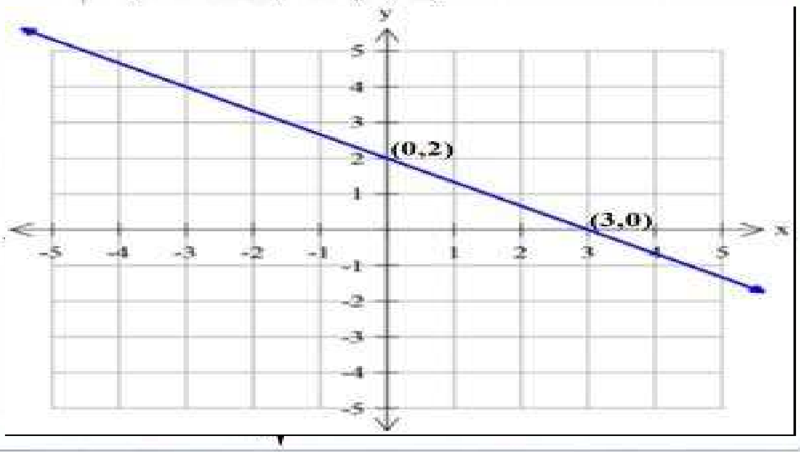
Find the intercepts and graph the line.

$$2x + 3y = 6$$

**1 $x-$intercept**
Let $y = 0$: $2x + 3(0) = 6 \Rightarrow x = 3$
$x-$intercept is $(3, 0)$

**2 $y-$intercept**
Let $x = 0$: $2(0) + 3y = 6 \Rightarrow y = 2$
$y-$intercept is $(0, 2)$

**3 Graph the line**
1. Plot $(3, 0)$ on the $x-$axis.
2. Plot $(0, 2)$ on the $y-$axis.
3. Draw a straight line through the two points.
4. Extend the line in both directions.




## Example 2 ($Ax + By = C$) Intercepts Method

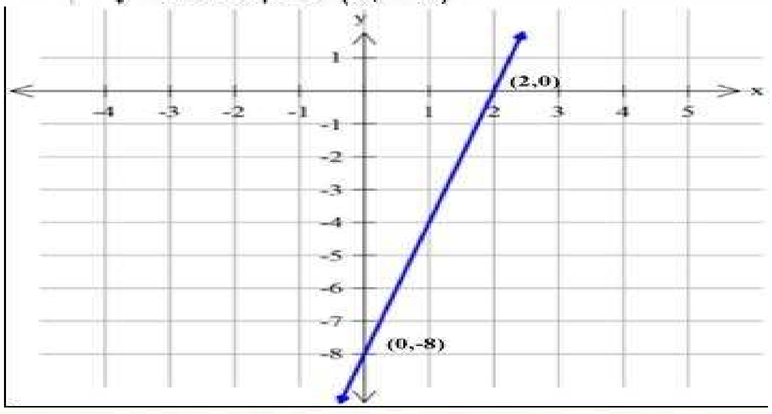
Find the intercepts and graph the line.

$$4x - y = 8$$

**1 $x-$intercept**
Let $y = 0$: $4x - 0 = 8 \Rightarrow x = 2$
$x-$intercept is $(2, 0)$

**2 $y-$intercept**
Let $x = 0$: $0 - y = 8 \Rightarrow y = -8$
$y-$intercept is $(0, -8)$

**3 Graph the line**
1. Plot $(2, 0)$ on the $x-$axis.
2. Plot $(0, -8)$ on the $y-$axis.
3. Draw a straight line through the two points.
4. Extend the line in both directions.




## Remember

- In standard form $Ax + By = C$, the intercepts are easy to find.
- The $x-$intercept is found by letting $y = 0$.
- The $y-$intercept is found by letting $x = 0$.
- Plot the intercepts correctly and draw the line through them.
- Always label the axes, units, and points.

## Common Mistakes

- Using the wrong intercept values.
- Swapping $x$ and $y$ intercepts.
- Misplacing points on the axes.
- Not extending the line in both directions.
- Forgetting to check your line by substituting a point.

## Key Tip

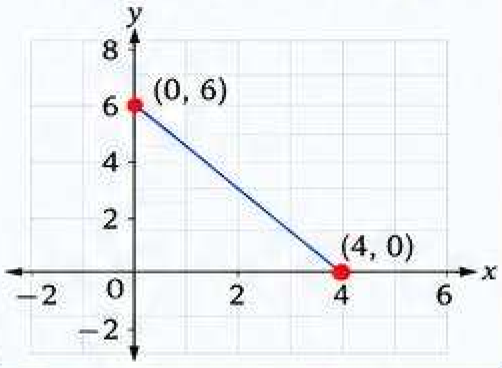
The intercepts give you two clear points on the line.
Plot them accurately and draw the line through them with a ruler.

## Example (Standard Form)

Find the intercepts and graph.

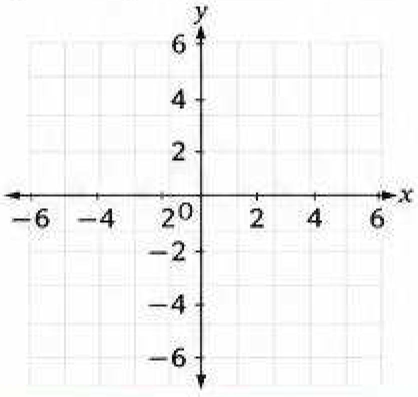
$$3x + 2y = 12$$

- $x-$intercept: Let $y = 0$
  $3x = 12 \Rightarrow x = 4 \Rightarrow (4, 0)$
- $y-$intercept: Let $x = 0$
  $2y = 12 \Rightarrow y = 6 \Rightarrow (0, 6)$

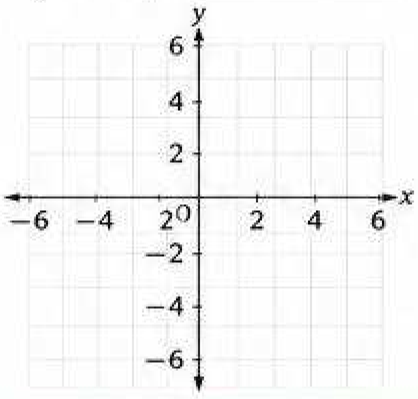



## Practice: Use Standard Form to Find Intercepts

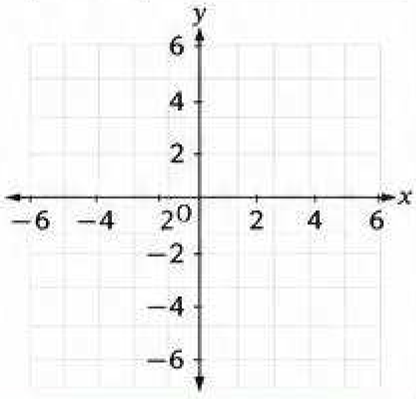
**1** $2x + 3y = 6$
$x-$intercept: ____________
$y-$intercept: ____________



**2** $4x - y = 8$
$x-$intercept: ____________
$y-$intercept: ____________



**3** $3x + 2y = 12$
$x-$intercept: ____________
$y-$intercept: ____________



## Check Yourself!

Verify by substitution.

1 Is the point $(2, 1)$ on the line $2x + 3y = 7$?
____________
____________

2 Is the point $(3, -2)$ on the line $3x - y = 11$?
____________
____________

3 Is the point $(4, 0)$ on the line $y = -\frac{3}{2}x + 6$?
____________



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# Writing Equations of Straight Lines from a Table or a Graph

We can find the equation of a straight line from a table of values or from two clear points on a graph.

## Example 6 From a Table of Values

The table shows the values of $x$ and $y$ that lie on a straight line.

Find the equation of the line.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y$ | $-5$ | $-3$ | $-1$ | $1$ | $3$ | $5$ |

**Step 1:** Find the slope using two values from the table.

Use $(0, -1)$ and $(2, 3)$

$$m = \frac{3-(-1)}{2-0} = \frac{4}{2} = 2$$

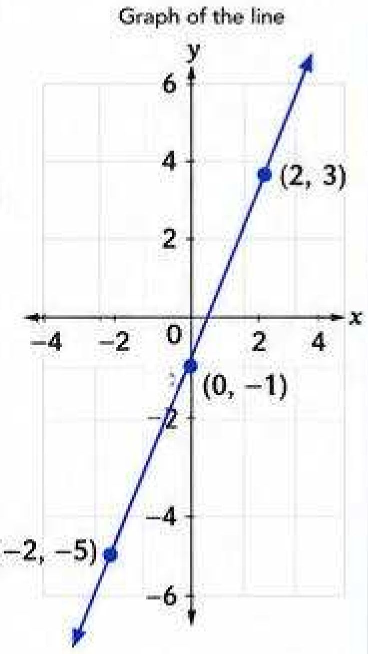
**Step 2:** Find the $y$-intercept.

Use $(0, -1)$ and $y = mx + b$

$-1 = 2(0) + b \Rightarrow b = -1$

**Step 3:** Write the equation.

$$y = 2x - 1$$




## Example 7 From a Graph

The line shown in the graph passes through points $A$ and $B$.

Find the equation of the line.

**Step 1:** Read the two points.

$A(-3, 2)$ and $B(3, -4)$

**Step 2:** Find the slope.

$$m = \frac{-4-2}{3-(-3)} = \frac{-6}{6} = -1$$

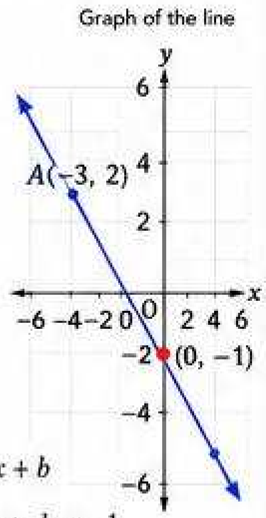
**Step 3:** Find the $y$-intercept.

Use point $A(-3, 2)$ and $y = mx + b$

$2 = -1(-3) + b \Rightarrow 2 = 3 + b \Rightarrow b = -1$

**Step 4:** Write the equation.

$$y = -x - 1$$




### Accuracy in Graphing

- Plot each point carefully.
- Count the units correctly.
- Draw a straight line through the points using a ruler.
- Extend the line in both directions with arrowheads.
- Label the axes, units, and points clearly.

### Common Mistakes

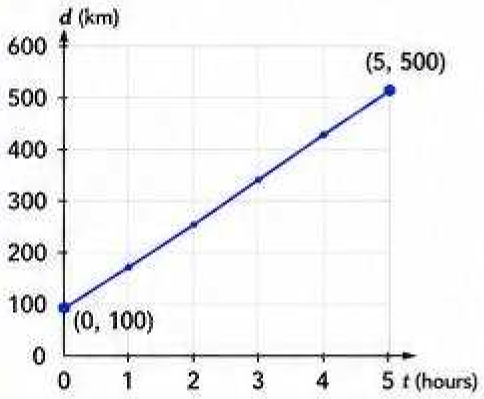
- Reading coordinates incorrectly.
- Calculating the slope incorrectly.
- Plotting points inaccurately.
- Forgetting the negative sign in the slope.
- Not extending the line in both directions.

# Equations from Real-Life Situations

## Example 8 A Car's Distance-Time Relationship

The distance $d$ (in km) of a car from its starting point is shown in the graph after $t$ hours.

Find an equation that represents the distance $d$ in terms of time $t$.




**Step 1:** Read two points.

$(0, 100)$ and $(5, 500)$

**Step 2:** Find the slope.

$$m = \frac{500-100}{5-0} = \frac{400}{5} = 80$$

(km per hour)

**Step 3:** Find the $y$-intercept.

When $t = 0$, $d = 100 \Rightarrow b = 100$

**Step 4:** Write the equation.

$$d = 80t + 100$$

The car travels at a constant speed of 80 km per hour and starts 100 km away from the starting point.

## Example 9 Taxi Fare Situation

A taxi company charges a fixed fee of 12 LE plus 6 LE for each kilometer traveled.

Write an equation for the total fare $F$ (in LE) in terms of the distance $x$ (in km).

**Step 1:** Rate per kilometer = 6 LE $\Rightarrow m = 6$

**Step 2:** Fixed fee = 12 LE $\Rightarrow b = 12$

**Step 3:** Write the equation.

$$F = 6x + 12$$

When the distance is 0 km, the fare is 12 LE.
The fare increases by 6 LE for each additional kilometer.

### Key Formula

Equation of a straight line in slope-intercept form:

$$y = mx + b$$

where:

$m$ = slope (rate of change)

$b$ = $y$-intercept (value of $y$ when $x = 0$)

## Your Turn! Find the Equation

**1** From the table:

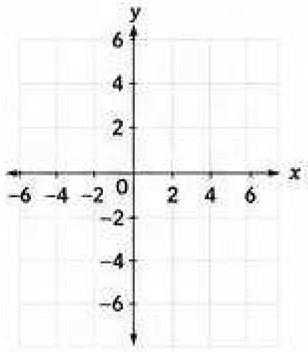
| $x$ | $-3$ | $-1$ | $1$ | $3$ | $5$ |
|---|---|---|---|---|---|
| $y$ | $-8$ | $-4$ | $0$ | $4$ | $8$ |

Slope $m$ = ____________

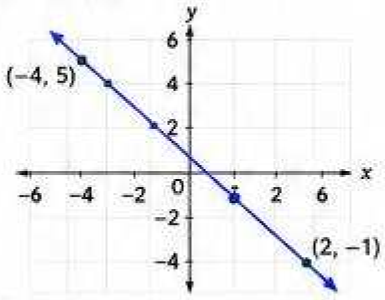
$y$-intercept $b$ = ____________

Equation: ____________

Graph the points and draw the line accurately.



**2** From the graph:




Slope $m$ = ____________

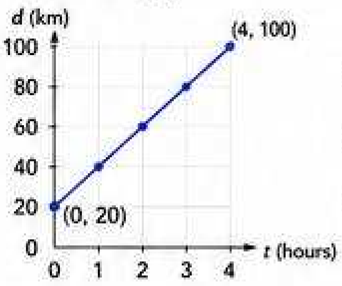
$y$-intercept $b$ = ____________

Equation: ____________

**3** Situation:

A boat moves at a constant speed.

The distance traveled $d$ (in km) after $t$ hours is shown in the graph.




Slope $m$ = ____________

$y$-intercept $b$ = ____________

Equation: ____________

## Challenge!

**1** Write an equation for the total cost $C$ (in LE) in terms of the number of items $x$ if each item costs 25 LE and there is a fixed fee of 10 LE.

$C$ = ____________

**2** A bicycle rental charges 15 LE for the first hour and 10 LE for each additional hour.

Let $t$ be the total time (in hours).

Write an equation for the total cost $R$ (in LE).

$R$ = ____________

## Check Yourself!

Verify by substitution.

1 Is the point $(2, 1)$ on the line $2x + 3y = 7$?

____________

2 Is the point $(3, -2)$ on the line $3x - y = 11$?

____________

3 Is the point $(4, 0)$ on the line $y = -\frac{3}{2}x + 6$?



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## Real–Life Applications and Graphing

### Example 10 Mobile Data Plan

A mobile company charges a fixed monthly fee of 50 LE and 5 LE for each GB of data used. Let $x$ be the number of GB used and $C$ be the total cost (in LE).
Write an equation for $C$ in terms of $x$ and graph it.

**Step 1:** Identify the slope and $y$–intercept.

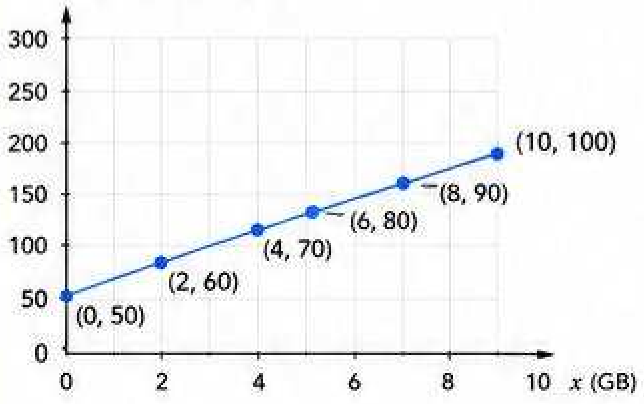
- Cost per GB $= 5 \Rightarrow m = 5$
- Fixed monthly fee $= 50 \Rightarrow b = 50$

**Step 2:** Write the equation.

$$C = 5x + 50$$

**Step 3:** Graph the equation.

Total cost $C$ (LE)




When no data is used ($x = 0$), the cost is 50 LE. The cost increases by 5 LE for each additional GB.

### Example 11 Fuel in a Car

A car has 40 liters of fuel in the tank. It uses 0.5 liter of fuel per kilometer.
Let $x$ be the distance traveled (in km) and $F$ be the fuel left (in liters).
Write an equation for $F$ in terms of $x$ and graph it.

**Step 1:** Identify the slope and $y$–intercept.

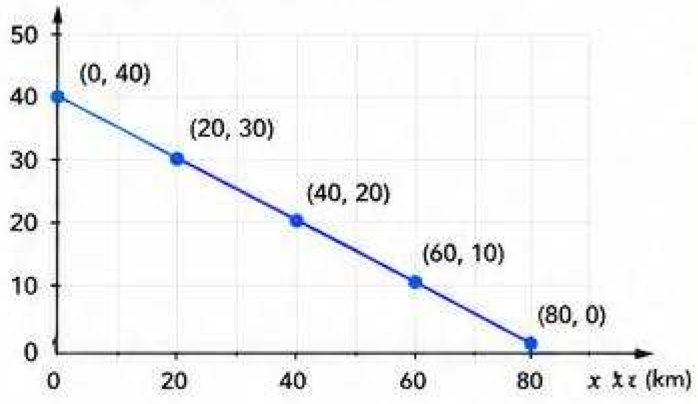
- Fuel decreases by 0.5 liter per km $\Rightarrow m = -0.5$
- Initial fuel $= 40 \Rightarrow b = 40$

**Step 2:** Write the equation.

$$F = -0.5x + 40$$

**Step 3:** Graph the equation.

Fuel left $F$ (liters)




The car will run out of fuel after 80 km. Fuel decreases by 0.5 liter for each kilometer.

### Excellent Tips!

- Determine the slope ($m$) and $y$–intercept ($b$) correctly.
- Plot at least two points that satisfy the equation.
- Use a ruler to draw a straight line through the points.
- Extend the line in both directions with arrowheads.
- Label the axes with correct units.

### Common Mistakes

- Using the wrong slope or $y$–intercept.
- Switching the roles of $x$ and $y$.
- Plotting points inaccurately.
- Not labeling the axes or units.
- Not extending the line in both directions.

### Key Formula

Equation of a straight line in slope–intercept form:

$$y = mx + b$$

where:

$m$ = slope (rate of change)

$b$ = $y$–intercept (value of $y$ when $x = 0$)

## Practice Time!

**A** Write the equation of the line in slope–intercept form.

1. Passes through (2, 3) and (6, 11). ________
2. Passes through (−3, −2) and (5, 4). ________
3. Slope = 3, $y$–intercept = −4. ________
4. Slope = $-\frac{1}{2}$, $y$–intercept = 6. ________
5. Passes through (0, 4) and (8, −4). ________
6. Passes through (1, −3) and (−3, 1). ________

**B** Write the equation of the line from the given information.

1. Slope = −2, $y$–intercept = 5. ________
2. Passes through (−2, 3) with slope 4. ________
3. Passes through (0, −6) with slope $-\frac{1}{3}$. ________
4. Passes through (3, 0) and has slope $\frac{2}{3}$. ________
5. Passes through (−4, −5) and (4, −1). ________
6. Passes through (1, 2) and (1, −6). ________

## Test Yourself!

### 1 Multiple Choice

Choose the correct answer.

1. The equation of the line with slope 2 and $y$–intercept −1 is:
   a) $y = 2x - 1$  b) $y = 2x + 1$
   c) $y = -2x - 1$  d) $y = -2x + 1$
2. The line that passes through (0, −2) and (4, 2) has slope:
   a) 1  b) −1
   c) $\frac{1}{2}$  d) 2

### 2 Fill in the Blanks

1. The equation of a line in slope–intercept form is ________.
2. The $y$–intercept is the point where the line crosses the ________.
3. The line with slope 0 is a ________ line.
4. The equation of the line that passes through (0, 3) and has slope 4 is ________.

### 3 True or False

Write (T) for True or (F) for False.

1. A line with negative slope rises from left to right. ( ____ )
2. The $y$–intercept is the value of $y$ when $x = 0$. ( ____ )
3. If the slope is 3 and the line passes through (0, −2), then the equation is $y = 3x - 2$. ( ____ )
4. Parallel lines have the same slope. ( ____ )

### 4 Problem Solving

1. A train travels 100 km at a constant speed and then continues at another constant speed.
   The distance $d$ (in km) after $t$ hours is given by $d = 80t + 100$.
   Find the distance after 6 hours. ________
2. A linear function has slope −3 and passes through (2, 5).
   Find the equation of the line. ________

## Challenge!

1. Write an equation for the total cost $P$ (in LE) in terms of the number of hours $h$ a parking lot is used. The fixed fee is 20 LE and the cost is 5 LE per hour.
   $P$ = ________
2. An internet company charges 60 LE per month plus 2 LE for each GB of data used. Let $x$ be the number of GB used and $C$ the total cost (in LE). Write an equation for $C$ in terms of $x$.
   $C$ = ________

## Check Yourself!

Verify by substitution.

1. Is the point (2, 1) on the line $2x + 3y = 7$? ________
2. Is the point (3, −2) on the line $3x - y = 11$? ________
3. Is the point (4, 0) on the line $y = -\frac{3}{2}x + 6$? ________



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## Exercises

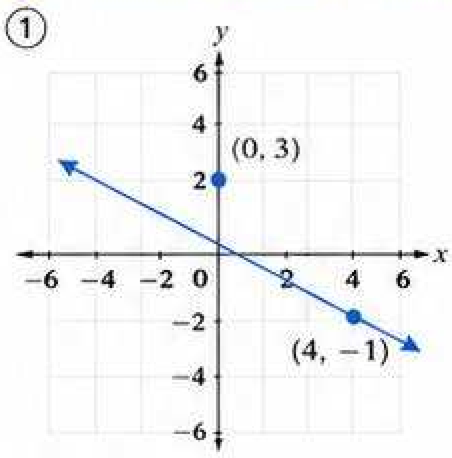
**A** Write the equation of the line in slope–intercept form.

1. Passes through $(2, 3)$ and $(6, 11)$. ________
2. Passes through $(-3, -2)$ and $(5, 4)$. ________
3. Slope $= 3$, $y-$intercept $= -4$. ________
4. Slope $= -\frac{1}{2}$, $y-$intercept $= 6$. ________
5. Passes through $(0, 4)$ and $(8, -4)$. ________
6. Passes through $(1, -3)$ and $(-3, 1)$. ________

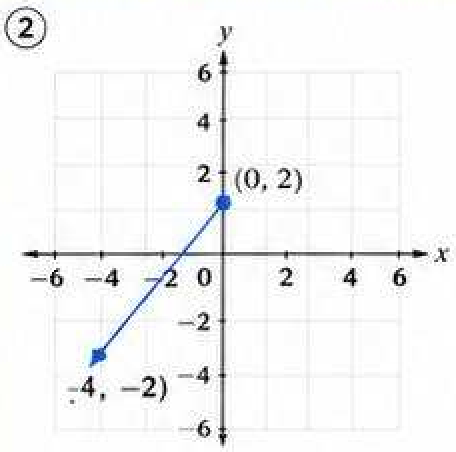
**B** Write the equation of the line from the given information.

1. Slope $= -2$, $y-$intercept $= 5$. ________
2. Passes through $(-2, 3)$ with slope $4$. ________
3. Passes through $(0, -6)$ with slope $= -\frac{1}{3}$. ________
4. Passes through $(3, 0)$ and has slope $\frac{2}{3}$. ________
5. Passes through $(-4, -5)$ and $(4, -1)$. ________
6. Passes through $(1, 2)$ and $(1, -6)$. ________

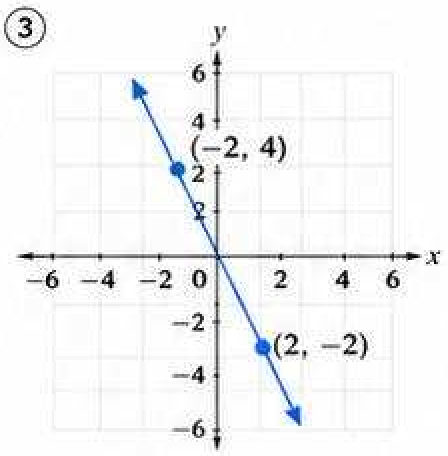
**C** Find the slope and $y-$intercept of the line, then write its equation.

① 

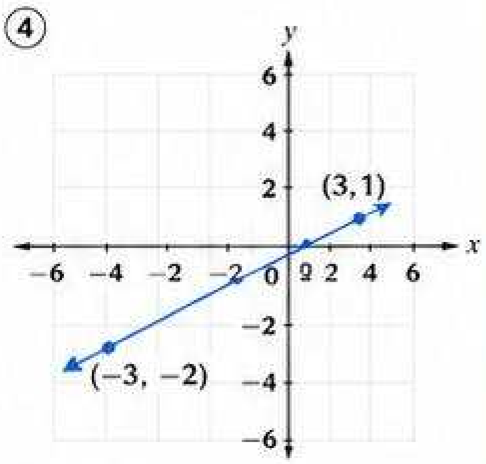

Slope $(m) =$ ________
$y-$intercept $(b) =$ ________
Equation: ________

② 


Slope $(m) =$ ________
$y-$intercept $(b) =$ ________
Equation: ________

③ 


Slope $(m) =$ ________
$y-$intercept $(b) =$ ________
Equation: ________

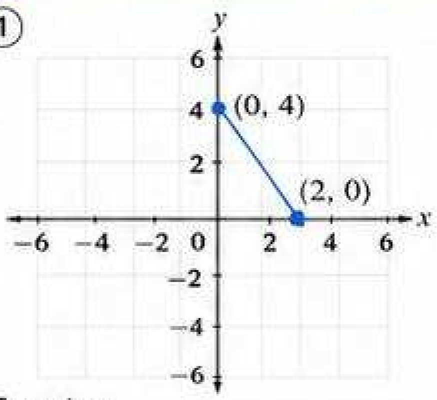
④ 


Slope $(m) =$ ________
$y-$intercept $(b) =$ ________
Equation: ________

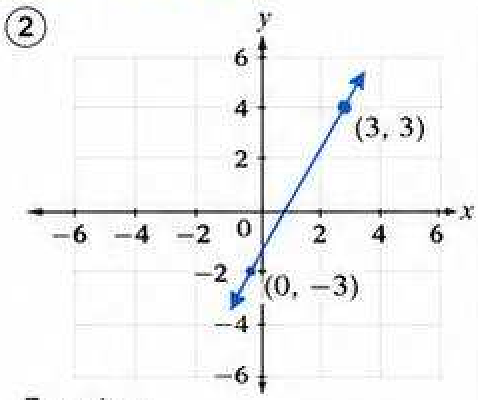
**D** Use the given intercepts to write the equation of the line.

1. $x-$intercept $= (5, 0)$, $y-$intercept $= (0, 3)$
2. $x-$intercept $= (-4, 0)$, $y-$intercept $= (0, -2)$
3. $x-$intercept $= (6, 0)$, $y-$intercept $= (0, -3)$
4. $x-$intercept $= (-2, 0)$, $y-$intercept $= (0, 4)$

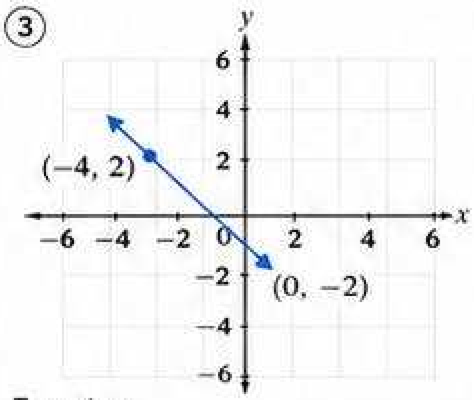
**E** Write the equation of the line shown in each graph.

① 

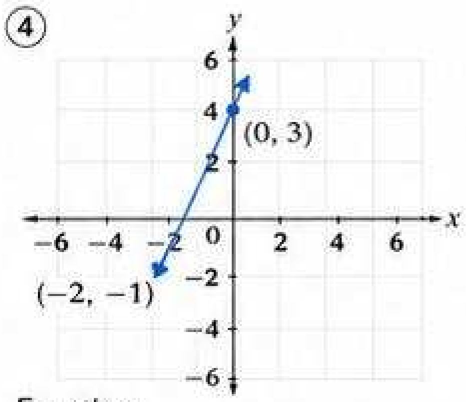

Equation: ________

② 


Equation: ________

③ 


Equation: ________

④ 


Equation: ________

**F** Solve for $x$ in each equation.

1. $2x + 3 = 11$ — $x =$ ________
2. $4x - 5 = 19$ — $x =$ ________
3. $3(x + 2) = 21$ — $x =$ ________
4. $-2x + 7 = -11$ — $x =$ ________
5. $\frac{1}{2}x - 3 = 5$ — $x =$ ________
6. $5 - x = -10$ — $x =$ ________

**G** Determine whether the given point is on the line. (Substitute the point in the equation.)

1. $(2, 1)$ on the line $2x + 3y = 7$
2. $(3, -2)$ on the line $3x - y = 11$
3. $(4, 0)$ on the line $y = -\frac{1}{2}x + 3$ ________
4. $(-1, 5)$ on the line $4x + y = 1$
5. $(0, -3)$ on the line $y = 2x - 3$
6. $(5, -2)$ on the line $x + y = 3$ ________

**H** Real–Life Problems

1. A company charges a fixed fee of 30 LE plus 4 LE for each hour of internet use. Let $x$ be the number of hours and $C$ be the total cost (in LE). Write an equation for $C$ in terms of $x$.

   $C =$ ________

2. A taxi company charges a fixed fee of 10 LE plus 5 LE for each kilometer. Let $x$ be the distance traveled (in km) and $F$ be the fare (in LE). Write an equation for $F$ in terms of $x$.

   $F =$ ________



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## Exercises

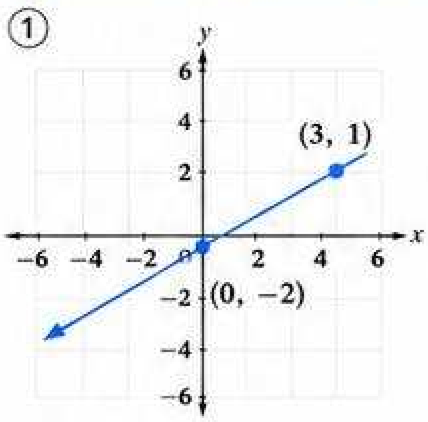
**A** Write the equation of the line in slope–intercept form.

1. Passes through $(4, 5)$ and $(8, 17)$. ________
2. Passes through $(-6, 2)$ and $(0, -10)$. ________
3. Slope $= -3$, $y$–intercept $= 7$. ________
4. Slope $= \frac{2}{3}$, $y$–intercept $= -4$. ________
5. Passes through $(-2, 4)$ and $(4, 10)$. ________
6. Passes through $(0, -3)$ and $(5, 7)$. ________

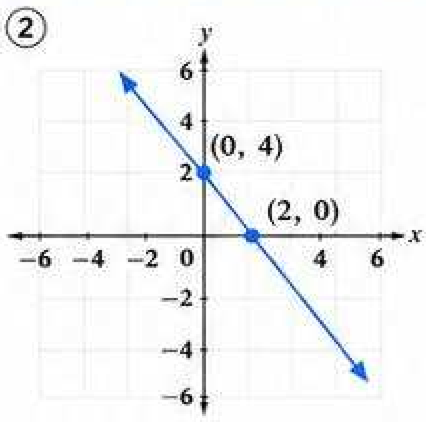
**B** Write the equation of the line from the given information.

1. Slope $= 4$, $y$–intercept $= -1$. ________
2. Passes through $(-3, 1)$ with slope $-2$. ________
3. Passes through $(0, 5)$ with slope $\frac{3}{5}$. ________
4. Passes through $(2, -4)$ with slope $\frac{1}{2}$. ________
5. Passes through $(-5, 6)$ and $(1, -2)$. ________
6. Passes through $(3, -1)$ and $(3, 7)$. (Vertical line) ________

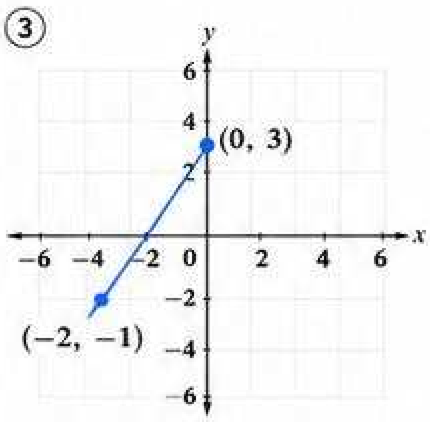
**C** Find the slope and $y$–intercept of the line, then write its equation.

①


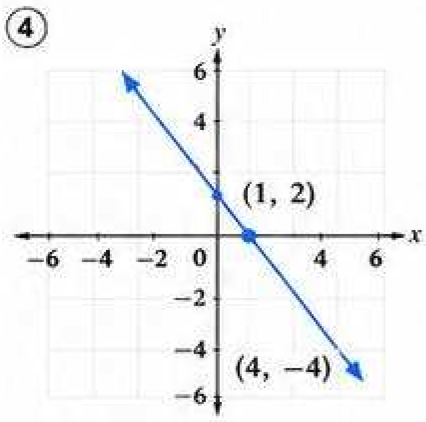

Slope $(m) =$ ________
$y$–intercept $(b) =$ ________
Equation: ________

②



Slope $(m) =$ ________
$y$–intercept $(b) =$ ________
Equation: ________

③



Slope $(m) =$ ________
$y$–intercept $(b) =$ ________
Equation: ________

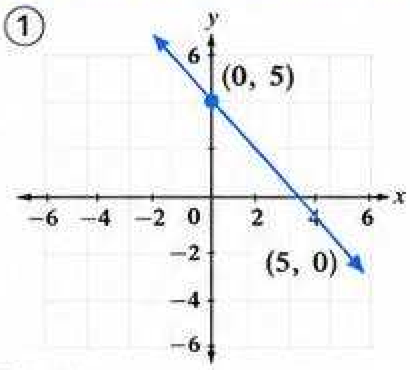
④



Slope $(m) =$ ________
$y$–intercept $(b) =$ ________
Equation: ________

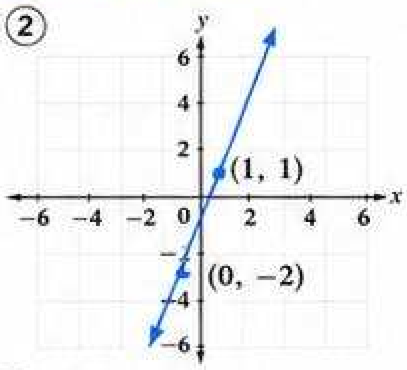
**D** Use the given intercepts to write the equation of the line.

1. $x$–intercept $= (8, 0)$, $y$–intercept $= (0, -2)$
2. $x$–intercept $= (-4, 0)$, $y$–intercept $= (0, 5)$
3. $x$–intercept $= (6, 0)$, $y$–intercept $= (0, -6)$
4. $x$–intercept $= (-3, 0)$, $y$–intercept $= (0, 3)$

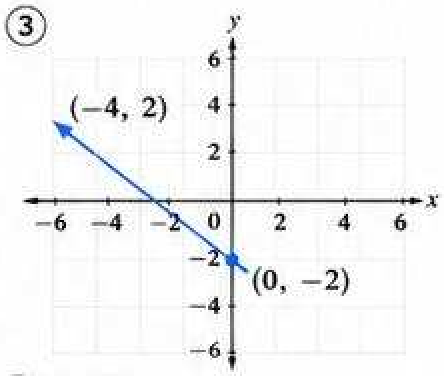
**E** Write the equation of the line shown in each graph.

①


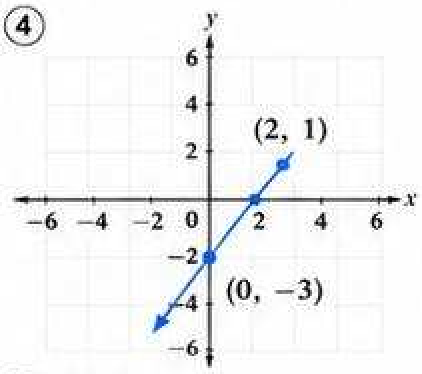

Equation: ________

②



Equation: ________

③



Equation: ________

④



Equation: ________

**F** Solve for $x$ in each equation.

1. $2x + 5 = 17$  $x =$ ________
2. $4x - 7 = 21$  $x =$ ________
3. $3(x - 4) = 12$  $x =$ ________
4. $-2x + 6 = -10$  $x =$ ________
5. $\frac{1}{2}x + 4 = 10$  $x =$ ________
6. $-3x - 2 = 16$  $x =$ ________

**G** Determine whether the given point is on the line. (Substitute the point in the equation.)

1. $(2, 3)$ on the line $2x + y = 7$ ______
2. $(-1, 4)$ on the line $3x - y = -7$ ______
3. $(0, -2)$ on the line $y = -\frac{1}{2}x + 6$ ______
4. $(4, -1)$ on the line $4x + y = 15$ ______
5. $(-2, 5)$ on the line $y = 2x + 9$ ______
6. $(3, 0)$ on the line $x - y = 3$ ______

**H** Real–Life Problems

1. A delivery service charges a fixed fee of 20 LE plus 6 LE for each kilometer traveled. Let $x$ be the distance (in km) and $C$ be the total cost (in LE). Write an equation for $C$ in terms of $x$.

$C =$ ________________

2. A water tank has 150 liters of water. Water flows out at a rate of 5 liters per minute. Let $t$ be the time (in minutes) and $W$ be the amount of water left (in liters). Write an equation for $W$ in terms of $t$.

$W =$ ________________



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## Exercises

### A Classify each equation.

Write (V) if the equation is linear. Write (N) if it is not linear.

1. $y = 4x + 7$ ( )
2. $2x - 3y = 6$ ( )
3. $y = x^2 + 2$ ( )
4. $y = \frac{3}{x} + 1$ ( )
5. $5x + y = 0$ ( )
6. $y = -7$ ( )
7. $y = \sqrt{x} + 4$ ( )
8. $-2x + y = 9$ ( )

### B Complete the table for each linear equation.

1. $y = 2x - 3$

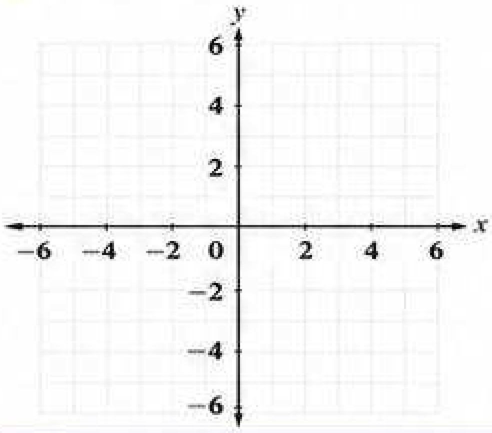
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

2. $y = -x + 4$

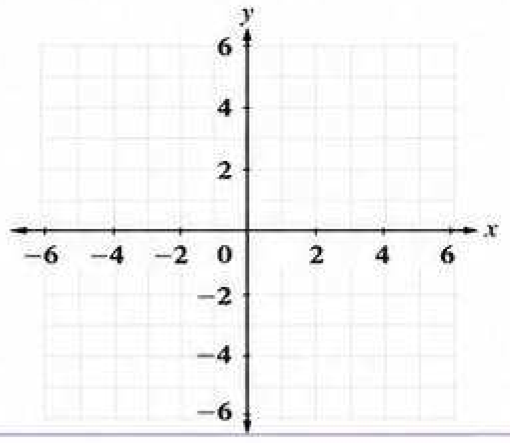
| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

### C Graph the line using the table from part (B).

1. $y = 2x - 3$



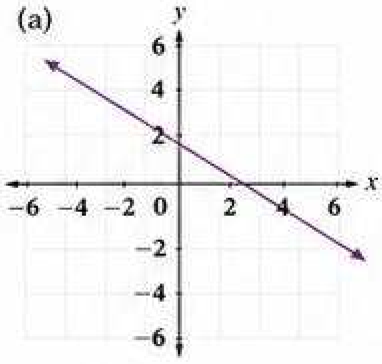

2. $y = -x + 4$



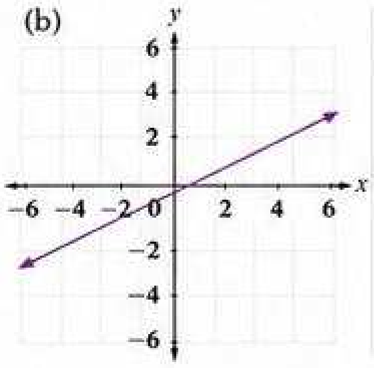

### D Match each equation with its graph.

1. $y = \frac{1}{2}x + 2$
2. $y = -2x + 1$
3. $y = x - 3$
4. $y = -\frac{1}{2}x - 1$

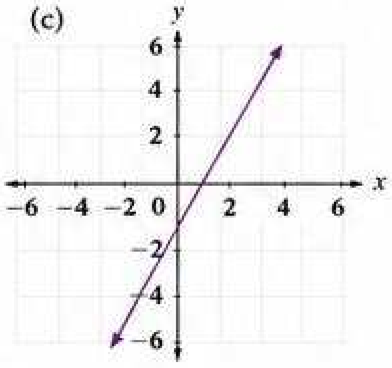
(a)



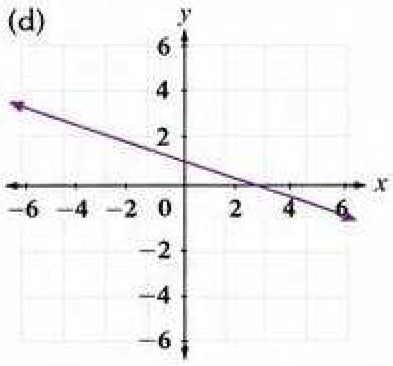
(b)



(c)



(d)



### E Find the slope of the line that passes through each pair of points.

1. (1, 2) and (5, 10) ______
2. (−3, −1) and (3, 5) ______
3. (0, −4) and (6, 2) ______
4. (2, 7) and (2, −5) ______
5. (−4, 3) and (−4, −2) ______
6. (1, −1) and (7, −1) ______

### F Find the value of $k$ so that the points are collinear.

1. (0, 1), (2, 5), ($k$, 13) $k =$ ______
2. (−2, 3), (1, $k$), (4, 15) $k =$ ______
3. (1, −1), (3, 3), ($k$, 11) $k =$ ______

### G Interpret the slope and $y$−intercept in each situation, then write an equation.

1. A plant grows 2 cm in height each week. The initial height of the plant is 5 cm. Let $t$ be the number of weeks and $h$ be the height (in cm).

   Slope (rate of change) = ______

   $y$−intercept (initial value) = ______

   Equation: ______

2. The temperature decreases by 3 °C each hour. The initial temperature is 18 °C. Let $t$ be the number of hours and $T$ be the temperature (in °C).

   Slope (rate of change) = ______

   $y$−intercept (initial value) = ______

   Equation: ______

3. A savings account earns 25 LE interest each month. The initial amount in the account is 150 LE. Let $m$ be the number of months and $S$ be the total amount (in LE).

   Slope (rate of change) = ______

   $y$−intercept (initial value) = ______

   Equation: ______

### H Real−Life Problem Solving

1. A movie theater charges a ticket fee of 40 LE plus 15 LE for each 3D glasses rental. Let $g$ be the number of glasses rented and $C$ be the total cost (in LE).
   a) Write an equation for $C$ in terms of $g$.
   b) How much will it cost to rent 6 glasses?

2. A cable company charges 120 LE for basic installation plus 35 LE for each additional channel. Let $x$ be the number of additional channels and $T$ be the total bill (in LE).
   a) Write an equation for $T$ in terms of $x$.
   b) Find the total bill if 8 additional channels are added.

3. A gym membership costs 200 LE to join plus 50 LE per month. Let $m$ be the number of months and $P$ be the total paid (in LE).
   a) Write an equation for $P$ in terms of $m$.
   b) How many months will it take if you paid a total of 950 LE?



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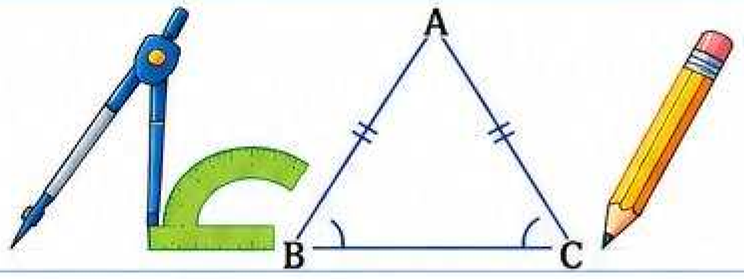
# Unit 3 Geometry Assessment




## Exercises

### 1 Choose the correct answer:

1. If $\tan X \sin 30° = \cos 30°$, then $X = ?$
   a) $30°$ b) $45°$ c) $60°$ d) $75°$
2. In an equilateral triangle, each angle measures:
   a) $45°$ b) $60°$ c) $90°$ d) $120°$
3. The slope of the line $2y = x - 4$ is:
   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $1$ d) $-1$
4. The midpoint of the segment joining $P(-2, 4)$ and $P'(-3, 6)$ is:
   a) $(-\frac{5}{2}, 10)$ b) $(-2.5, 5)$ c) $(-\frac{5}{2}, 5)$ d) $(-2.5, 4)$

### 2 Fill in the blanks:

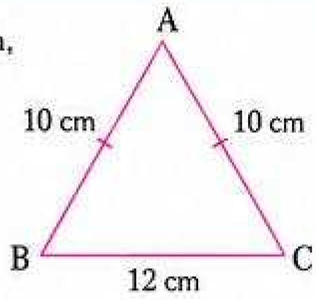
1) In an isosceles triangle, the base angles are ________.
2) The equation of a line in standard form is ________.
3) The slope of a line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ is ________.
4) If BC bisects $\angle ABD$, then $\angle ABC =$ ________.
5) The coordinates of the midpoint of a segment are the ________ of the $x$-coordinates and the ________ of the $y$-coordinates.

### 3 Solve the following:

1. In equilateral $\triangle ABC$, if $AC = AB = 10$ cm, $BC = 12$ cm. Find $\cos\frac{A}{2}$, $m(\angle B)$.

   $\cos\frac{A}{2} =$ ........................

   $m(\angle B) =$ ........................

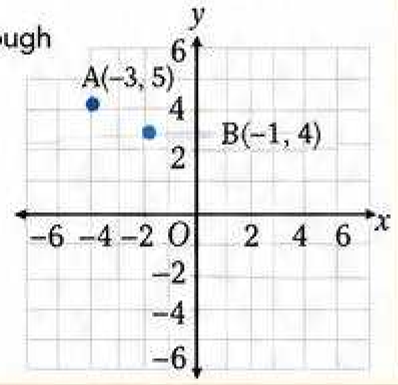



2. Find the equation of the line passing through $A(-3, 5)$ and $B(-1, 4)$.

   ........................

   ........................

   ........................

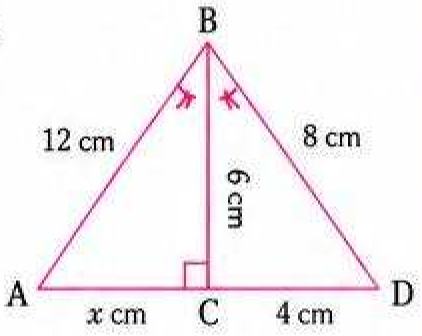



3. In the figure: If $\overrightarrow{BC}$ bisects $\angle ABD$, prove that $\triangle ABC \sim \triangle DBC$ and find the value of $x$.

   ........................

   ........................

   ........................

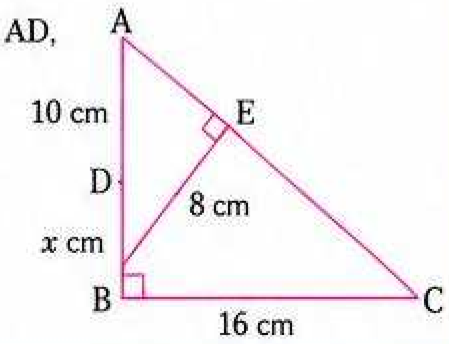



4. In $\triangle ABD$ right-angled at $B$, $BE \perp AD$, $AB = 10$ cm, $BE = 8$ cm. Find the value of $x$.

   ........................

   ........................

   ........................



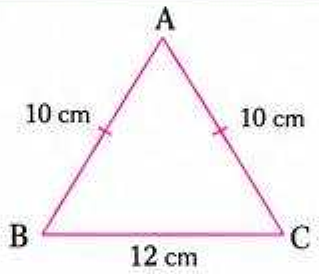

### 4 Answer the following:

1. In $\triangle ABC$, $AC = AB = 10$ cm, $BC = 12$ cm. Find $\cos\frac{A}{2}$, $m(\angle B)$.

   ........................

   ........................

   ........................




2. Find the equation of the line passing through the points $A(-3, 5)$ and $B(-1, 4)$.

   ........................

   ........................

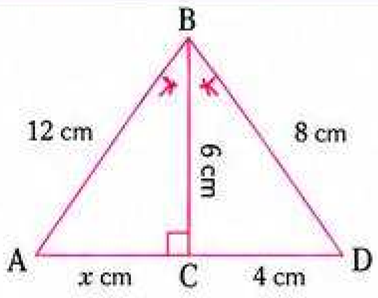
   ........................

3. In the figure: If $\overrightarrow{BC}$ bisects $\angle ABD$, prove that $\triangle ABC \sim \triangle DBC$ and find the value of $x$.

   ........................

   ........................

   ........................




4. In $\triangle ABD$ right-angled at $B$, $BE \perp AD$, $AB = 10$ cm, $BE = 8$ cm. Find the value of $x$.

   ........................

   ........................

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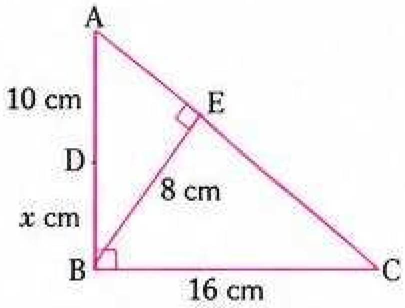


### 5 Coordinates and Midpoint:

1) Find the midpoint of the segment joining $P(-2, 4)$ and $P'(-3, 6)$.

   ........................

2) If $B(2, 4)$ is the image of $A$ by a dilation centered at $O(0, 0)$ with scale factor 2, find the coordinates of $A$.

   ........................

### 6 Mixed Problems:

1) In triangle ABC, $AB = AC = 10$ cm, $BC = 12$ cm. Find $m(\angle B)$. ........................
2) Find the slope of the line passing through $(1, -2)$ and $(5, 6)$.

   ........................

3) In right triangle ABC right-angled at $B$, if $AB = 6$ cm, $BC = 8$ cm, find $\tan A$. ........................



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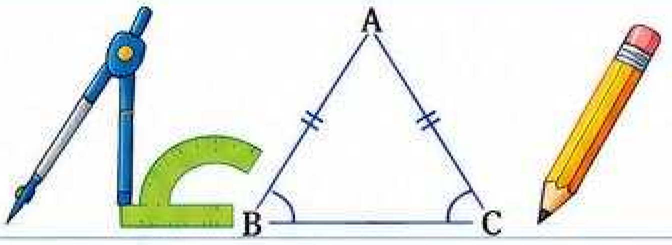
# Unit 3 Geometry Assessment




## Exercises

### 1 Choose the correct answer:

1) If the coordinates of $A$ are $(3, -2)$ and $B$ are $(-1, 4)$, then the midpoint of $\overline{AB}$ is:

a) $(1, 1)$ b) $(1, -1)$ c) $(2, 1)$ d) $(-2, 2)$

2) If $BC$ bisects $\angle ABD$, then $\angle ABC = \frac{1}{2}\angle ABD$.
If $\angle ABD = 70°$, then $\angle ABC = ?$

a) $20°$ b) $30°$ c) $35°$ d) $40°$

3) The slope of the line $3x - 6y + 12 = 0$ is:

a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) $2$ d) $-2$

4) In right triangle $ABC$ right-angled at $B$, if $AB = 6$ cm and $BC = 8$ cm, then $AC = ?$

a) 10 cm b) 8 cm c) 6 cm d) 4 cm

### 2 Fill in the blanks:

1) In an equilateral triangle, each angle measures ________.

2) The slope of a horizontal line is ________.

3) The slope of a vertical line is ________.

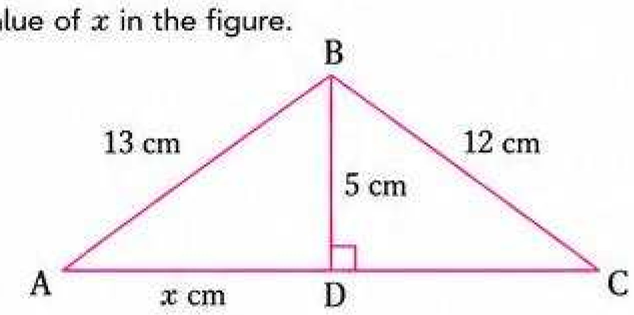
4) In $\triangle ABC$ right-angled at $B$, if $AB = 9$ cm and $AC = 15$ cm, then $BC =$ ________ cm.

5) The equation of a line passing through $(x_1, y_1)$ with slope $m$ is ________.

6) If the midpoint of $\overline{PQ}$ is $(4, -1)$ and $P(2, 1)$, then the coordinates of $Q$ are ________.

### 3 Solve the following:

1) Find the value of $x$ in the figure.




$x =$ ..............................

2) Find the equation of the line passing through $A(2, -1)$ and $B(4, 5)$.

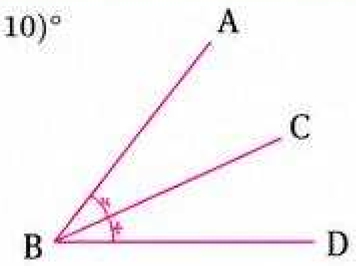
..............................

..............................

..............................

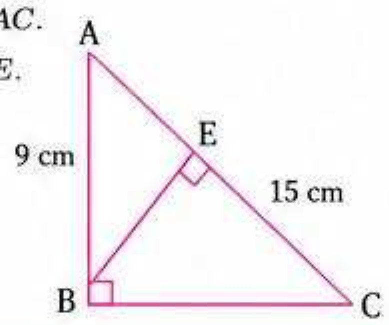
..............................

3) If $BC$ bisects $\angle ABD$, $\angle ABC = (2x + 10)°$ and $\angle DBC = (3x - 5)°$.
Find the value of $x$.




$x =$ ..............................

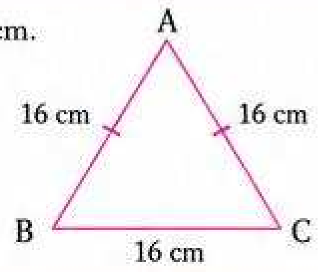
4) In $\triangle ABC$ right-angled at $B$, $BE \perp AC$.
If $AB = 9$ cm, $AC = 15$ cm, find $BE$.




$BE =$ ..............................

### 4 Answer the following:

1) In equilateral $\triangle ABC$, $AB = BC = CA = 16$ cm.
Find $m(\angle A)$ and $m(\angle B)$.




$m(\angle A) =$ ..............................

$m(\angle B) =$ ..............................

2) Find the equation of the line parallel to the line $2x - 3y + 5 = 0$ and passing through $(1, -2)$.

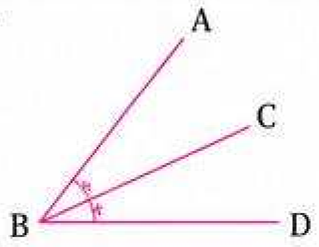
..............................

..............................

..............................

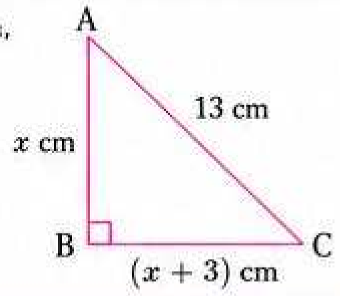
..............................

3) If $BC$ bisects $\angle ABD$, $\angle ABC = (x + 15)°$ and $\angle DBC = (4x - 5)°$.
Find the value of $x$.




$x =$ ..............................

4) In $\triangle ABC$ right-angled at $B$, if $AB = x$ cm, $BC = (x + 3)$ cm and $AC = 13$ cm.
Find the value of $x$.




$x =$ ..............................

### 5 Coordinates:

1) Find the midpoint of the segment joining $P(-4, 6)$ and $Q(2, -2)$.

..............................

2) Find the coordinates of the point that divides the segment joining $A(2, 6)$ and $B(8, 0)$ in the ratio $1 : 3$ (from $A$ to $B$).

..............................

### 6 Geometry and Trigonometry:

1) In right triangle $ABC$ right-angled at $B$, $AB = 5$ cm and $BC = 12$ cm.
Find $\cos A$ and $\tan A$.

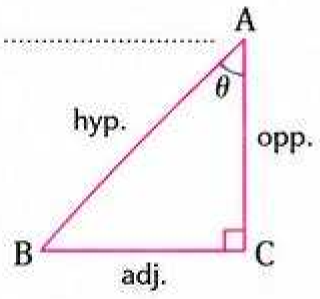
$\cos A =$ .............................. , $\tan A =$ ..............................

2) In right triangle $ABC$ right-angled at $C$, $\tan A = \frac{3}{4}$.
If $AC = 6$ cm, find $BC$.

$BC =$ ..............................

3) If $\sin\theta = \frac{3}{5}$ and $\theta$ is an acute angle, find $\cos\theta$.

$\cos\theta =$ ..............................



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# Unit 4 Statistics

## Lesson 1 Measures of Dispersion

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### In this lesson, you will learn:

- Define measures of dispersion.
- Find the Range of a data set.
- Interpret the meaning of the range.

### Warm-Up

Imagine two classes have the same average score. Which class has scores that are more spread out? Measures of dispersion help us answer this question.

### Common Mistakes

- Using the first and last numbers before ordering the data.
- Forgetting to subtract.
- Confusing Range with Mean.

## 1. Range (R)

The **Range** is the difference between the greatest value and the smallest value in a data set.

$$R = \text{Maximum} - \text{Minimum}$$

**Where:**

Maximum = largest value.
Minimum = smallest value.
$R$ = Range.

**Important Note** The range depends only on the largest and smallest values, so extreme values affect it greatly.

### Example 1

Find the range of the data set:
18, 22, 20, 31, 24, 27, 25

**Step 1:** Find the maximum.
Maximum = 31

**Step 2:** Find the minimum.
Minimum = 18

**Step 3:** Find the range.
$R = 31 - 18 = 13$

**Answer:** Range = 13

### Example 2

Use the stem-and-leaf plot below to find the range.

| Stem | Leaves |
|---|---|
| 1 | 2 5 8 |
| 2 | 1 4 7 9 |
| 3 | 0 2 |

Key: 2 | 4 means 24

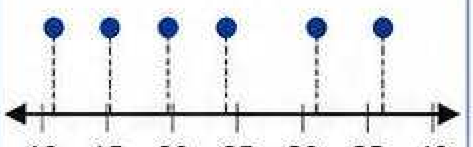
**Solution:**
Minimum = 12 (1 | 2)
Maximum = 32 (3 | 2)
**Range** = 32 − 12 = **20**

**Answer:** Range = 20

### Example 3

Find the range from the number line.

12, 16, 20, 24, 31, 36




**Solution:**
Minimum = 12
Maximum = 36
**Range** = 36 − 12 = **24**

**Answer:** Range = 24

**Tip** Always identify the maximum and minimum values carefully before finding the range.

### Practice Yourself

**1** Find the range of the data set:
14, 17, 23, 18, 29, 31

Answer: ____________

**2** Find the range of the data set:
9, 12, 15, 19, 21, 25, 27

Answer: ____________

**3** Find the range from the stem-and-leaf plot.

| Stem | Leaves |
|---|---|
| 0 | 3 7 9 |
| 1 | 0 2 8 |
| 2 | 1 4 9 |
| 3 | 2 |

Answer: ____________

**4** A student's quiz scores are:
18, 21, 25, 27, 30
Find the range.

Answer: ____________



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## 2. Interquartile Range (IQR)

The **Interquartile Range (IQR)** is the difference between the third quartile ($Q_3$) and the first quartile ($Q_1$).

$$IQR = Q_3 - Q_1$$

$Q_1$ is the median of the lower half of the data.
$Q_3$ is the median of the upper half of the data.
IQR shows the spread of the middle 50% of the data and is less affected by extreme values.

### Example 4

Find the three quartiles and the IQR for the data set:

6, 10, 12, 15, 18, 23, 24, 26, 27

**Step 1:** Order the data (already ordered).

**Step 2:** Find the median ($Q_2$).
There are 9 values, so the median is the 5th value → $Q_2 = 18$

**Step 3:** Lower half: 6, 10, 12, 15 → $Q_1$ = median = $(10 + 12)/2 = 11$

**Step 4:** Upper half: 23, 24, 26, 27 → $Q_3$ = median = $(24 + 26)/2 = 25$

**Step 5:** $IQR = Q_3 - Q_1 = 25 - 11 = 14$

| Lower half |
|---|
| 6, 10, 12, 15 |
| **Upper half** |
| 23, 24, 26, 27 |

**Answer:** $Q_1 = 11$, $Q_2 = 18$, $Q_3 = 25$, $IQR = 14$

### Example 5

Find the IQR for the data using the stem-and-leaf plot.

| Stem | Leaves |
|---|---|
| 1 | 2 4 8 |
| 2 | 1 3 5 7 9 |
| 3 | 0 2 6 8 |

**Step 1:** Write the data.
12, 14, 18, 21, 23, 25, 27, 29, 30, 32, 36, 38

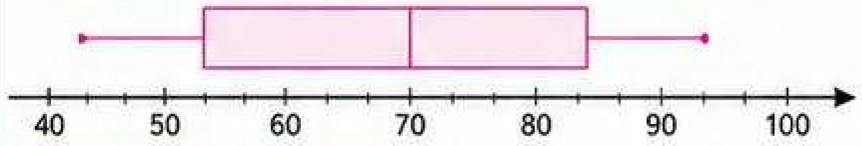
**Step 2:** There are 12 values.

- $Q_2$ = median = $(25 + 27)/2 = 26$
- Lower half: 12, 14, 18, 21, 23, 25 → $Q_1 = (18 + 21)/2 = 19.5$
- Upper half: 27, 29, 30, 32, 36, 38 → $Q_3 = (30 + 32)/2 = 31$
- $IQR = Q_3 - Q_1 = 31 - 19.5 = 11.5$

**Answer:** $Q_1 = 19.5$, $Q_2 = 26$, $Q_3 = 31$, $IQR = 11.5$

### Example 6

The box plot below shows the scores of a class. Find the IQR.




From the box plot:
$Q_1 = 55$  $Q_3 = 80$
$IQR = Q_3 - Q_1 = 80 - 55 = 25$

**Answer:** $IQR = 25$

### Summary

- Range (R) = Maximum − Minimum
- $IQR = Q_3 - Q_1$
- Range uses only two values and is affected by extremes.
- IQR describes the spread of the middle 50% of the data and is more reliable.

### Key Point

Use Range for a quick idea of spread.
Use IQR for a better idea of spread when there are outliers.

## Practice Yourself

**1** Find the range of each data set.

a) 11, 16, 19, 23, 27

Answer: ____________

b) 5, 8, 12, 15, 22, 25

Answer: ____________

c) 30, 34, 37, 41, 43, 47, 50

Answer: ____________

**2** Find the IQR for each data set.

a) 7, 9, 12, 15, 18, 21, 24

Answer: ____________

b) 2, 4, 6, 8, 10, 12, 14, 16

Answer: ____________

**3** Use the stem-and-leaf plot below to find the IQR.

| Stem | Leaves |
|---|---|
| 0 | 8 9 |
| 1 | 2 4 6 8 |
| 2 | 1 3 7 9 |
| 3 | 0 2 5 |

Answer: ____________

**4** The test scores of 10 students are:
55, 60, 62, 68, 70, 72, 75, 78, 82, 90

Find:

a) The range.

Answer: ____________

b) The IQR.

Answer: ____________



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## 3. Mean Deviation (MD)

The **Mean Deviation** (MD) from the mean ($\overline{x}$) is the average of the absolute deviations of the data values from the mean.

$$MD = \frac{\sum |x - \overline{x}|}{n}$$

Where:

$\overline{x}$ = Mean of the data.
$x$ = Each value of the data.
$|x - \overline{x}|$ = Absolute deviation of $x$ from the mean.
n = Number of data values.

**Note:** Absolute deviation means we ignore the sign (use positive values only).

### Example 7

Find the mean deviation from the mean for the data:

12, 15, 18, 15, 12

**Step 1:** Find the mean.

$$\overline{x} = \frac{12 + 15 + 18 + 15 + 12}{5} = \frac{72}{5} = 14.4$$

**Step 2:** Find $|x - \overline{x}|$ for each value.
**Step 3:** Add the absolute deviations.
**Step 4:** Divide by the number of values.

| $x$ | $x - \overline{x}$ | $\lvert x - \overline{x}\rvert$ |
|---|---|---|
| 12 | 12 – 14.4 = –2.4 | 2.4 |
| 15 | 15 – 14.4 = 0.6 | 0.6 |
| 18 | 18 – 14.4 = 3.6 | 3.6 |
| 15 | 15 – 14.4 = 0.6 | 0.6 |
| 12 | 12 – 14.4 = –2.4 | 2.4 |
| Total | | 9.6 |

$$MD = \frac{9.6}{5} = 1.92$$

**Answer:** Mean Deviation = 1.92

### Example 8

Find the mean deviation from the mean for the data:

8, 10, 12, 14, 16, 20

**Step 1:** Find the mean.

$$\overline{x} = \frac{8 + 10 + 12 + 14 + 16 + 20}{6} = \frac{80}{6} \approx 13.33$$

**Step 2:** Find $|x - \overline{x}|$ for each value.
**Step 3:** Add the absolute deviations.
**Step 4:** Divide by the number of values.

| $x$ | $x - \overline{x}$ | $\lvert x - \overline{x}\rvert$ |
|---|---|---|
| 8 | 8 – 13.33 = –5.33 | 5.33 |
| 10 | 10 – 13.33 = –3.33 | 3.33 |
| 12 | 12 – 13.33 = –1.33 | 1.33 |
| 14 | 14 – 13.33 = 0.67 | 0.67 |
| 16 | 16 – 13.33 = 2.67 | 2.67 |
| 20 | 20 – 13.33 = 6.67 | 6.67 |
| Total | | 19.33 |

$$MD = \frac{19.33}{6} \approx 3.22$$

**Answer:** Mean Deviation ≈ 3.22

### Example 9

Use the frequency table to find the mean deviation from the mean.

| Value ($x$) | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| Frequency ($f$) | 2 | 3 | 4 | 3 | 2 |

**Step 1:** $\overline{x} = \frac{\sum f \cdot x}{\sum f} = \frac{(5\times2)+(10\times3)+(15\times4)+(20\times3)+(25\times2)}{2+3+4+3+2}$

$= \frac{180}{14} \approx 12.86$

**Step 2:** Find $|x - \overline{x}|$ for each value.
**Step 3:** Find $f \cdot |x - \overline{x}|$ and add.
**Step 4:** $MD = \frac{\sum f|x-\overline{x}|}{\sum f} = \frac{144}{14} \approx 10.29$

**Answer:** Mean Deviation ≈ 10.29

## Practice Yourself

**1** Find the mean deviation from the mean for the data.

a) 6, 8, 10, 12, 14

Answer: ________

b) 20, 22, 24, 26, 28, 30

Answer: ________

**2** Find the mean deviation from the mean for the data.

a) 3, 5, 7, 9, 11

Answer: ________

b) 2, 4, 6, 8, 10, 12, 14

Answer: ________

**3** Use the frequency table below to find the mean deviation from the mean.

| Value ($x$) | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|
| Frequency ($f$) | 1 | 2 | 3 | 2 | 1 |

Answer: ________

**4** The weekly rainfall (in mm) in a city for 6 days is:

2, 4, 5, 7, 8, 10

Find the mean deviation from the mean.

Answer: ________

**5** A group of students scored:

45, 50, 55, 60, 65, 70, 75

Find the mean deviation from the mean.

Answer: ________

**Think!**

Which data set has greater spread?

A: 10, 12, 14, 16, 18
B: 2, 8, 14, 20, 26



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# Lesson 1 Measures of Dispersion



## 4. Variance ($\sigma^2$) and Standard Deviation ($\sigma$)

**Variance** and **Standard Deviation** measure how spread out the data values are from the mean.

**Variance ($\sigma^2$)**

$$\sigma^2 = \frac{\sum(x-\overline{x})^2}{n}$$

**Standard Deviation ($\sigma$)**

$$\sigma = \sqrt{\frac{\sum(x-\overline{x})^2}{n}}$$

**Where:**

- $\overline{x}$ = Mean of the data.
- $x$ = Each value of the data.
- $x-\overline{x}$ = Deviation of $x$ from the mean.
- $(x-\overline{x})^2$ = Squared deviation.
- n = Number of data values.

**Note:** When the data are far from the mean, variance and standard deviation become larger.

### Example 10

Find the variance and the standard deviation for the data:

4, 7, 10, 13, 16

**Step 1:** Find the mean.

$$\overline{x} = \frac{4+7+10+13+16}{5} = \frac{50}{5} = 10$$

**Step 2:** Find $x-\overline{x}$ for each value.

**Step 3:** Square each deviation.

**Step 4:** Add the squared deviations.

**Step 5:** Divide by n to get the variance.

**Step 6:** Take the square root to get the standard deviation.

| $x$ | $x-\overline{x}$ | $(x-\overline{x})^2$ |
|---|---|---|
| 4 | 4 − 10 = −6 | 36 |
| 7 | 7 − 10 = −3 | 9 |
| 10 | 10 − 10 = 0 | 0 |
| 13 | 13 − 10 = 3 | 9 |
| 16 | 16 − 10 = 6 | 36 |
| | Total | 90 |

Variance: $\sigma^2 = \frac{90}{5} = 18$

Standard Deviation: $\sigma = \sqrt{18} \approx 4.24$

**Answer:** Variance = 18, Standard Deviation ≈ 4.24

### Example 11

Find the variance and the standard deviation for the data:

2, 4, 6, 8, 10, 12

**Step 1:** Find the mean.

$$\overline{x} = \frac{2+4+6+8+10+12}{6} = \frac{42}{6} = 7$$

**Step 2:** Find $x-\overline{x}$ for each value.

**Step 3:** Square each deviation.

**Step 4:** Add the squared deviations.

**Step 5:** Divide by n to get the variance.

**Step 6:** Take the square root to get the standard deviation.

| $x$ | $x-\overline{x}$ | $(x-\overline{x})^2$ |
|---|---|---|
| 2 | 2 − 7 = −5 | 25 |
| 4 | 4 − 7 = −3 | 9 |
| 6 | 6 − 7 = −1 | 1 |
| 8 | 8 − 7 = 1 | 1 |
| 10 | 10 − 7 = 3 | 9 |
| 12 | 12 − 7 = 5 | 25 |
| | Total | 70 |

Variance: $\sigma^2 = \frac{70}{6} \approx 11.67$

Standard Deviation: $\sigma = \sqrt{11.67} \approx 3.42$

**Answer:** Variance ≈ 11.67, Standard Deviation ≈ 3.42

### Key Point

- Range uses only the minimum and maximum values.
- IQR uses the middle 50% of the data.
- Mean Deviation uses the average of absolute deviations.
- Variance and Standard Deviation use squared deviations and are more reliable for analysis.

| Measure | What it measures |
|---|---|
| Range (R) | Largest – Smallest |
| IQR | Spread of the middle 50% |
| MD | Average of absolute deviations |
| Variance ($\sigma^2$) | Average of squared deviations |
| Standard Deviation ($\sigma$) | Square root of variance |

## Practice Yourself

**1** Find the variance and the standard deviation for the data.

6, 9, 12, 15, 18

Answer:

Variance = ___________

Standard Deviation = _________

**2** Find the variance and the standard deviation for the data.

3, 6, 9, 12, 15

Answer:

Variance = ___________

Standard Deviation = _________

**3** Find the variance and the standard deviation for the data.

5, 7, 9, 11, 13, 15

Answer:

Variance = ___________

Standard Deviation = _________

**4** Find the variance and the standard deviation for the data.

1, 3, 5, 7

Answer:

Variance = ___________

Standard Deviation = _________

**5** A test scores of 6 students are:

45, 50, 55, 60, 65, 70

Find the variance and the standard deviation.

Answer:

Variance = ___________

Standard Deviation = _________

### Challenge!

Two classes have the following test scores:

Class A: 60, 62, 64, 66, 68

Class B: 40, 50, 60, 70, 80

Which class has a greater spread? Justify your answer using the range and the standard deviation.

Answer: ______________________________

______________________________



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# Lesson 1 Measures of Dispersion

## Summary of Measures of Dispersion

| Measure | Formula | What it tells us | When to use |
|---|---|---|---|
| Range (R) | R = Maximum – Minimum | Overall spread based on extremes only. | Quick idea of spread. |
| Interquartile Range (IQR) | $IQR = Q_3 - Q_1$ | Spread of the middle 50%. | Less affected by extreme values. |
| Mean Deviation (MD) | $MD = \frac{\sum \lvert x - \bar{x} \rvert}{n}$ | Average of absolute deviations from the mean. | Good for comparing variability. |
| Variance ($\sigma^2$) | $\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$ | Average of squared deviations from the mean. | More reliable for further analysis. |
| Standard Deviation ($\sigma$) | $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$ | Square root of variance. In same units as data. | Most commonly used measure of spread. |

**Remember:** The larger the value of a measure of dispersion, the more spread out the data.

## Practice More

**1** Find each of the following for the data:

9, 12, 15, 18, 21

a) Range
b) IQR
c) Mean Deviation from the mean
d) Variance
e) Standard Deviation

Answer:
a) ________ b) ________
c) ________ d) ________
e) ________

**2** The following stem-and-leaf plot represents the ages (in years) of 15 people.

| Stem | Leaves |
|---|---|
| 2 | 1 3 4 7 |
| 3 | 0 2 5 6 8 |
| 4 | 1 4 5 |

Find:
a) Range b) IQR
c) Mean Deviation
d) Variance e) Standard Deviation

Answer:
a) ______ b) ______ c) ______
d) ______ e) ______

**3** The scores of 8 students in a test are:

55, 60, 65, 70, 75, 80, 85, 90

Find the variance and the standard deviation.

Answer:
Variance = ____________
Standard Deviation = ____________

**4** The daily temperatures (in °C) in a city for 7 days are:

18, 21, 20, 19, 22, 18, 20

Find:
a) Range b) IQR c) Mean Deviation from the mean
d) Variance e) Standard Deviation

Answer:
a) ________ b) ________ c) ________
d) ________ e) ________

**5** The following frequency table shows the number of hours 20 students spent studying in a week.

| Hours (x) | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| Frequency (f) | 2 | 3 | 6 | 5 | 4 |

Find the mean deviation from the mean.

Answer: ____________________

## Think & Solve

The heights (in cm) of students in two classes are as follows:

Class A: 145, 148, 150, 152, 155, 158, 160
Class B: 145, 150, 155, 160, 165, 170, 175

Which class has a greater spread? Justify your answer using range and standard deviation.

Answer: ____________________________________________

## Check Yourself

### 1 Choose the correct answer:

1. The range of the data 8, 12, 17, 21 is:
a) 13 b) 14 c) 15 d) 16

2. If $Q_1 = 12$ and $Q_3 = 24$, then IQR = ............
a) 12 b) 24 c) 36 d) 48

3. The mean deviation is the average of:
a) Squared deviations.
b) Absolute deviations.
c) Deviations without absolute.
d) None of the above.

4. If the variance of a data set is 81, then the standard deviation is:
a) 8 b) 9 c) 18 d) 81

5. Which of the following is least affected by extreme values?
a) Range b) IQR
c) Mean Deviation d) Variance

### 2 Fill in the blanks:

a) Range = Maximum – __________.
b) $IQR = Q_3 -$ __________.
c) $MD = \frac{\sum \lvert x - \bar{x} \rvert}{____}$
d) Variance $= \frac{\sum (x - \bar{x})^2}{____}$.
e) Standard Deviation $= \sqrt{____}$

### 3 True or False:

a) Range uses only the maximum and minimum values. ( )
b) IQR describes the spread of the middle 50% of the data. ( )
c) Mean deviation uses squared deviations. ( )
d) Standard deviation is the square root of the variance. ( )

## Quick Tips

- Always order the data before finding quartiles.
- Use absolute value in mean deviation.
- Squaring removes the effect of positive and negative signs.
- Standard deviation is in the same units as the original data.



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## 3. Shortcut Formulas for Frequency Distributions

If the data is arranged in groups with frequencies, we can use shortcut formulas to find the variance and standard deviation.

$$\sigma = \sqrt{\frac{\sum f\,(x_m - \bar{x})^2}{\sum f}}$$

where :
- $x_m$ = midpoint of the class
- $f$ = frequency of the class
- $\bar{x}$ = mean
- $\sum f$ = total frequency

$$\bar{x} = \frac{\sum (f \,.\, x_m)}{\sum f}$$

Mean for grouped data

### Key Notes

- These formulas save time and work.
- The midpoint ($x_m$) of each class is:

$$x_m = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

- Use midpoints with frequencies to compute mean, variance, and standard deviation.
- These methods are very useful for large data sets.

### Example 6 – Grouped Frequency Distribution (14 ages)

The frequency distribution of the ages of 14 people working in a restaurant is given below:

| Age (years) | 2 | 5 | 10 | 12 | 16 | 34 |
|---|---|---|---|---|---|---|
| Frequency ($f$) | 1 | 2 | 5 | 2 | 3 | 1 |

Find the variance ($\sigma^2$) and the standard deviation ($\sigma$).

**Solution:**

| $x$ | $f$ | $x_m$ | $f \,.\, x_m$ | $x_m - \bar{x}$ | $(x_m - \bar{x})^2$ | $(x_m - \bar{x})^2$ |
|---|---|---|---|---|---|---|
| 2 | 1 | 2 | 2 | −10 | 100 | 100 |
| 5 | 2 | 5 | 10 | −7 | 49 | 98 |
| 10 | 5 | 10 | 50 | −2 | 4 | 20 |
| 12 | 2 | 12 | 24 | 0 | 0 | 0 |
| 16 | 3 | 16 | 48 | 4 | 16 | 48 |
| 34 | 1 | 34 | 34 | 22 | 484 | 484 |
| Total | 14 | | 168 | | | 750 |

$$\bar{x} = \frac{\sum (f \,.\, x_m)}{\sum f} = \frac{168}{14} = 12$$

$$\sigma = \sqrt{\frac{\sum f\,(x_m - \bar{x})^2}{\sum f}} = \sqrt{\frac{750}{14}} \approx 7.32$$

**Answer:**
Variance ($\sigma^2$) = $\frac{750}{14} \approx 53.57$ , Standard Deviation ($\sigma$) ≈ 7.32

### Example 7 – Grouped Data by Classes (50 people)

The following table shows the weights (in kg) of 50 people:

| Weight (kg) | 60 – | 70 – | 80 – | 90 – | 100 – |
|---|---|---|---|---|---|
| Frequency ($f$) | 5 | 11 | 18 | 9 | 7 |

Find the variance ($\sigma^2$) and the standard deviation ($\sigma$).

**Solution:**

| Class (kg) | Midpoint ($x_m$) | Frequency ($f$) | $f \,.\, x_m$ | $x_m - \bar{x}$ | $(x_m - \bar{x})^2$ | $f\,(x_m - \bar{x})^2$ |
|---|---|---|---|---|---|---|
| 60 – | 65 | 5 | 325 | −20.4 | 416.16 | 2080.80 |
| 70 – | 75 | 11 | 825 | −10.4 | 108.16 | 1189.76 |
| 80 – | 85 | 18 | 1530 | −0.4 | 0.16 | 2.88 |
| 90 – | 95 | 9 | 855 | 9.6 | 92.16 | 829.44 |
| 100 – | 105 | 7 | 735 | 19.6 | 384.16 | 2689.12 |
| Total | | 50 | 4270 | | | 6792 |

$$\bar{x} = \frac{\sum (f \,.\, x_m)}{\sum f} = \frac{4270}{50} = 85.4$$

$$\sigma = \sqrt{\frac{\sum f\,(x_m - \bar{x})^2}{\sum f}} = \sqrt{\frac{6792}{50}} \approx 11.66$$

**Answer:**
Variance ($\sigma^2$) = $\frac{6792}{50} \approx 135.84$ , Standard Deviation ($\sigma$) ≈ 11.66

### Tips for Success

- Always find the mean first before calculating variance or standard deviation.
- Use midpoints ($x_m$) when data is grouped into classes.
- Organize your table carefully to avoid mistakes.
- Check your totals ($\sum f$ and $\sum f \,.\, x_m$) before final steps.

### Quick Check

1. If $\sum f\,(x_m - \bar{x})^2 = 900$ and $\sum f = 25$.
   Find $\sigma^2$ and $\sigma$.
   $\sigma^2$ = ________ , $\sigma$ = ________
2. In the grouped data table, what is the formula for the midpoint ($x_m$)?
   $x_m$ = ____________________



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## 4. Comparison of Measures of Dispersion

Each measure of dispersion gives us different information about how the data are spread.

| Measure | What it tells us | Based on | Sensitive to |
|---|---|---|---|
| Range (R) | Overall spread using only the smallest and largest values. | Maximum and Minimum | Very sensitive (affected by extreme values). |
| IQR ($Q_3 - Q_1$) | Spread of the middle 50% of the data. | Middle values ($Q_1$ and $Q_3$) | Less sensitive (ignores extremes). |
| Mean Deviation (MD) | Average of absolute deviations from the mean. | All values | Moderately sensitive. |
| Variance ($\sigma^2$) | Average of squared deviations from the mean. | All values (squared deviations) | Sensitive (penalizes large deviations). |
| Standard Deviation ($\sigma$) | Square root of variance; shows typical distance from the mean. | All values (same units as data) | Sensitive (penalizes large deviations). |

## Practice Yourself

**1** Find the range for each set of data.

a) 7, 11, 15, 20, 25
Answer: __________

b) 3, 9, 12, 18, 24, 30
Answer: __________

c) 14, 16, 19, 22, 28, 31, 35
Answer: __________

d) 2, 5, 8, 11, 14
Answer: __________

**2** Find the IQR ($Q_3 - Q_1$) for each set of data.

a) 4, 7, 9, 12, 16, 20, 22
Answer: __________

b) 1, 3, 5, 7, 9, 11, 13, 15
Answer: __________

**3** Find the mean deviation from the mean for each set.

a) 5, 8, 10, 13
Answer: __________

b) 2, 4, 6, 8, 10
Answer: __________

**4** Find the variance ($\sigma^2$) and standard deviation ($\sigma$) for:

a) 4, 9, 11, 15
$\sigma^2$ = __________ , $\sigma$ = __________

b) 6, 8, 12, 14, 16
$\sigma^2$ = __________ , $\sigma$ = __________

**5** The following frequency table shows the heights (in cm) of 20 students.

| Height (cm) | 140 – 150 | 150 – 160 | 160 – 170 | 170 – 180 | 180 – 190 |
|---|---|---|---|---|---|
| Frequency ($f$) | 2 | 4 | 6 | 5 | 3 |

Find the mean, variance ($\sigma^2$), and standard deviation ($\sigma$) using shortcut formulas for grouped data.

Answer:
Mean ($\bar{x}$) = __________
Variance ($\sigma^2$) = __________ , Standard Deviation ($\sigma$) = __________

**6** The weekly number of hours spent studying by a student over 10 weeks is:

3, 4, 5, 6, 7, 5, 4, 6, 8, 7

Find: (a) Range (b) IQR (c) Mean Deviation from the mean (d) Variance (e) Standard Deviation

Answer:
a) Range = __________
b) IQR = __________
c) MD = __________
d) $\sigma^2$ = __________
e) $\sigma$ = __________

**7** The test scores of 6 students are:

42, 48, 55, 60, 62, 70

Find the variance and standard deviation.

Answer:
Variance ($\sigma^2$) = __________
Standard Deviation ($\sigma$) = __________

**8** Two classes took the same test.

Class A scores: 60, 62, 64, 66, 68
Class B scores: 40, 50, 60, 70, 80

Which class has greater dispersion? Justify your answer using range and standard deviation.

Answer: __________

## Homework

**1** Find the range for:
a) 13, 16, 22, 28, 33
b) 5, 9, 14, 19, 24, 29
c) 2, 4, 6, 8, 10, 12
d) 7, 11, 15, 19, 23, 27, 31

**2** Find the IQR for:
a) 6, 8, 10, 12, 14, 16, 18
b) 1, 2, 3, 4, 5, 6, 7, 8, 9
c) 3, 5, 8, 10, 12, 15
d) 10, 12, 14, 16, 18, 20

**3** Find the mean deviation from the mean for:
a) 4, 6, 8, 10
b) 3, 5, 7, 9, 11
c) 2, 4, 6, 8, 10, 12

**4** Find the variance ($\sigma^2$) and standard deviation ($\sigma$) for:
a) 2, 4, 6, 8
b) 5, 7, 9, 11, 13
c) 1, 3, 5, 7, 9, 11

**5** The following table shows the number of books read by 15 students in a month.

| Books | 0–2 | 2–4 | 4–6 | 6–8 | 8–10 |
|---|---|---|---|---|---|
| Frequency | 1 | 3 | 5 | 4 | 2 |

Find the mean, variance, and standard deviation.



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## Exercises

**1** Find the range for each set of data.

a) 8, 14, 21, 7, 16

Answer: ____________

b) 3, 9, 12, 18, 24, 30

Answer: ____________

**2** Find the $IQR$ $(Q_3 - Q_1)$ for each set of data.

a) 4, 7, 9, 12, 16, 20, 22

Answer: ____________

b) 1, 3, 5, 7, 9, 11, 13, 15

Answer: ____________

**3** Find the mean deviation from the mean for each set.

a) 5, 8, 10, 13

Answer: ____________

b) 2, 4, 6, 8, 10

Answer: ____________

**4** Find the variance $(\sigma^2)$ and standard deviation $(\sigma)$ for each set.

a) 4, 9, 11, 15

$\sigma^2$ = ____________, $\sigma$ = ____________

b) 6, 8, 12, 14, 16

$\sigma^2$ = ____________, $\sigma$ = ____________

**5** The following frequency table shows the heights (in cm) of 20 students.

| Height (cm) | 140 – 150 | 150 – 160 | 160 – 170 | 170 – 180 | 180 – 190 |
|---|---|---|---|---|---|
| Frequency (f) | 2 | 4 | 6 | 5 | 3 |

Find the mean, variance $(\sigma^2)$, and standard deviation $(\sigma)$ using shortcut formulas for grouped data.

Mean $(\bar{x})$ = ____________

Variance $(\sigma^2)$ = ____________

Standard Deviation $(\sigma)$ = ____________

**6** The weekly number of hours spent studying by a student over 10 weeks is:

3, 4, 5, 6, 7, 5, 4, 6, 8, 7

Find: (a) Range (b) IQR (c) Mean Deviation from the mean (d) Variance (e) Standard Deviation

a) Range = ____________ b) IQR = ____________

c) MD = ____________ d) $\sigma^2$ = ____________

e) $\sigma$ = ____________

**7** The test scores of 6 students are:

42, 48, 55, 60, 62, 70

Find the variance and standard deviation.

Variance $(\sigma^2)$ = ____________

Standard Deviation $(\sigma)$ = ____________

**8** Two classes took the same test.

Class A scores: 60, 62, 64, 66, 68

Class B scores: 40, 50, 60, 70, 80

Which class has greater dispersion? Justify your answer using range and standard deviation.

Answer: ____________

**9** Find the range, $IQR$, mean deviation from the mean, variance and standard deviation for the data:

6, 9, 12, 15, 18

Range = ____________

IQR = ____________

MD = ____________

$\sigma^2$ = ____________

$\sigma$ = ____________

**10** Find the mean, variance and standard deviation for the frequency distribution.

| Value (x) | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| Frequency (f) | 2 | 3 | 4 | 5 | 6 |

Mean $(\bar{x})$ = ____________

Variance $(\sigma^2)$ = ____________

Standard Deviation $(\sigma)$ = ____________

**11** If the mean of a data set is 12 and the variance is 18, find the standard deviation.

Answer: ____________

**12** If the standard deviation of a data set is 7, find the variance.

Answer: ____________

**13** Complete the table.

| Data Set | Range | IQR | MD (from mean) | $\sigma^2$ | $\sigma$ |
|---|---|---|---|---|---|
| a) 2, 6, 8, 10, 14 | | | | | |
| b) 1, 3, 5, 7, 9, 11 | | | | | |

**14** The ages (in years) of 12 students are:

13, 14, 15, 16, 16, 17, 18, 18, 19, 20, 21, 22

Find: (a) Range (b) IQR (c) Mean deviation from the mean (d) Variance (e) Standard deviation

a) Range = ____________ b) IQR = ____________

c) MD = ____________ d) $\sigma^2$ = ____________

e) $\sigma$ = ____________



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# Lesson 1 Measures of Dispersion

## Exercises

**1** Find the range for each set of data.

a) 8, 14, 21, 7, 16

Answer: ________

b) 3, 9, 12, 18, 24, 30

Answer: ________

**2** Find the $IQR$ $(Q_3 - Q_1)$ for each set of data.

a) 4, 7, 9, 12, 16, 20, 22

Answer: ________

b) 1, 3, 5, 7, 9, 11, 13, 15

Answer: ________

**3** Find the mean deviation from the mean for each set.

a) 5, 8, 10, 13

Answer: ________

b) 2, 4, 6, 8, 10

Answer: ________

**4** Find the variance ($\sigma^2$) and standard deviation ($\sigma$) for each set.

a) 4, 9, 11, 15

$\sigma^2$ = ________, $\sigma$ = ________

b) 6, 8, 12, 14, 16

$\sigma^2$ = ________, $\sigma$ = ________

**5** The following frequency table shows the heights (in cm) of 20 students.

| Height (cm) | 140 – 150 | 150 – 160 | 160 – 170 | 170 – 180 | 180 – 190 |
|---|---|---|---|---|---|
| Frequency (f) | 2 | 4 | 6 | 5 | 3 |

Find the mean, variance ($\sigma^2$), and standard deviation ($\sigma$) using shortcut formulas for grouped data.

Mean ($\bar{x}$) = ________

Variance ($\sigma^2$) = ________

Standard Deviation ($\sigma$) = ________

**6** The weekly number of hours spent studying by a student over 10 weeks is:

3, 4, 5, 6, 7, 5, 4, 6, 8, 7

Find: (a) Range (b) IQR (c) Mean Deviation from the mean (d) Variance (e) Standard Deviation

a) Range = ________ b) IQR = ________

c) MD = ________ d) $\sigma^2$ = ________

e) $\sigma$ = ________

**7** The test scores of 6 students are:

42, 48, 55, 60, 62, 70

Find the variance and standard deviation.

Variance ($\sigma^2$) = ________

Standard Deviation ($\sigma$) = ________

**8** Two classes took the same test.

Class A scores: 60, 62, 64, 66, 68

Class B scores: 40, 50, 60, 70, 80

Which class has greater dispersion? Justify your answer using range and standard deviation.

Answer: ________________

________________

**9** Find the range, $IQR$, mean deviation from the mean, variance and standard deviation for the data:

6, 9, 12, 15, 18

Range = ________

IQR = ________

MD = ________

$\sigma^2$ = ________

$\sigma$ = ________

**10** Find the mean, variance and standard deviation for the frequency distribution.

| Value (x) | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| Frequency (f) | 2 | 3 | 4 | 5 | 6 |

Mean ($\bar{x}$) = ________

Variance ($\sigma^2$) = ________

Standard Deviation ($\sigma$) = ________

**11** If the mean of a data set is 12 and the variance is 18, find the standard deviation.

Answer: ________

**12** If the standard deviation of a data set is 7, find the variance.

Answer: ________

**13** Complete the table.

| Data Set | Range | IQR | MD (from mean) | $\sigma^2$ | $\sigma$ |
|---|---|---|---|---|---|
| a) 2, 6, 8, 10, 14 | | | | | |
| b) 1, 3, 5, 7, 9, 11 | | | | | |

**14** The ages (in years) of 12 students are:

13, 14, 15, 16, 16, 17, 18, 18, 19, 20, 21, 22

Find: (a) Range (b) IQR (c) Mean deviation from the mean (d) Variance (e) Standard deviation

a) Range = ________ b) IQR = ________

c) MD = ________ d) $\sigma^2$ = ________

e) $\sigma$ = ________



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## Exercises

**1** Find the range for each set of data.

a) 12, 18, 25, 7, 30

Answer: ________

b) 4, 9, 15, 20, 26, 31

Answer: ________

**2** Find the $IQR$ $(Q_3 - Q_1)$ for each set of data.

a) 6, 7, 10, 12, 14, 18, 20

Answer: ________

b) 2, 4, 6, 8, 10, 12, 14, 16

Answer: ________

**3** Find the mean deviation from the mean for each set.

a) 3, 6, 9, 12

Answer: ________

b) 4, 5, 7, 8, 10

Answer: ________

**4** Find the variance $(\sigma^2)$ and standard deviation $(\sigma)$ for each set.

a) 3, 6, 9, 12

$\sigma^2$ = ________, $\sigma$ = ________

b) 7, 11, 13, 17, 19

$\sigma^2$ = ________, $\sigma$ = ________

**5** The following frequency table shows the marks (out of 100) of 20 students in a math test.

| Marks | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 | 90 – 100 |
|---|---|---|---|---|---|---|
| Frequency (f) | 1 | 3 | 6 | 5 | 3 | 2 |

Find the mean, variance $(\sigma^2)$, and standard deviation $(\sigma)$ using shortcut formulas for grouped data.

Mean $(\bar{x})$ = ________

Variance $(\sigma^2)$ = ________

Standard Deviation $(\sigma)$ = ________

**6** The following table shows the number of goals scored by a football team in 12 matches.

| Goals | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency (f) | 1 | 2 | 4 | 3 | 1 | 1 |

Find: (a) Range (b) IQR (c) Mean Deviation from the mean (d) Variance (e) Standard Deviation

a) Range = ________ b) IQR = ________

c) MD = ________ d) $\sigma^2$ = ________

e) $\sigma$ = ________

**7** The marks of 7 students in a quiz are:

38, 42, 45, 49, 50, 53, 58

Find the variance and standard deviation.

Variance $(\sigma^2)$ = ________

Standard Deviation $(\sigma)$ = ________

**8** Two classes took the same science test.

Class A scores: 55, 60, 62, 64, 66, 68, 70, 72

Class B scores: 59, 59, 60, 61, 61, 62, 62, 63

Which class has greater dispersion? Justify your answer using range and standard deviation.

Answer: ________

________

**9** Find the range, $IQR$, mean deviation from the mean, variance and standard deviation for the data:

9, 12, 13, 15, 18, 20

Range = ________

IQR = ________

MD = ________

$\sigma^2$ = ________

$\sigma$ = ________

**10** Find the mean, variance and standard deviation for the frequency distribution.

| Value (x) | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| Frequency (f) | 3 | 5 | 4 | 2 | 1 |

Mean $(\bar{x})$ = ________

Variance $(\sigma^2)$ = ________

Standard Deviation $(\sigma)$ = ________

**11** If the mean of a data set is 25 and the variance is 49, find the standard deviation.

Answer: ________

**12** If the standard deviation of a data set is 9, find the variance.

Answer: ________

**13** Complete the table.

| Data Set | Range | IQR | MD (from mean) | $\sigma^2$ | $\sigma$ |
|---|---|---|---|---|---|
| a) 5, 9, 12, 16, 21 | | | | | |
| b) 2, 4, 6, 8, 10, 12, 14 | | | | | |

**14** The weights (in kg) of 10 students are:

45, 50, 52, 54, 55, 57, 60, 62, 64, 68

Find: (a) Range (b) IQR (c) Mean deviation from the mean (d) Variance (e) Standard deviation

a) Range = ________ b) IQR = ________

c) MD = ________ d) $\sigma^2$ = ________

e) $\sigma$ = ________



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# Lesson 1
## Measures of Dispersion

### Exercise

1 The following table shows the mass, in kilograms, of 50 persons.
Find the **standard deviation** of the data.

| Mass (kg) | 60 – 70 | 70 – 80 | 80 – 90 | 90 – 100 | 100 – 110 |
|---|---|---|---|---|---|
| Number of persons ($f$) | 5 | 11 | 18 | 9 | 7 |

### Solution

**Step 1:** Complete the table.

| Class (kg) | Frequency ($f$) | Midpoint ($x_m$) | Deviation ($x_m - \overline{x}$) | Squared deviation $(x_m - \overline{x})^2$ | $f \cdot x_m$ | $f \cdot (x_m - \overline{x})^2$ |
|---|---|---|---|---|---|---|
| 60 – 70 | | | | | | |
| 70 – 80 | | | | | | |
| 80 – 90 | | | | | | |
| 90 – 100 | | | | | | |
| 100 – 110 | | | | | | |
| **Total** | | | | | | |

**Remember:**

- Mean, $\overline{x} = \dfrac{\sum f\, x_m}{N}$
- Standard deviation, $\sigma = \sqrt{\dfrac{\sum f\,(x_m - \overline{x})^2}{N}}$

where $N$ is the total frequency.



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**2** The following table shows the scores of 60 students in a final test.
Find the **standard deviation** of the data.

| Scores | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 |
|---|---|---|---|---|---|
| Number of students ($f$) | 6 | 10 | 20 | 15 | 9 |

**3** The following table shows the time (in minutes) spent by 50 workers to complete a certain task.
Find the **standard deviation** of the data.

| Time (min) | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 |
|---|---|---|---|---|---|
| Number of workers ($f$) | 4 | 8 | 15 | 13 | 10 |

**4** The following table shows the heights (in cm) of 40 students.
Find the **standard deviation** of the data.

| Height (cm) | 140 – 145 | 145 – 150 | 150 – 155 | 155 – 160 | 160 – 165 |
|---|---|---|---|---|---|
| Number of students ($f$) | 3 | 7 | 12 | 11 | 7 |

## Space for Your Answers

| Class (Interval) | Frequency ($f$) | Midpoint ($x_m$) | Deviation ($x_m - \overline{x}$) | Squared deviation $(x_m - \overline{x})^2$ | $f \cdot x_m$ | $f \cdot (x_m - \overline{x})^2$ |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| Total | | | | | | |

**Remember:**

- Mean, $\overline{x} = \dfrac{\sum f\, x_m}{N}$
- Standard deviation, $\sigma = \sqrt{\dfrac{\sum f\,(x_m - \overline{x})^2}{N}}$

where $N$ is the total frequency.



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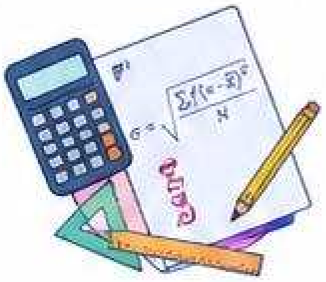
# Lesson 1
# Measures of Dispersion

## Exercise

**1** The following frequency distribution shows the ages of **14** people working in a restaurant.

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 16 | 12 | 10 | 5 | 2 | Age ($X$) |
| 1 | 3 | 2 | 5 | 2 | 1 | Frequency ($f$) |

Find the **standard deviation** of the ages of the employees.
Assume that the age ($X$) is the frequency ($f$).



## Solution

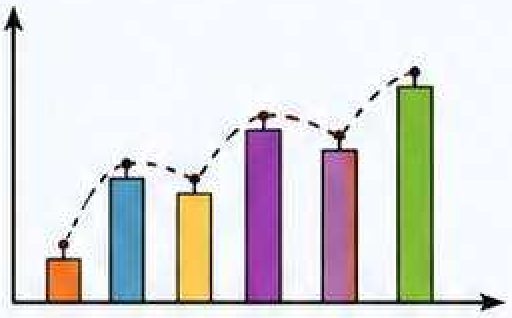
**Step 1:** Complete the following table.

| $X$ | $f$ | $f\,.\,X$ | $X-\overline{x}$ | $(X-\overline{x})^2$ | $f\,.\,(X-\overline{x})^2$ |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| **Total** | | | | | |

**Remember:**

- Mean, $\overline{x} = \dfrac{\sum f\,x}{N}$
- Standard deviation, $\sigma = \sqrt{\dfrac{\sum f(X-\overline{x})^2}{N}}$

where $N$ is the total frequency.





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# Lesson 1
## Measures of Dispersion

### Exercise

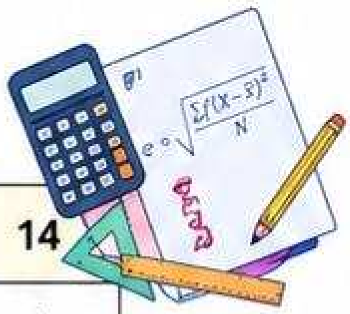
**2** The following table shows the daily sales (in dollars) of **14** days in a small shop.

Find the **standard deviation** of the daily sales.

| Sales ($X$) | 50 | 45 | 40 | 35 | 30 | 25 | 20 |
|---|---|---|---|---|---|---|---|
| Frequency ($f$) | 1 | 2 | 3 | 4 | 2 | 1 | 1 |

**3** The following table shows the number of hours that **14** students spent studying in a week.

Find the **standard deviation** of the data.



| Hours ($X$) | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|---|---|
| Frequency ($f$) | 1 | 3 | 2 | 5 | 2 | 1 | 1 |

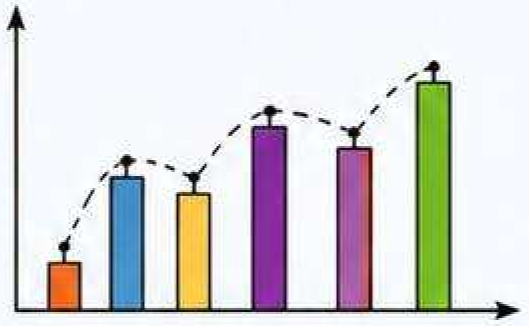
### Space for Your Answers

| $X$ | $f$ | $f \cdot X$ | $X - \overline{x}$ | $(X - \overline{x})^2$ | $f \cdot (X - \overline{x})^2$ |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| **Total** | | | | | |

**Remember:**

- Mean, $\overline{x} = \dfrac{\sum f x}{N}$
- Standard deviation, $\sigma = \sqrt{\dfrac{\sum f (X - \overline{x})^2}{N}}$

where $N$ is the total frequency.





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# Lesson 1
# Measures of Dispersion

## Exercise

For each of the following grouped data, find the **standard deviation**.

1. The following table shows the marks (out of 100) of 50 students in a test.

| Marks (out of 100) | 60 – 70 | 70 – 80 | 80 – 90 | 90 – 100 | 100 – 110 |
|---|---|---|---|---|---|
| Number of students | 5 | 11 | 18 | 9 | 7 |

2. The following table shows the weights (in kg) of 50 boxes in a warehouse.

| Weight (kg) | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 |
|---|---|---|---|---|---|
| Number of boxes | 4 | 8 | 15 | 14 | 9 |

3. The following table shows the daily sales (in dollars) of 50 days in a shop.

| Sales (dollars) | 100 – 200 | 200 – 300 | 300 – 400 | 400 – 500 | 500 – 600 |
|---|---|---|---|---|---|
| Number of days | 6 | 10 | 17 | 11 | 6 |

4. The following table shows the temperatures (in °C) recorded in a city over 50 days.

| Temperature (°C) | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 | 30 – 35 |
|---|---|---|---|---|---|
| Number of days | 7 | 9 | 16 | 11 | 7 |

5. The following table shows the time (in minutes) taken by 50 runners in a race.

| Time (min) | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 |
|---|---|---|---|---|---|
| Number of runners | 3 | 9 | 18 | 13 | 7 |

6. The following table shows the number of pages read by 50 students in a week.

| Number of pages | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| Number of students | 5 | 12 | 15 | 10 | 8 |

## Space for Your Answers

| Class (Interval) | Frequency ($f$) | Midpoint ($x_m$) | Deviation ($x_m - \bar{x}$) | Squared deviation $(x_m - \bar{x})^2$ | $f \cdot x_m$ | $f \cdot (x_m - \bar{x})^2$ |
|---|---|---|---|---|---|---|
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| | | | | | | |
| Total | | | | | | |



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# Lesson 1
# Measures of Dispersion

## Exercise

For each of the following grouped data, find the **standard deviation**.

1. The following table shows the heights (in cm) of 50 students in a school.

| Height (cm) | 120 – 130 | 130 – 140 | 140 – 150 | 150 – 160 | 160 – 170 |
|---|---|---|---|---|---|
| Number of students | 6 | 12 | 15 | 11 | 6 |

2. The following table shows the monthly expenses (in dollars) of 50 families.

| Expenses (dollars) | 100 – 200 | 200 – 300 | 300 – 400 | 400 – 500 | 500 – 600 |
|---|---|---|---|---|---|
| Number of families | 5 | 10 | 17 | 12 | 6 |

3. The following table shows the scores (out of 100) of 50 students in a final exam.

| Scores (out of 100) | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 |
|---|---|---|---|---|---|
| Number of students | 4 | 9 | 16 | 13 | 8 |

4. The following table shows the amount of rainfall (in mm) recorded in a city over 50 days.

| Rainfall (mm) | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| Number of days | 8 | 14 | 12 | 10 | 6 |

5. The following table shows the waiting time (in minutes) of 50 customers in a bank.

| Waiting time (min) | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| Number of customers | 7 | 13 | 15 | 9 | 6 |

6. The following table shows the number of goals scored by 50 players in a league.

| Number of goals | 0 – 2 | 2 – 4 | 4 – 6 | 6 – 8 | 8 – 10 |
|---|---|---|---|---|---|
| Number of players | 9 | 14 | 11 | 9 | 7 |

## Space for Your Answers

| Class (Interval) | Frequency ($f$) | Midpoint ($x_m$) | Deviation ($x_m - \bar{x}$) | Squared deviation $(x_m - \bar{x})^2$ | $f \cdot x_m$ | $f \cdot (x_m - \bar{x})^2$ |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| Total | | | | | | |



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# Unit 4 Assessment

**1** The table below shows the number of days each of 10 students exercised last week.

| Number of days | 5 | 4 | 3 | 2 | 5 |
|---|---|---|---|---|---|
| Number of students | 2 | 2 | 1 | 1 | 3 |

Find the mode and the median of these data.

..............................................................................................................

..............................................................................................................

**2** List the following data in ascending order:

21 , 16 , 2 , 3 , 5 , 22 , 15 , 17

..............................................................................................................

**3** From the following box plot, answer the questions.

(1) Find the minimum, maximum, median, and the upper quartile. ..............................

(2) Find the first quartile and the third quartile. ..............................

(3) Find the interquartile range. ..............................

(4) Find the range. ..............................

**4** Choose the correct answer from the given choices:

(1) From the stem–and–leaf plot, the median is:

| Stem | Leaves |
|---|---|
| 14 | 1 4 |
| 27 | 4 6 7 |
| 28 | 7 8 9 |
| 29 | 2 3 |

**Key:** 27 | 6 means 27.6

(a) 3.4 (b) 3.6 (c) 3.2 (d) 3.0

(2) If the data is $7 + k$ , $7 - k$ , $7 + 6$ , $k$ , $7 - 3k$ and the mean is 18, then k equals:

(a) 4 (b) 3 (c) 2 (d) 1

(3) The smallest number of correct answers if 3 is the mode is:

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{3}$

(4) Which of the following is an arithmetic mean?

(a) 2 , 6 , 20 , 18 (b) 5 , 5 , 5 , 5 , 5 (c) 3 , 4 , 5 , 6 (d) 8 , 9 , 7 , 10



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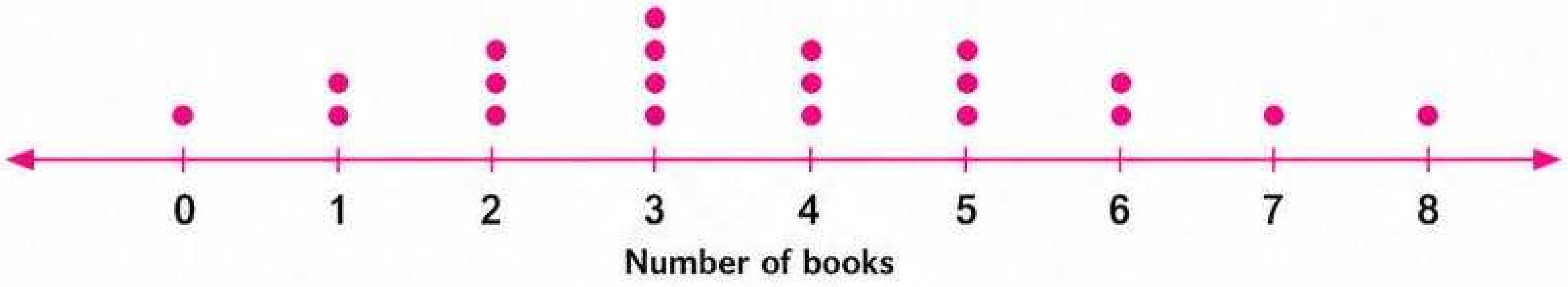
# Unit 4 Assessment

**5** Complete the following table using the listed data:

18 , 22 , 17 , 22 , 20 , 18 , 25 , 17 , 22 , 15 , 20

| Class interval | 15 – 17 | 18 – 20 | 21 – 23 | 24 – 26 |
|---|---|---|---|---|
| Tally marks | | | | |
| Frequency | | | | |

**6** The following dot plot shows the number of books read by each student in a class.

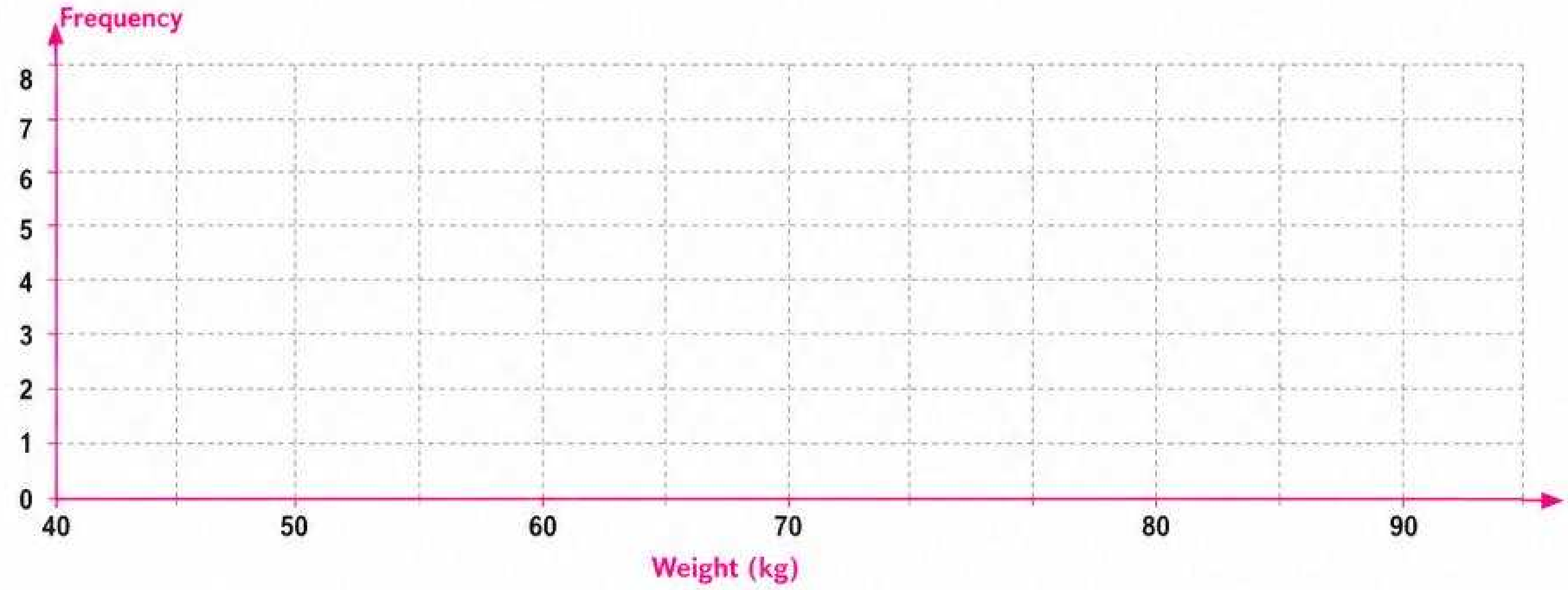



1. How many students are in the class? ..........................................
2. What is the mode of the data? ..........................................
3. What is the median of the data? ..........................................
4. What is the mean number of books read? ..........................................

**7** The following frequency table shows the weights (in kg) of 20 players.

| Weight (kg) | 40 – 49 | 50 – 59 | 60 – 69 | 70 – 79 | 80 – 89 |
|---|---|---|---|---|---|
| Frequency | 1 | 3 | 6 | 7 | 3 |

Draw a histogram to represent the data.




**8** Use the given data to draw a stem-and-leaf plot.

14 , 17 , 19 , 21 , 22 , 24 , 25 , 27 , 28 , 28 , 31 , 33

**Key:** 2 | 4 means 24

| Stem | Leaves |
|---|---|
| 1 | |
| 2 | |
| 3 | |



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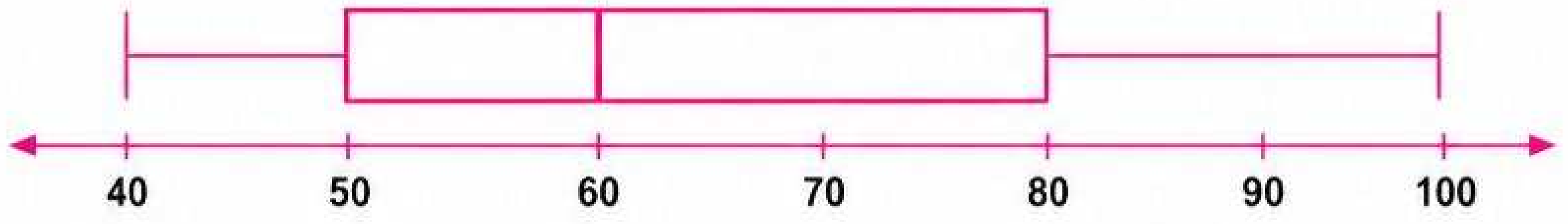
**9** The table below shows the number of goals scored by a football team in 15 matches.

| Number of goals | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Number of matches | 2 | 3 | 4 | 3 | 2 | 1 |

1. How many matches did the team play? ..................................
2. What is the mode of the data? ..................................
3. What is the median of the data? ..................................
4. How many matches did the team score 2 goals or more? ..................................

**10** The box plot below represents the test scores of a group of students.




1. What is the minimum score? ..................................
2. What is the first quartile? ..................................
3. What is the median score? ..................................
4. What is the third quartile? ..................................
5. What is the interquartile range? ..................................
6. What is the range? ..................................

**11** A survey was conducted to find the number of hours students spend studying per week. The results are shown in the following frequency table.

| Hours (h) | 1 – 3 | 4 – 6 | 7 – 9 | 10 – 12 | 13 – 15 |
|---|---|---|---|---|---|
| Number of students | 3 | 6 | 9 | 7 | 5 |

1. How many students were surveyed? ..................................
2. In which class interval is the median likely to lie? ..................................
3. How many students spend more than 6 hours studying? ..................................
4. What is the modal class interval? ..................................

**12** Use the data to answer the following questions.

12 , 14 , 14 , 16 , 18 , 18 , 20 , 22 , 22 , 24 , 26 , 26 , 28 , 30 , 30 , 32

1. What is the mode of the data? ..................................
2. What is the median of the data? ..................................
3. Find the lower quartile ($Q_1$) and the upper quartile ($Q_3$).
   $Q_1$ = .................................. , $Q_3$ = ..................................
4. Find the interquartile range. ..................................



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